FLUCTUATIONS AND SCATTERING OF ELECTROMAGNETIC WAVES IN ANTIFERROMAGNETS

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Fluctuations and scattering of electromagnetic waves in ferromagnets are studied. Expressions are obtained for the correlators of the fluctuations of quantities that describe the antiferromagnet (the magnetic moment, the magnetic field, the density, the displacement vector), both far away from magnetoacoustic resonance and close to resonance.

It is shown that the correlation functions have sharp maxima near the frequencies of characteristic oscillations of the crystal—spin and acoustic waves. Far from magnetoacoustic resonance, there are on spin waves large fluctuations of magnetic quantities (the magnetic field, the magnetic moment), and on acoustic waves large fluctuations of nonmagnetic quantities (the density, the displacement vector); near magnetoacoustic resonance, there are large fluctuations both of magnetic and of nonmagnetic quantities. Scattering of electromagnetic waves in antiferromagnets is studied, with allowance for the coupling between elastic waves and oscillations of the magnetic moment. It is shown that far from magnetoacoustic resonance, there appear in the spectrum of the scattered radiation four pairs of lines: longitudinal and transverse sound satellites, caused by scattering of the electromagnetic wave on elastic oscillations, and two pairs of magnon satellites, caused by scattering of the electromagnetic wave on the two branches of the spin waves. An appreciable contribution to the intensity of the magnon satellites comes from interaction of the electromagnetic wave with the fluctuations of elastic quantities that accompany the spin wave. It is shown that in the vicinity of magnetoacoustic resonance, there appears in the spectrum of the scattered radiation, instead of the pair of weak lines, an additional pair of bright lines; but the total intensity of the scattered radiation does not change.

INTRODUCTION

In this paper, a study is made of combinational scattering of electromagnetic waves in antiferromagnetic crystals. In such crystals, along with sound waves, still another type of weakly attenuated oscillations is possible—spin waves (magnons). Hence in the spectrum of radiation scattered in an antiferromagnet, along with sound satellites, there occur also magnon satellites, distant from the fundamental line by the spin-wave frequency.[1)]

In studying the scattering of electromagnetic waves in magnetically ordered crystals, it is necessary to take account of the coupling between elastic waves and oscillations of the magnetic moment, by virtue of which a spin wave is always accompanied by oscillations of the elastic quantities and, conversely, a sound wave is accompanied by oscillations of the magnetic moment.[2)] This is due primarily to the fact that two processes contribute to the scattering cross section of electromagnetic waves in magnetically ordered crystals: interaction of these waves with oscillations of the deformation tensor and interaction of the electromagnetic waves with oscillations of the magnetic moment; and an electromagnetic wave interacts more strongly with elastic oscillations than with oscillations of the magnetic moment. Consequently, as is shown in the present paper, the intensity of the magnon satellites is in many cases determined by

[1)]A qualitative treatment of combinational scattering of light in magnetically ordered crystals was given by Shen and Bloembergen [1].

[2)]Coupled magnetoelastic waves and magnetoacoustic resonance in antiferromagnets were treated by Peletminskiĭ [2] and by Savchenko [3].

the scattering of the electromagnetic waves on the elastic oscillations that accompany the spin wave (and not on the oscillations of magnetic moment themselves).

The coupling between elastic waves and oscillations of the magnetic moment manifests itself particularly strongly in the vicinity of magnetoacoustic resonance. Thanks to this, as we show, the character of the distribution of scattered radiation changes appreciably on approach to the resonance point. Specifically, instead of a pair of bright lines (sound satellites) and a pair of weak lines (magnon satellites), there appear in the spectrum of the scattered radiation two (or three) pairs of bright lines, caused by scattering of the electromagnetic wave on coupled magnetoelastic oscillations. What changes, however, is the distribution of the scattered radiation, not its total intensity.

The intensity of scattering of electromagnetic waves in crystals is determined, as is known, by the level of fluctuations in them. In this paper, therefore, along with the scattering of electromagnetic waves, we study the fluctuations of the quantities that describe the antiferromagnet, with allowance for the coupling between elastic waves and oscillations of the magnetic moment.

