EFFECT OF ELECTRON SCATTERING ON GIANT QUANTUM OSCILLATIONS OF SOUND AND ELECTROMAGNETIC WAVES IN METALS

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The necessary and sufficient conditions have been found for the existence of giant oscillations for finite electron path lengths. It is shown that in the limit of absolute zero temperature the oscillations are gigantic if the wavelength 1/k is large in comparison with the radius of the electron orbit R and small in comparison with the path length l. The amplitude and shape of the resonance lines are studied for finite temperature, when the energy of thermal motion is much smaller than the distance between Landau levels. In the low-frequency region the amplitude of the oscillations of the wave absorption is $k_z l$ times greater than the amplitude of oscillations of the Shubnikov-de Haas type. For high frequencies, the condition $kR \ll 1$ is practically sufficient for the existence of giant oscillations.

1. INTRODUCTION

IN the propagation of sound waves in metals at low temperatures in a strong magnetic field, the wave absorption experiences gigantic quantum oscillations.[1] For large values of the conduction electron mean free path, their interaction with the wave can be regarded as the direct absorption of quanta. From the laws of conservation of energy and the component of the momentum in the direction of the magnetic field **H** it follows that only those electrons can absorb the wave, for which

$$p_z = P \equiv \frac{m}{k_z}\left(\omega - \frac{\hbar k_z^2}{2m}\right), \tag{1.1}$$

where m is the effective mass of the electron, ω the frequency of the wave, k_z and p_z the projections of the wave vector **k** and the electron momentum on the axis z ∥ H.

On the other hand, the electrons that take part in the absorption of the wave, are those located near the Fermi surface. Quantization of the transverse energy of these electrons in a magnetic field causes their longitudinal momentum p_z to take on the discrete values

$$p_{zn} = [2m(\varepsilon_F - \hbar\Omega n)]^{1/2}, \tag{1.2}$$

where ϵ_F is the Fermi energy; $\Omega = eH/mc$ is the cyclotron frequency; the magnetic quantum number is n = 0, 1, 2, ...; c is the velocity of light.

When one of the p_{zn} is identical with P, the wave absorption coefficient has a resonance maximum. If none of the p_{zn} coincides with P, then the absorption is small. Therefore, the absorption undergoes strong oscillations upon change in the magnetic field. The effect has been observed experimentally in many metals (zinc [2], bismuth [3-6], gallium [7], antimony [8]) and is used at the present time for the study of the Fermi surface, for the measurement of the mean free path, the study of electron-phonon interactions, etc.

The character and shape of the resonance maxima depends materially on the temperature and the electron path length. The role of the thermal motion, which leads to a washing out of the steps of the Fermi distribution and to a corresponding broadening of the resonance maxima, is comparatively easy to take into account. Scattering of these electrons violates the law of conservation of energy, from which it follows that only electrons with $p_z = P$ can absorb the wave. This also smears out and smoothes the resonance peaks. The effect of electron scattering on the giant oscillations is very considerable even in the case of long free path lengths, inasmuch as the absorption maxima are very narrow. Therefore, the problem of the role of electron collisions under quantization conditions is very important. Unfortunately, there is no single point of view in the literature today relative to the necessary and sufficient conditions for the existence of giant oscillations of the wave absorption for a finite free path length. These divergences are associated in appreciable measure with the misunderstandings brought about by the researches of

Quinn and Rodriguez,[9] Quinn,[10] and Tosima, Quinn and Lampert,[11] Furthermore, there has been to date no theoretical investigation of the effect of scattering on the shape of the resonance peaks. In this connection, it is expedient to recall the idea of the derivation of the exact formula for the absorbed energy Q (or conductivity $\sigma_{\alpha\beta}$) with account of electron scattering.[12] With its help one can investigate the shape of the lines and establish exact criteria for the existence of giant oscillations for the case of finite path length.

2. DERIVATION OF THE EXACT FORMULA FOR ABSORPTION

Let us consider the case in which the electrons are scattered by randomly distributed fixed impurities. The Hamiltonian of the electron in an external magnetic field and in the field of these impurities is

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \sum_j V(\mathbf{r} - \mathbf{r}_j), \tag{2.1}$$

Where $\hat{\mathcal{H}}_0$ is the Hamiltonian in the absence of the impurities, $V(\mathbf{r} - \mathbf{r}_j)$ is the potential of the impurity located at the point $\mathbf{r}_j$.

For fixed locations of the impurity centers there exist strictly stationary states of the electron $|A\rangle$, which are eigenfunctions of the operator

$$\hat{\mathcal{H}}|A\rangle = \varepsilon_A|A\rangle. \tag{2.2}$$

The eigenfunctions $|A\rangle$ and the energy eigenvalues ϵ_A depend on the location of the impurity $\mathbf{r}_j$. Of course, we cannot find the explicit form of $|A\rangle$ and ϵ_A; for us, only the fact of the existence of strictly stationary states is important. As a consequence of this, the expression for the energy Q absorbed by the electrons can be represented in the form

$$Q = \frac{4\pi}{\hbar V_0}\Big\langle \sum_{A,B} (\varepsilon_B - \varepsilon_A)\,|\langle B|U|A\rangle|^2 \times \delta(\varepsilon_A - \varepsilon_B + \hbar\omega)[f(\varepsilon_A) - f(\varepsilon_B)]\Big\rangle, \tag{2.3}$$

where $\hat{U} = \exp(i\mathbf{k}\cdot\mathbf{r})\,\hat{U}_0$ is the operator which describes the absorption of the wave by the electron: the outer angle brackets denote averaging over the locations of the impurities $\mathbf{r}_j$; $f(\epsilon)$ is the Fermi function of argument $(\epsilon - \epsilon_F)/T$, T is the temperature in energy units; V_0 is the volume of the crystal. Equation (2.3) can be conveniently rewritten in another way, which clearly indicates the temperature dependence of the procedure of averaging over the $\mathbf{r}_j$:

$$Q = \int_0^\infty d\varepsilon \left[\frac{f(\varepsilon) - f(\varepsilon + \hbar\omega)}{\hbar\omega}\right] Q(\varepsilon), \tag{2.4}$$

where

$$Q(\varepsilon) = \frac{4\pi\hbar\omega^2}{V_0}\Big\langle \sum_{A,B} |\langle B|U|A\rangle|^2\,\delta(\varepsilon - \varepsilon_A)\,\delta(\varepsilon + \hbar\omega - \varepsilon_B)\Big\rangle \tag{2.5}$$

represents the energy of the wave absorbed by electrons with given energy ϵ. Thus the interaction with the impurities does not affect the exact law of energy conservation; the energy of the wave quantum $\hbar\omega$ enters in Eqs. (2.4) and (2.5).