1. DETERMINATION OF THE CORRELATION FUNCTIONS

The scattering cross section of electromagnetic waves in magnetically ordered crystals is expressed, as is known (see [4]), in terms of the correlators of the fluctuations of the permittivity of the crystal and of the magnetic-moment density. Therefore we shall determine, first of all, the correlators of the fluctuations of the quantities that describe the antiferromagnet. In accordance with the general method of fluctuation theory, based on the fluctuation-dissipation theorem, it is necessary for this purpose to introduce into the equations that describe the system under consideration additional auxiliary quantities—the so-called "random forces" (Landau and Lifshitz[5, 6]). On introducing random forces $\mathbf{w}$ and $\mathbf{y}$ into the equations of motion of the magnetic moments of the sublattices and into the equation of elasticity, we get

$$\partial\boldsymbol{\mu}_\nu / \partial t = g[\boldsymbol{\mu}_\nu \mathbf{H}^e] - (\rho_0\mu_0^2\tau_m)^{-1}[\boldsymbol{\mu}_\nu[\boldsymbol{\mu}_\nu \mathbf{H}_\nu^e]] + \mathbf{w}_\nu,\quad \partial^2\mathbf{u}/\partial t^2 = \mathbf{f} - \dot{\mathbf{u}}/\tau_s + \mathbf{y}_0 \quad (\dot{\mathbf{u}} = \partial\mathbf{u}/\partial t), \tag{1*}$$

where $\boldsymbol{\mu}_\nu$ is the magnetic moment per unit mass associated with the ν-th sublattice ($\nu = 1, 2$), $\mathbf{H}_\nu^e$ is the effective field, $\mathbf{u}$ is the displacement vector, $\mathbf{f}$ is the force acting on unit mass, g is the gyromagnetic ratio, μ_0^2 and ρ_0 are the equilibrium values of the square of the magnetic moment associated with each of the sublattices and of the density of the crystal, and τ_m and τ_s are relaxation constants (a more exact form of the relaxation terms is not important for us, since in the final results the limiting process $\tau_m \to \tau_s \to \infty$ will be carried out). On taking into account that the square of the magnetic moment per unit mass of each of the sublattices is an integral of the motion, $\mu_1^2 = \mu_2^2 = \mu_0^2$, we can express the random force $\mathbf{w}_\nu$ in terms of the transverse random force $\mathbf{y}_\nu$: $\mathbf{w}_\nu = \mu_0^{-1}[\boldsymbol{\mu}_\nu \times \mathbf{y}_\nu]$.

The expressions for the effective fields and the force f, with allowance for the coupling between the oscillations of the magnetic moments of the sublattices and the elastic oscillations, have the form (Bar'yakhtar and Gann[7])

$$\begin{aligned}
\mathbf{H}_1^e &= \mathbf{H} + \alpha\rho_0\Delta\boldsymbol{\mu}_1 + \alpha'\rho_0\Delta\boldsymbol{\mu}_2 + \beta\rho_0\mathbf{n}(\boldsymbol{\mu}_1\mathbf{n}) + \beta'\rho_0\mathbf{n}(\boldsymbol{\mu}_2\mathbf{n}) - \eta\rho_0\boldsymbol{\mu}_2 + {}^1/_2\mathbf{h},\\
\mathbf{H}_2^e &= \mathbf{H} + \alpha\rho_0\Delta\boldsymbol{\mu}_2 + \alpha'\rho_0\Delta\boldsymbol{\mu}_1 + \beta\rho_0\mathbf{n}(\boldsymbol{\mu}_2\mathbf{n}) + \beta'\rho_0\mathbf{n}(\boldsymbol{\mu}_1\mathbf{n}) - \eta\rho_0\boldsymbol{\mu}_1 - {}^1/_2\mathbf{h},\\
f_i &= \Lambda_{ik,i'k'}\partial^2 u_{i'}/\partial x_k\partial x_{k'} + f_i^M,\\
\mathbf{h} &= -f\rho_0\{\nabla(\boldsymbol{\lambda}^0\mathbf{u}) + (\boldsymbol{\lambda}^0\nabla)\mathbf{u}\} + (\beta-\beta')\rho_0(\mathbf{n}\nabla)\{\mathbf{n}(\boldsymbol{\lambda}^0\mathbf{u}) + \mathbf{u}(\boldsymbol{\lambda}^0\mathbf{n})\},\\
\mathbf{f}^M &= {}^1/_2 f\rho_0\{\boldsymbol{\lambda}^0\operatorname{div}\boldsymbol{\lambda} + (\boldsymbol{\lambda}^0\nabla)\boldsymbol{\lambda}\} - {}^1/_2(\beta-\beta')\rho_0(\mathbf{n}\nabla)\{\boldsymbol{\lambda}^0(\boldsymbol{\lambda}\mathbf{n}) + \boldsymbol{\lambda}(\boldsymbol{\lambda}^0\mathbf{n})\},
\end{aligned} \tag{2}$$