We rewrite (2.5) in operator form:

$$Q(\varepsilon) = \frac{4\pi\hbar\omega^2}{V_0}\Big\langle \mathrm{Sp}\,\hat{U}\frac{1}{2\pi i}[(\varepsilon - \hat{\mathcal{H}} - i\eta)^{-1} - (\varepsilon - \hat{\mathcal{H}} + i\eta)^{-1}]\,\hat{U}^+\frac{1}{2\pi i}[(\varepsilon + \hbar\omega - \hat{\mathcal{H}} - i\eta)^{-1} - (\varepsilon + \hbar\omega - \hat{\mathcal{H}} + i\eta)^{-1}]\Big\rangle, \tag{2.6}$$

where $\eta \to +0$, the operator U^+ is the Hermitian conjugate of the operator U.

The trace of the product of operators, as is well known, can be computed in any representation. We transform in (2.6) to the representation of the Hamiltonian

$$\mathcal{H}_0 = \varepsilon\left(\frac{\hbar}{i}\frac{\partial}{\partial x}; \frac{\hbar}{i}\frac{\partial}{\partial y} + \frac{eHx}{c}; \frac{\hbar}{i}\frac{\partial}{\partial z}\right) \tag{2.7}$$

where $\epsilon(p_x, p_y, p_z)$ is the dispersion law for the conduction electron. We have chosen the vector potential $\mathbf{A}_0$ in the form

$$A_{0x} = A_{0z} = 0, \quad A_{0y} = Hx. \tag{2.8}$$

Inasmuch as $\hat{\mathcal{H}}_0$ does not contain the coordinates y and z explicitly, the dependence of its own wave functions $|a\rangle$ on these coordinates is described by plane waves:

$$|a\rangle = \Psi_{n p_z s_z}(x, X)\exp[i(p_z z/\hbar) - i\varkappa X y], \qquad \varkappa = eH/\hbar c. \tag{2.9}$$

Here a represents the set of quantum numbers n, p_z, s_z and X which characterize the eigenfunctions of the operator $\hat{\mathcal{H}}_0$; n is the magnetic quantum number, p_z and s_z are the projections of the momentum and the spin on the direction of the magnetic field, and X is the coordinate of the center of rotation; the explicit form of the function Ψ plays no significent role for us. The energy levels ϵ_α of the electron do not depend on X and have the form

$$\varepsilon_{n p_z s_z} = \varepsilon_n(p_z) + s_z g\mu_0 H, \quad \mu_0 = e\hbar/2m_0c, \quad s_z = \pm 1/2, \tag{2.10}$$

where m_0 is the mass of the free electron and g is the effective g factor.

In what follows, we shall be interested in the state of the electron with large values of the quantum number n and small values of p_z, for

which

$$\varepsilon_n(p_z) = \hbar\Omega n + p_z^2/2m_{\parallel}. \tag{2.11}$$

Here

$$\Omega = \frac{eH}{mc}, \quad m_\iota = \frac{1}{2\pi}\frac{\partial S(\varepsilon, p_z)}{\partial \varepsilon}\Big|_{\substack{\varepsilon=\varepsilon_F \\ p_z=0}},$$
$$\frac{1}{m_{\parallel}} = \frac{\partial^2 \varepsilon_n(p_z)}{\partial p_z^2}\Big|_{p_z=0} \tag{2.12}$$

$S(\epsilon, p_Z)$ is the area of the section of the constant-energy surface in the plane p_Z = const. The transverse quantized energy of the electron is determined by the cyclotron mass m, while the mass $m_{||}$ enters into the longitudinal kinetic energy.

In the representation of the operator $\hat{\mathcal{H}}_0$, the matrix elements $\langle b | U | a \rangle$ do not depend on the impurities. Therefore, for the calculation of $Q(\epsilon)$ it is necessary to find the two-particle Green's function averaged over the positions of the impurities r_j:

$$G^{aa'}_{b'b}(\varepsilon) = \left\langle \left\langle a \left| \frac{1}{\varepsilon - \hat{\mathcal{H}} - i\eta} \right| b' \right\rangle \right.$$
$$\left. \times \left\langle b \left| \frac{1}{\varepsilon + \hbar\omega - \hat{\mathcal{H}} + i\eta} \right| a' \right\rangle \right\rangle. \tag{2.13}$$

This function was studied in [12]. The simplest case was considered—the case of short-range acting impurities, the radius of action of which is small in comparison with the distances between them and the electron wavelength. It was found that the two-particle Green's function (2.13) reduces to the product of the mean one-particle Green's functions:

$$G^{aa'}_{b'b}(\varepsilon) = \frac{\delta_{ab'}}{\varepsilon - \varepsilon_a - {}^1/_2 i\hbar\nu(\varepsilon)} \frac{\delta_{ba'}}{\varepsilon + \hbar\omega - \varepsilon_b + {}^1/_2 i\hbar\nu(\varepsilon + \hbar\omega)}, \tag{2.14}$$

where $\nu(\epsilon)$ is the frequency of electron collisions with energy ϵ with the impurities. Equation (2.14) is valid for all ϵ with the exception of the small range of values

$$\Delta < 1/N,$$

where

$$\varepsilon = \Omega\hbar(N + \Delta), \quad N = [\varepsilon/\hbar\Omega], \tag{2.15}$$

N is an integer, while Δ is the fractional part of the ratio $\epsilon/\hbar\Omega$.

We note that the collision frequency $\nu(\epsilon)$ is an oscillating function of the magnetic field. However, for $\Delta > 1/N$, these changes of $\nu(\epsilon)$ cannot be taken into account, and we assume $\nu(\epsilon) \approx \nu$, where ν is the collision frequency of electrons with impurities in the absence of a magnetic field.

Using Eq. (2.14) for the two-particle Green's function, we can represent (2.6) in the form

$$Q(\varepsilon) = \frac{4\pi\hbar\omega^2}{V_0} \sum_{a,b} |\langle b | U | a \rangle|^2 D(\varepsilon - \varepsilon_a) D(\varepsilon + \hbar\omega - \varepsilon_b), \tag{2.16}$$

where

$$D(\varepsilon) = \frac{1}{2\pi} \frac{\hbar\nu}{\varepsilon^2 + (\hbar\nu/2)^2}. \tag{2.17}$$

Consequently, the interaction of the electrons with the impurities leads to the broadening of the electron levels. It indicates the effect on the probability of absorption of a quantum of the wave and on the width of the region of accessible values of the projection of the momentum of the electrons that interacts most effectively with the wave. However, scattering does not change the energy quantum $\hbar\omega$ absorbed by the electron and the expression (2.16) is proportional to $(\hbar\omega)^2$ and not to $(\epsilon_b - \epsilon_a)^2$ (cf.[10,11]). This circumstance plays an unusually important role in the determination of the amplitude and shape of the lines of the quantum oscillations in the case of a finite path length $l = v/\nu$.

The matrix element $\langle b | U | a \rangle$ contains the Kronecker delta symbol, which expresses the laws of conservation of the z component of the momentum, the projection of the spin and coordinate of the center of rotation:

$$\langle b | U | a \rangle = U_{nn'}(p_z, X)\, \delta_{p_z', p_z + \hbar k_z} \delta_{s_z', s_z} \delta_{\varkappa X', \varkappa X - k_y}. \tag{2.18}$$

We substitute (2.18) in (2.16) and carry out summation over p_Z', s_Z' and X' with account of the δ symbols. Furthermore, change from summation over p_Z and X to integration:

$$\sum_{p_z, X} \ldots \to \frac{\varkappa}{(2\pi)^2\hbar} \int dp_z \int \frac{dX}{L_x},$$

where L_X is the dimension of the crystal along the x axis. As a result, we obtain

$$Q(\varepsilon) = \frac{\varkappa\omega^2}{\pi} \sum_{n,n'=0}^{\infty} \sum_{s_z} \int_{-\infty}^{\infty} dp_z |U_{nn'}(p_z)|^2 D[\varepsilon - \varepsilon_{ns_z}(p_z)]$$
$$\times\ D[\varepsilon + \hbar\omega - \varepsilon_{n's_z}(p_z + \hbar k_z)]. \tag{2.19}$$

The giant oscillations take place under conditions of strong spatial dispersion and strong magnetic field, when the wavelength is small in comparison with the path length l and large in comparison with the characteristic dimension of the orbit of the electron in the magnetic field $R = v/\Omega$:

$$R \ll 1/k_z \ll l. \tag{2.20}$$

The integrand in (2.19) has a sharp maximum when the arguments of the D functions are identical, i.e., for

$$\Omega(n'-n)=\omega-\frac{\hbar k_z^2}{2m_\parallel}-\frac{k_z p_z}{m_\parallel}. \qquad (2.21)$$

The frequency on the right hand side of (2.21) is of the order of or smaller than $k_z v$. Therefore, in the case $kR \ll 1$, condition (2.21) is satisfied only when

$$n'=n, \quad p_z=P. \qquad (2.22)$$

The velocity of the wave is usually much less than the Fermi velocity of the electron v, and the condition $p_z = P$ corresponds to small values of p_z. Consequently, the principal contribution is made by components with $n' = n \approx N$. Under the conditions of quasiclassical quantization $N \gg 1$, the matrix element $U_{nn}(p_z)$ is a smooth function of n and p_z and it can be taken from under the integral over p_z and summed over n:

$$Q(\varepsilon)=\frac{\varkappa\omega^2}{\pi}|U_{NN}(P)|^2\sum_{n=0}^{\infty}\sum_{s_z}\int_{-\infty}^{\infty}dp_z\cdot D(\varepsilon-\varepsilon_{ns_z}(p_z))\times D(\varepsilon+\hbar\omega-\varepsilon_{ns_z}(p_z+\hbar k_z)). \qquad (2.23)$$

It is convenient to rewrite this formula somewhat differently, expressing the quantity $Q(\epsilon)$ in terms of its limiting classical value Q_0. In the classical case, $\hbar\Omega \to 0$, the sum over n transforms into an integral, while the D functions can be replaced by δ functions. This corresponds to the fact that for $k_z l \gg 1$ the absorption has a non-collision character and is determined by electrons in the gas moving with the wave, for which the right side of (2.21) vanishes (Landau damping). As a result, we have[1)]

$$Q_0=\frac{\omega^2 m m_\parallel}{\pi\hbar^3|k_z|}|U_{NN}(P)|^2. \qquad (2.24)$$

In the quasiclassical approximation the diagonal matrix element $U_{NN}(P)$ is equal to the mean value of the classical quantity $U(\epsilon, p_z, \varphi)$:

$$U_{NN}(P)=\frac{1}{2\pi}\int_0^{2\pi}d\varphi U(\varepsilon_F,P,\varphi)\equiv\overline{U}(\varepsilon_F,P). \qquad (2.25)$$