where $\mathbf{H}$ is the magnetic field, satisfying the equations of magnetostatics

$$\operatorname{div}(\mathbf{H} + 4\pi\rho_0\boldsymbol{\mu}) = 0, \qquad \operatorname{rot}\mathbf{H} = 0; \tag{3}$$

$\boldsymbol{\mu} = \boldsymbol{\mu}_1 + \boldsymbol{\mu}_2$ is the magnetic moment of unit mass; $\boldsymbol{\lambda} = \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2$; $\boldsymbol{\lambda}^0$ is the equilibrium value of the vector $\boldsymbol{\lambda}$; α, α', and η are exchange-interaction constants; β and β' are magnetic anisotropy constants; $\mathbf{n}$ is a unit vector along the anisotropy axis (the z axis); f is a magnetostriction constant; and Λ is the elastic-constant tensor, which we shall hereafter choose in the simplest form

$$\Lambda_{ik,i'k'} = (s_l^2 - 2s_t^2)\delta_{ik}\delta_{i'k'} + s_t^2(\delta_{ik'}\delta_{i'k} + \delta_{ii'}\delta_{kk'})$$

where s_l and s_t are the velocities of longitudinal and of transverse sound (the external constant magnetic field is supposed to be zero).

Following the general method of fluctuation theory, we must now determine the time derivative of the internal energy of the system and express it in the form

$$\dot{U} = \sum_{\nu=0,1,2}\int \mathbf{x}_\nu(\mathbf{r},t)\mathbf{X}_\nu(\mathbf{r}\ t)\,d\mathbf{r},$$

where $\mathbf{x}$ are the "generalized thermodynamic velocities" and $\mathbf{X}$ are the corresponding "generalized thermodynamic forces" (the index $\nu = 0$ refers to

*$[\mu_\nu \mathbf{H}_\nu^e] \equiv \mu_\nu \times \mathbf{H}_\nu^e$.

elastic quantities, the indices $\nu = 1$ and 2 to magnetic quantities connected respectively with the first and second sublattices). On differentiating with respect to time the expression for the internal energy of an antiferromagnet,[7]

$$\dot{U} = \int \Big\{ \frac{H^2}{8\pi} + \rho \dot{u}^2 + \frac{1}{2}\rho\Lambda_{ij;\,i'j'}\frac{\partial u_i}{\partial x_j}\frac{\partial u_{i'}}{\partial x_{j'}} + \eta\rho^2\mu_1\mu_2 + \frac{1}{2}\alpha\rho^2\Big[\Big(\frac{\partial \mu_1}{\partial x_i}\Big)^2 + \Big(\frac{\partial \mu_2}{\partial x_i}\Big)^2\Big] + \alpha'\rho^2\frac{\partial \mu_1}{\partial x_i}\frac{\partial \mu_2}{\partial x_i} - \frac{1}{2}\beta\rho^2[(\mu_1\mathbf{n})^2 + (\mu_2\mathbf{n})^2] - \beta'\rho^2(\mu_1\mathbf{n})(\mu_2\mathbf{n}) + \frac{1}{2}f\rho^2\lambda_i\lambda_j\frac{\partial u_i}{\partial x_j} - \frac{1}{2} \times (\beta - \beta')\rho^2(\lambda\mathbf{n})\lambda_i(\mathbf{n}\nabla)u_i \Big\} d\mathbf{r}, \tag{4}$$