Here φ is the canonical conjugate of the energy of the angular variable, which characterizes the absorption of the electron over its orbit in the magnetic field. Expressing $|U_{NN}^2(P)|$ in terms of Q_0 and substituting (2.23) in (2.4), we finally obtain

$$\frac{Q}{Q_0}=\frac{\hbar\Omega k_z}{2m_\parallel\omega}\sum_{s_z}\sum_{n=0}^{\infty}\int_0^{\infty}d\varepsilon[f(\varepsilon)-f(\varepsilon+\hbar\omega)]\times\int_{-\infty}^{\infty}dp_z D[\varepsilon-\varepsilon_{ns_z}(p_z)]D[\varepsilon+\hbar\omega-\varepsilon_{ns_z}(p_z+\hbar k_z)]. \qquad (2.26)$$

It is obvious that the giant oscillations take place when the matrix element $U_{NN}(P)$ differs from zero. If the vectors k and H are not parallel, then $U_{NN} \neq 0$ for both electromagnetic and sound waves. The absorption of the sound waves by the electrons in the case (2.20) is determined by the deformation interaction of the electrons with the lattice:

$$U_{NN}=\tfrac{1}{2}\overline{\Lambda}_{ik}u_{ik}^{(0)}, \qquad (2.27)$$

where $u_{ik}^{(0)}$ is the amplitude value of the deformation tensor:

$$u_{ik}=(\partial u_i/\partial x_k+\partial u_k/\partial x_i)/2,$$

u is the displacement vector in the sound wave; $\Lambda_{ik}(\epsilon, p_z, \varphi)$ are the components of the deformation potential; summation is indicated by repetition of the vector indices (i, k = x, y, z).

The damping of the helical electromagnetic wave in an isotropic metal is determined by the conductivity $\sigma_{xx}(\omega, k)$:[13]

$$Q_0=\tfrac{1}{2}\operatorname{Re}\sigma_{xx}^{(0)}|E_x^2|, \qquad (2.28)$$

the x axis is directed transverse to the vectors k and H; the angle Φ between them is different from zero; E_x is the projection of the electric field of the wave on the x axis, and the limiting classical value of $\sigma_{xx}^{(0)}$ has the form [13]

$$\sigma_{xx}^{(0)}=\frac{3\pi}{8}\frac{n_0 ec}{H}k_z R\operatorname{tg}^2\Phi, \qquad (2.29)$$

n_0 is the concentration of electrons. The quantum oscillations of the damping of the helical wave were investigated in [14].

Giant oscillations are absent under those special circumstances in which the collision-free Landau damping disappears ($Q_0 = 0$). Thus, if the vectors k and H are parallel to an axis of symmetry of higher order, then the damping of the transverse waves in the case $kR < 1$ is due to electron scattering.

3. SHAPE OF THE ABSORPTION LINES AT ABSOLUTE ZERO TEMPERATURE

Let us consider the effect of electron scattering on the character of the absorption at absolute zero temperature. For simplicity, we shall neglect the spin splitting of the energy levels. The ratio ω/ν can be either larger or small in

[1)] We note that if the Eqs. (2.16) and (2.23) were proportional to $[\epsilon_n(p_z+\hbar k_z)-\epsilon_n(p_z)]^2$, as Quinn maintained, [10] then all the electrons would have made an important contribution to the absorption, and the expression for Q would have contained an additional component of the order of $Q_0[k_z v\nu/(\omega^2+\nu^2)]$, which is actually not present.

comparison with unity. We shall study both limiting cases.

1. Low frequencies ($\omega \ll \nu$). In this range of frequencies the quantity $\omega - \hbar k_z^2/2m_{||}$ in the argument of the second D function in (2.26) can be neglected in comparison with ν. The difference of the Fermi functions is equal to unity in the range

$$\varepsilon_F - \hbar\omega < \varepsilon < \varepsilon_F \tag{3.1}$$

and vanishes for other values of ϵ. Inasmuch as the width of the interval of integration over ϵ is much less than the characteristic width of the D functions, integration over ϵ in (2.26) reduces to multiplication by $\hbar\omega$ and replacement of ϵ by ϵ_F in the arguments of the D functions:

$$\frac{Q}{Q_0} = \frac{\hbar^2\Omega k_z}{m_{||}} \sum_{n=0}^{N} \int_{-\infty}^{\infty} dp_z D[\varepsilon_F - \varepsilon_n(p_z)] \times D\left[\varepsilon_F - \varepsilon_n(p_z) - \frac{\hbar k_z p_z}{m_{||}}\right]. \tag{3.2}$$

Completing the elementary integration over p_z, we get

$$\frac{Q}{Q_0} = \frac{\hbar\Omega}{\pi}\left(\frac{2k_z^2}{m}\right)^{1/2} \sum_{n=0}^{N} \frac{\nu}{\sqrt{\nu^2 + 2k_z^2 m_{||}^{-1}(\varepsilon_F - \hbar\Omega n)}} \times \mathrm{Re}\left(\varepsilon_F - \hbar\Omega n + \frac{i}{2}\hbar\nu\right)^{-1/2} \tag{3.3}$$

For those values of the magnetic field for which $\epsilon_F = N\hbar\Omega$, the component with $n = N$ has a sharp maximum. We separate this basic component and replace the sum of the remaining components approximately by an integral. Then

$$\frac{Q}{Q_0} = 1 - \frac{2}{\pi}\operatorname{arctg} A + \frac{1}{\pi}\frac{A}{1 + A^2\Delta}\mathrm{Re}\left(\Delta + \frac{i\nu}{2\Omega}\right)^{-1/2}, \tag{3.4}$$

where

$$\Delta = \frac{\varepsilon_F}{\hbar\Omega} - \left[\frac{\varepsilon_F}{\hbar\Omega}\right], \quad A = \frac{k_z l_{||}}{N^{1/2}}, \quad l_{||} = \left(\frac{2\varepsilon_F}{m_{||}\nu^2}\right)^{1/2}. \tag{3.5}$$