using equations (1)–(3), and taking into account that because of the closed character of the system the energy flow across the surface vanishes, we get

$$\dot{U} = -\int \Big\{ \frac{\rho_0}{\mu_0}\sum_{\nu=1,2}[\mu_\nu\mathbf{H}_\nu^e]\Big(\frac{1}{\rho_0\tau_m\mu_0}[\mu_\nu\mathbf{H}_\nu^e] - \mathbf{y}_\nu\Big) + \rho_0\mathbf{u}\Big(\frac{\dot{\mathbf{u}}}{\tau_s} - \mathbf{y}\Big)\Big\} d\mathbf{r}. \tag{5}$$

We choose as "generalized thermodynamic forces and velocities" the quantities

$$\mathbf{x}_\nu = -(\rho_0\mu_0\tau_m)^{-1}[\mu_\nu\mathbf{H}_\nu^e] + \mathbf{y}_\nu,\ \mathbf{X}_\nu = \rho_0\mu_0^{-1}[\mu_\nu\mathbf{H}_\nu^e]\ (\nu = 1, 2),\quad \dot{\mathbf{x}}_0 = -\tau_s^{-1}\dot{\mathbf{u}} + \mathbf{y}_0,\quad \mathbf{X}_0 = \rho_0\dot{\mathbf{u}}, \tag{6}$$

and put $\dot{\mathbf{x}}$ into the form

$$\dot{\mathbf{x}}_\nu = -\gamma_\nu\mathbf{X}_\nu + y_\nu \quad (\nu = 0, 1, 2),\ \gamma_1 = \gamma_2 = \rho_0^{-2}\tau_m^{-1},$$

$$\gamma_0 = (\rho_0\tau_s)^{-1}.$$

The "kinetic coefficients" γ_ν directly determine, as is known, the normalization of the random forces. Allowing for the connection between the quantities $\mathbf{w}_\nu$ and $\mathbf{y}_\nu$, we get

$$\langle w_i^\nu w_j^{\nu'}\rangle_{\mathbf{q},\omega} = 2(\delta_{ij} - \lambda_i^0\lambda_j^0/4\mu_0^2)\,\delta_{\nu\nu'}\hbar\omega(N_\omega + 1)\rho_0^{-2}\tau_m^{-1},$$

$$\langle y_i y_j\rangle_{\mathbf{q},\omega} = 2\delta_{ij}\hbar\omega(N_\omega + 1)(\rho_0\tau_s)^{-1};$$

$$\langle w_i^\nu y_j\rangle_{\mathbf{q},\omega} = \langle y_i w_j^\nu\rangle_{\mathbf{q},\omega} = 0. \tag{7}$$

where $N_\omega = (e^{h\omega/T} - 1)^{-1}$ is the Planck distribution function.

To find the correlation functions, we must now express the magnetic moment vectors, the displacement vector, the density, and the other quantities that describe the crystal in terms of the random forces and then carry out an averaging over the random forces with the aid of formulas (7). Without quoting the complicated general expressions for the correlation functions, we shall discuss only the most interesting cases of fluctuations near magnetoacoustic resonance and fluctuations in the nonresonant range.

2. FLUCTUATIONS AND SCATTERING OF ELECTROMAGNETIC WAVES FAR FROM MAGNETOACOUSTIC RESONANCE

The correlators of the fluctuations of quantities describing the system have, as is known, sharp maxima for values of the frequency and of the wave vector that satisfy the dispersion equation of the characteristic oscillations of the system. For this reason sharp maxima, due to the possibility of propagation of characteristic oscillations in the system, occur also in the scattering cross section for electromagnetic waves.