We shall consider the question of the amplitude and shape of the lines of the quantum oscillations Q. As has been noted in the previous section, Eq. (2.14) for the two-particle Green's function is valid for $\Delta > 1/N$. Therefore, the maximum value of the last term in (3.4) has a value of the order

$$AN^{1/2}(1 + N\nu/2\Omega)^{-1/2}, \tag{3.6}$$

while the difference of the first two is less than unity. Consequently, the necessary and sufficient condition that the quantum oscillations of the absorption be gigantic is that

$$k_z l_{||} \gg \left(1 + \frac{\nu}{2\Omega}\frac{\varepsilon_F}{\hbar\Omega}\right)^{1/2}. \tag{3.7}$$

In the case of not too strong a magnetic field, when the quantity $\alpha = (\hbar\Omega^2/\nu\epsilon_F)^{1/2}$ is small in comparison with unity, the relative amplitude of the oscillations is of the order $\alpha k_z l_{||}$. The parameter α represents the relative amplitude of the static quantum oscillations of the density of states at $T = 0$.[15] Consequently, the oscillations of the absorption of the variable field is seen to be $k_z l$ times greater than the static. They are gigantic if

$$k_z l_{||}\hbar\Omega/(\hbar\nu\varepsilon_F)^{1/2} \gg 1. \tag{3.8}$$

In the region of strong magnetic fields ($\alpha^2 \gg 1$), condition (3.7) reduces to the inequality

$$AN^{1/2} \equiv k_z l_{||} \gg 1, \tag{3.9}$$

which is identical with the condition for strong spatial inhomogeneity of the variable field over the path length (2.20).

The shape of the line depends materially on the value of A^2. If

$$A^2 \gg 1, \tag{3.10}$$

then $Q_{min} \sim Q_0/A$, i.e., it is much less than the limiting classical value of Q_0. The shape of the resonance maximum for $\Delta > \nu/\Omega$ is described by the function

$$\Delta^{-1/2}(1 + A^2\Delta)^{-1}. \tag{3.11}$$

In the case (3.10), the characteristic width of the maximum $\Delta \sim A^{-2}$, while for $A^2 \ll 1$, it is of the order of A^2.

Thus, for absolute zero temperature and low frequencies $\omega \ll \nu$, the condition (3.10) is not necessary. The necessary and sufficient condition for the existence of giant quantum oscillations of the absorption is seen to be condition (3.7).

2. High frequencies ($\omega \gg \nu$). In this case, the range of integration over ϵ in (2.26) is much greater than the characteristic width of the D function. Integration over p_z and ϵ leads to a rather complicated and very cumbersome expression which we shall not write out. The result of the exact calculation is seen to be approximately the same as when one D function is replaced by a δ function and the width of the other D function is doubled. We make such a substitution in (2.26) and integrate over ϵ with account of the δ function. Then

$$\frac{Q}{Q_0} = \frac{\Omega}{\omega}\frac{k_z}{m_{||}}\int_{-\infty}^{\infty} dp_z \sum_{n=0}^{\infty} \frac{\nu/\pi}{\nu^2 + (\omega - \omega_{||} - k_z v_z)^2}[f(\varepsilon_n(p_z)) - f(\varepsilon_n(p_z) + \hbar\omega)], \tag{3.12}$$

where

$$\omega_{||} = \hbar k_z^2/2m_{||}, \quad v_z = p_z/m_{||}. \tag{3.13}$$

Carrying out the integration over p_z, we obtain

$$\frac{Q}{Q_0}=\frac{\Omega}{\pi\omega}\left\{\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta}-\omega+\omega^{\parallel}}{\nu}\right)+\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta}+\omega-\omega_{\parallel}}{\nu}\right)-\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta-\omega/\Omega}-\omega+\omega_{\parallel}}{\nu}\right)-\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta-\omega/\Omega}+\omega-\omega_{\parallel}}{\nu}\right)\right\}+\frac{2}{\pi A}. \tag{3.14}$$

Here

$$v_1=(2\hbar\Omega/m_{\parallel})^{1/2}. \tag{3.15}$$

This formula is valid for $\Delta > \omega/\Omega$. If the difference $\Delta - \omega/\Omega$ is negative, then the square root $(\Delta-\omega/\Omega)^{1/2}$ must be set equal to zero.

Analysis of Eq. (3.14) shows that the amplitude and shape of the resonance maxima depends appreciably on the ratio $\nu/\omega_{\parallel}$, which can be either greater or less than unity.

A. In the region of not too high frequencies,

$$\omega_{\parallel}\ll\nu \tag{3.16}$$

the absorption maxima correspond to values of the magnetic field for which

$$\Delta=\Delta_0,\quad \Delta_0=\omega^2/2\omega_{\parallel}\Omega\equiv(\omega/\nu A)^2. \tag{3.17}$$

It is obvious in this case that the quantity Δ_0 is much greater than ω/Ω[2]. Therefore, the differences in the arctangents in (3.14) can be expanded in powers of $\omega/\Omega\Delta$, in which we limit ourselves to the linear term of the expansion. This gives

$$\frac{Q}{Q_0}=\frac{2}{\pi A}+\frac{A}{2\pi\sqrt{\Delta}}\{[1+A^2(\sqrt{\Delta}-\sqrt{\Delta_0})^2]^{-1}+[1+A^2(\sqrt{\Delta}+\sqrt{\Delta_0})^2]^{-1}\}. \tag{3.18}$$

The first term in (3.18) represents the integrated contribution of all the nonresonant electrons with $n < N$. This part of the absorption, which depends smoothly on the magnetic field, is small in comparison with unity by virtue of the conditions $\Delta_0 < 1$ and $\nu < \omega$ (see (3.17)). The second component describes the contribution of electrons with $n = N$, which lie close to the extremal cross section of the Fermi surface. The height of the maximum

$$\frac{Q_{max}}{Q_0}=\frac{A^2\nu}{2\pi\omega}\equiv\frac{\omega}{2\pi\nu\Delta_0}\gg 1, \tag{3.19}$$

and its relative width $|\Delta-\Delta_0|$ is of the order Q_0/Q_{max}. The absorption at the minimum for $\Delta\approx 1/2$ is the same in order of magnitude as the first term in (3.18).

As is seen from Eq. (3.18), Q has an additional maximum, located at $\Delta\sim\omega/\Omega$. It is due to the singularity in the density of electron states in the magnetic field (oscillations of the Shubnikov-de Haas type). The nature of these oscillations is different from the nature of the giant quantum oscillations, and is not connected with the resonance character of the interaction of the electrons with the wave. The height of this second maximum is

$$Q_2\approx\frac{A}{\pi}\left(\frac{\nu}{\omega}\right)^2\left(\frac{\Omega}{\omega}\right)^{1/2}Q_0, \tag{3.20}$$

and its width is much less than the width of the maximum of the giant oscillations. In contrast with the low-frequency case $\omega \ll \nu$, when the two maxima are superposed on one another, these maxima here should be clearly separated (see Fig. 1).

B. In the region of frequencies

$$\nu\ll\omega_{\parallel}\ll\omega \tag{3.21}$$

it is impossible to neglect the frequency $\omega_{\parallel}$. The location of the center of the resonance line is determined by the same Eq. (3.17) as in case A. The amplitude of the oscillations reaches its limiting value

$$Q_{max}=\frac{\Omega}{\omega}Q_0, \tag{3.22}$$

which is characteristic for giant oscillations in the absence of scattering.

Close to the center of the maximum, where $|k_z v_1\sqrt{\Delta}-\omega|<\omega_{\parallel}$, the shape of the line is described by the equation

$$\frac{Q}{Q_0}=\frac{\Omega}{\pi\omega}\left\{\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta}-|\omega-\omega_{\parallel}|}{\nu}\right)-\operatorname{arctg}\left(\frac{k_z v_1\sqrt{\Delta}-\omega-\omega_{\parallel}}{\nu}\right)\right\}. \tag{3.23}$$

In this case the absorption is almost rectangular with a plane vertex with slightly rounded corners

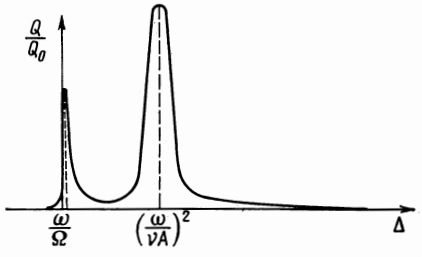

FIG. 1.

[2]We note that for $kR \ll 1$, the value of Δ_0 is small in comparison with unity everywhere with the exception of a small region of angles Φ close to $\pi/2$.

and almost vertical sides. The relative width of the maximum is equal to ω/Ω. The absorption on the wings of the line for $|k_z v_1\sqrt{\Delta} - \omega| > \omega_{||}$ is given by the formula

$$\frac{Q}{Q_0} = \frac{2}{\pi}\left(\frac{\omega_{||}}{\Omega}\right)^{1/2} \frac{\nu\Omega(k_z^2 v_1^2\Delta + \omega^2)}{\Delta(k_z^2 v_1^2\Delta - \omega^2)^2}. \quad (3.24)$$

We recall that in the absence of scattering the quantum maxima are strictly rectangular.[14]

So far as the second maximum for $\Delta \sim \omega/\Omega$ is concerned, its amplitude is determined as previously by Eq. (3.20) and turns out to be much less than (3.22).

C. We can investigate the line shape in the region

$$\omega_{||} \gg \omega \quad (3.25)$$

in similar fashion (for a sound wave, this region corresponds to much higher frequencies). The center of the maximum is located at the point

$$\Delta_0 = \omega_{||}/4\Omega, \quad (3.26)$$

and its shape is described by Eq. (3.23) as before.

Thus, for the existence of giant oscillations of the absorption at absolute zero temperature, satisfaction of the conditions of a strong magnetic field and a strong spatial dispersion (2.20) is sufficient. In the case of (3.16), the scattering of the electrons strongly spreads out the quantum maxima and has a material effect on their amplitude and shape. In the region of much higher frequencies $\omega_{||} > \nu$, the role of scattering is reduced only to the smoothing out of the angles in the rectangular maxima and to the appearance of a small absorption in the regions in between them.

Let us make clear the physical meaning of the inequality $\omega_{||} > \nu$. In the absence of electron scattering, the conservation laws for the absorption of a quantum of the wave can be written in the form

$$\omega_{||} = \omega' \equiv \omega - k_z v_z. \quad (3.27)$$

On the right-hand side of this equation, there is the frequency of the wave in a set of coordinates moving with the electron with velocity v_z. On the left side is the frequency corresponding to the change in the longitudinal energy of the electron in this system. The scattering violates the conservation law (3.27). For finite path length, those electrons absorb the wave most effectively whose longitudinal velocity satisfies the condition $|\omega' - \omega_{||}| \sim \nu$. If the longitudinal energy of the electron $\hbar\omega^{||}$ is small in comparison with the width of the level $\hbar\nu$, then the scattering essentially destroys the law of conservation of energy (3.27). The scatter of frequencies $\delta\omega'$ effectively absorbed by the electrons is seen to be greater than the frequency $\omega_{||}$. It is natural that in this case the electron collisions strongly affect the amplitude and shape of the resonance maxima. In the opposite case $\omega_{||} \gg \nu$ the violation of the law of energy conservation is insignificant, and the scattering weakly affects the shape of the absorption line.

4. GIANT OSCILLATIONS AT FINITE TEMPERATURE

The limiting case of zero temperature considered above requires the satisfaction of the inequality $T \ll \hbar(\omega + \nu)$. For example, if $\nu < \omega$, then at $T = 0.1°K$ the frequency of the field ω should be much greater than 10^{10} sec^{-1}. Along with the difficulties of obtaining such high frequencies and low temperatures, it is necessary to use extraordinarily large magnetic fields $H \gg 10^5$ Oe (in order to satisfy the condition $kR \ll 1$). Therefore, the opposite limiting case is the most realistic one at the present time:

$$\hbar(\omega + \nu) \ll T \ll \hbar\Omega, \quad (4.1)$$

when the energy of thermal motion of the electrons is much greater than the energy of the quanta of the variable field or the width of the energy levels, but is less than the separation between the Landau levels. For $\hbar\Omega \ll T$, the difference in the Fermi functions in Eq. (3.12) can be expanded in powers of $\hbar\omega$, limiting outselves to the first nonvanishing term:

$$\frac{Q}{Q_0} = \frac{\hbar\Omega}{4T}\frac{k_z}{m_{||}}\int_{-\infty}^{\infty} dp_z \frac{\nu/\pi}{\nu^2 + (\omega - \omega_{||} - k_z v_z)^2} \times \sum_{n=0}^{\infty} \mathrm{ch}^{-2}\left(\frac{\varepsilon_n(p_z) - \varepsilon_F}{2T}\right). \quad (4.2)$$

As in the previous section, we consider the limiting cases of low and high frequency separately.

1. Low frequencies ($\omega \ll \nu$). In this region of frequencies the quantity $\omega - \omega_{||}$ can be neglected. The integrand in (4.2) contains the product of two rapidly changing functions:

$$D_0(p) = \frac{\nu/\pi}{\nu^2 + (k_z p/m_{||})^2},$$

$$F(p) = \sum_{n=0}^{\infty} \mathrm{ch}^{-2}\left[\frac{p^2 - 2m_{||}(\varepsilon_F - n\hbar\Omega)}{4m_{||}T}\right]. \quad (4.3)$$

Graphs of these functions for $p > 0$ are shown in Fig. 2. The maximum of the function $D_0(p)$ is

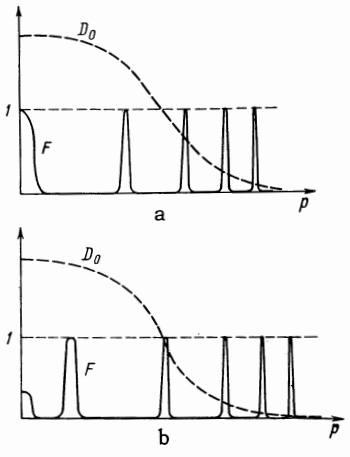

FIG. 2.

located at $p = 0$ and has the width

$$\delta p = m_{\parallel} v / k_z \approx p_F / k_z l_{\parallel}. \tag{4.4}$$

Far away from the maximum, the function $D_0(p)$ falls off according to the power law p^{-2}.

The function $F(p)$ is a set of peaks of unit height. The width of the peaks and the distance between them depend on their position (the continuous curves in the drawing). In their turn, the position of the maxima is determined by the value of the magnetic field. Figure 2a shows the function $F(p)$ for such a value of H that the Landau level with $p_z = 0$ and $n = N$ is identical with the Fermi energy. The central maximum of the function F corresponding to this level has the width $\Delta P = p_1 (T/\hbar\Omega)^{1/2}$, where $p_1 = (2m_{\parallel}\hbar\Omega)^{1/2}$. The next peak is located at a distance p_1 from the central peak and has the width of order $p_1 T/\hbar\Omega$. The distance of the s-th peak from the central peak is equal to $p_1 s^{1/2}$, and its width is $p_1 T/\hbar\Omega s^{1/2}$. Thus for $\hbar\Omega \gg T$, the central maximum of the function $F(p)$ is very broad, while the distances between the maxima are much greater than their width.

Upon decrease in the magnetic field, the central maximum of the function F is shifted in the direction of larger p, and its width is decreased. At the origin there is a new maximum with an exponentially small amplitude (Fig. 2b). Upon further decrease in H, this peak rapidly increases and its amplitude becomes equal to unity when the corresponding Landau level with $p_z = 0$ falls on the Fermi surface. The decrease of the magnetic field shows the greatest effect on the location and width of the central, very broad maximum of the function F. The quantum oscillations of the absorption are gigantic in the case in which the contribution of this maximum to the integral (4.2) is greater than the total contribution of all the remaining peaks of the function F.

We estimate the value of this total contribution, assuming that the width δp subtends several peaks of the function F ($A < 1$). It follows from the estimates given above that the number of such peaks is

$$M \approx (\delta p / p_1)^2 \approx A^{-2}. \tag{4.5}$$

Their total width is of the order

$$\sum_{s=1}^{M} \frac{p_1 T}{\hbar\Omega} s^{-1/2} \sim p_1 \frac{T}{\hbar\Omega} \frac{1}{A}. \tag{4.6}$$

The contribution of these peaks to the absorption Q will be small in comparison with the contribution of the central maximum of the function F if its width $p_1 (T/\hbar\Omega)^{1/2}$ is greater than the value of (4.6), i.e., for

$$A (\hbar\Omega / T)^{1/2} \gg 1 \quad \text{or} \quad k_z l_{\parallel} \hbar\Omega / (T\varepsilon_F)^{1/2} \gg 1. \tag{4.7}$$

The parameter $\hbar\Omega/(T\epsilon_F)^{1/2}$ represents the relative amplitude of oscillations of the Shubnikov-de Haas type for $T \gg \hbar\nu$. It follows from (4.7) that the amplitude of the oscillations of the wave absorption is $k_z l_{\parallel}$ times greater than the amplitude of static oscillations—the same as at absolute zero temperature (cf. (3.8)).

The inequality (4.7) is the necessary and sufficient condition for the existence of giant oscillations at finite temperature and finite path length in the region of low frequencies. A more stringent condition $A^2 \gg 1$, which is sufficient but not necessary, was cited in [1,12]. Criteria of the sort obtained in the works of Quinn [9,10] are incorrect.

Analysis of Eq. (4.2) confirms these qualitative considerations. In the calculation of Q in (4.2), it is convenient to isolate the specific terms with $n = N$ and $n = N + 1$, and to replace the rest of the sum by an integral. This gives

$$Q/Q_0 = q + q', \tag{4.8}$$

where

$$q = \frac{A}{2\pi}\left(\frac{\hbar\Omega}{T}\right)^{1/2} \int_0^{\infty} \frac{dx}{1 + B^2 x^2} \operatorname{ch}^{-2}\left[\frac{x^2}{2} - \frac{\hbar\Omega}{2T}\zeta(\Delta)\right], \tag{4.9}$$

$$\zeta(\Delta) = \begin{cases} \Delta & \text{for } \Delta \ll 1 \\ \Delta - 1 & \text{for } 1 - \Delta \ll 1 \end{cases}$$

is the resonance part of the absorption, and

$$q' = 1 - \frac{2}{\pi} \operatorname{arctg} A \tag{4.10}$$

is the nonresonance part, which is substantial only at great distances from the maximum of the giant oscillations. The quantity

$$B^2 = A^2 T/\hbar\Omega = (k_z l_{||})^2 T/\varepsilon_F \qquad (4.11)$$

characterizes the ratio of the width of the central maximum of the F function to the width of the function D_0. It is equal to the ratio of the thermal energy T to the scatter of energy of the resonance electrons as the result of their scattering. Actually, the scatter of the projection of the momentum $|p_z| \sim m_{||}\nu/k_z$ (see (4.3)) and the corresponding scatter of the longitudinal energy is $p_z^2/2m_{||} \approx \epsilon_F/(k_z l_{||})^2$.

Equation (4.9) describes the shape of the resonance line in the low frequency case considered. In the case $B^2 \ll 1$, the maximum value is

$$q_{max} \approx \frac{2A}{3\pi}\left(\frac{\hbar\Omega}{T}\right)^{1/2}. \qquad (4.12)$$

In the opposite limiting case $B^2 \gg 1$ in the calculation of the resonance part of the absorption q, the function D_0 can be replaced by delta functions. Here, $A^2 \gg 1$ and Eqs. (4.9) and (4.10) yield

$$\frac{Q}{Q_0} = \frac{\hbar\Omega}{4T}\left\{\mathrm{ch}^{-2}\left(\frac{\hbar\Omega\Delta}{2T}\right) + \mathrm{ch}^{-2}\left(\frac{\hbar\Omega(1-\Delta)}{2T}\right)\right\} + \frac{2}{\pi A}. \qquad (4.13)$$

The first two terms on the right side of Eq. (4.13) are due to electrons close to the extremal cross section of the Fermi surface. This part of the absorption does not depend on the free path length of the electrons. The latter component in (4.13) is inversely proportional to $l_{||}$ and depends smoothly on H.

Figure 3 shows the curves $Q(H)/Q_0$ which characterize the effect of scattering on the shape of the resonance maxima. Curve 1 is the absorption curve in the limit of infinitely long path ($A \to \infty$). Curve 2 corresponds to the value $A^2 = 7$, while curve 3 represents $A^2 = 1/7$. The ratio $\hbar\Omega/T$ for all three curves is equal to 50. The asymmetry of curves 2 and 3 relative to the point $\zeta = 0$ is due to the fact that, for finite path length, the singularity in the density of electron states is superimposed on the giant oscillations as $\Delta \to +0$.

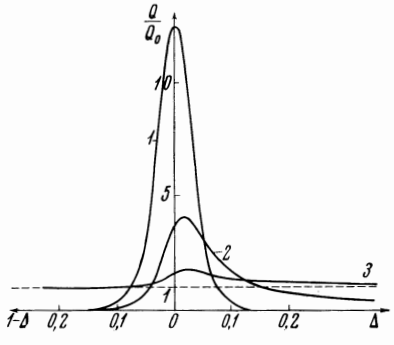

FIG. 3.

2. High frequencies ($\omega \gg \nu$). We limit ourselves to the consideration of not very high frequencies, when $\omega_{||} \ll \omega$. In Sec. 3A it was noted that the value of $\Delta_0 = (\omega/\nu_A)^2$ is much less than unity. Therefore, the contribution to the absorption from the nonresonance electrons with $n < N$ is determined as before by Eq. (4.10) for large values of A.

The expression for the resonance part of the absorption ($n = N$) can be rewritten in the form

$$q = \frac{A}{4\pi}\left(\frac{\hbar\Omega}{T}\right)^{1/2}\int_{-\infty}^{\infty}\frac{dx}{1+(Bx-\omega/\nu)^2}\,\mathrm{ch}^{-2}\left[\frac{1}{2}\left(x^2 - \frac{\hbar\Omega\Delta}{T}\right)\right]. \qquad (4.14)$$

The small region of values of x near $x_0 = (\hbar\Omega\Delta/T)^{1/2}$ plays a special role in this integral. We expand the argument of the hyperbolic cosine in (4.14) in powers of $x - x_0$ and limit ourselves to the linear term in the expansion. Then the expression for q, which describes the shape of the resonance line, takes the form

$$q = \frac{\hbar\Omega}{4\pi T}\int_{-\infty}^{\infty}\frac{dz\,\mathrm{ch}^{-2}(z\hbar\Omega\Delta^{1/2}/AT)}{1+[z+A(\Delta^{1/2}-\Delta_0^{1/2})]^2} \qquad (4.15)$$

This function has a maximum at $\Delta = \Delta_0$. The corresponding value of q depends on the value of the parameter $B^2\nu/\omega$. We write out the interpolation formula for q_{max}:

$$q_{max} = \left(\frac{2\pi\omega}{\nu A^2} + \frac{4T}{\hbar\Omega}\right)^{-1}. \qquad (4.16)$$

It follows from (4.16) and the inequality $A > \omega/\nu$ that q_{max} is always large in comparison with unity. In other words, the conditions

$$\omega \gg \nu, \quad kR \ll 1, \quad T \ll \hbar\Omega \qquad (4.17)$$

are sufficient for the existence of giant oscillations.

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