In antiferromagnets, far from magnetoacoustic resonance, characteristic oscillations of two types can be propagated: spin waves and sound waves. We consider first the fluctuations and scattering of electromagnetic waves on spin waves.

Spin waves in a crystal with anisotropy of the easy-axis type. In an antiferromagnet with anisotropy of the easy-axis type, there can be propagated two branches of spin waves, with dispersion laws $\omega_1(\mathbf{q})$ and $\omega_2(\mathbf{q})$, where

$$\omega_1^2(\mathbf{q}) = (gM_0)^2\{(\beta - \beta') + (\alpha - \alpha')q^2\}\cdot \times\{2\eta + (\beta + \beta') + (\alpha + \alpha')q^2 + 8\pi\sin^2\chi\},$$

$$\omega_2^2(q) = (gM_0)^2\{(\beta - \beta') + (\alpha - \alpha')q^2\} \times \{2\eta + (\beta + \beta') + (\alpha + \alpha')q^2\}, \tag{8}$$

where $M_0 = \rho_0\mu_0$ and χ is the angle between the wave vector $\mathbf{q}$ and the anisotropy axis. On these waves the large fluctuations are those of the magnetic quantities (the magnetic moment, the vector $\boldsymbol{\lambda}$, the magnetic field). The nonvanishing components of the tensors $\langle\mu_i\mu_j\rangle$ and $\langle\lambda_i\lambda_j\rangle$ have the form

$$\langle\mu_x^2\rangle_{\mathbf{q},\omega} = \langle\lambda_y^2\rangle_{\mathbf{q},\omega}\Big(\frac{\omega}{2gM_0\eta}\Big)^2 = 2\pi\hbar|N_\omega + 1|\rho_0^{-2}\omega^2\eta^{-1}\delta(\omega^2 - \omega_1^2),$$

$$\langle\mu_y^2\rangle_{\mathbf{q},\omega} = \langle\lambda_x^2\rangle_{\mathbf{q},\omega}\Big(\frac{\omega}{2gM_0\eta}\Big)^2 = 2\pi\hbar|N_\omega + 1|\rho_0^{-2}\omega^2\eta^{-1}\delta(\omega^2 - \omega_2^2), \tag{9}$$

where the y axis is chosen perpendicular to the (n, q) plane. (We have taken into account that $\eta \gg \beta, \beta'$ and $\alpha q^2 \sim \alpha' q^2 \sim \eta(aq)^2 \ll \eta$, where a is the lattice constant.) The fluctuations of the magnetic field are connected with the fluctuations of the magnetic moment by the obvious relation

$$\langle H_iH_j\rangle_{\mathbf{q},\omega} = (4\pi\rho_0)^2 q_iq_jq^{-2}\sin^2\chi\langle\mu_x^2\rangle_{\mathbf{q},\omega}. \tag{10}$$

The correlators of the crystal density and of the displacement vector $\mathbf{u}^t = \mathbf{u} - q^{-2}\mathbf{q}(\mathbf{q}\cdot\mathbf{u})$ have the form

$$\begin{aligned}
\langle\rho^2\rangle_{\mathbf{q},\omega} &= 2\pi\hbar|N_\omega+1|(gM_0)^2\eta M_0^2q^4(\omega^2-s_l^2q^2)^{-2}\\
&\quad\times\sin^2 2\chi(2f-\beta+\beta')^2\delta(\omega^2-\omega_2^2),\\
\langle u_y^2\rangle_{\mathbf{q},\omega} &= 8\pi\hbar|N_\omega+1|(gM_0)^2\eta M_0^2q^2\rho_0^{-2}(\omega^2-s_t^2q^2)^{-2}\\
&\quad\times\cos^2\chi(f-\beta+\beta')^2\delta(\omega^2-\omega_1^2),\\
\langle(u_z^t)^2\rangle_{\mathbf{q},\omega} &= \operatorname{tg}^2\chi\langle(u_x^t)^2\rangle_{\mathbf{q},\omega} = -\operatorname{tg}\chi\langle u_x^tu_z^t\rangle_{\mathbf{q},\omega}\\
&= 8\pi\hbar|N_\omega+1|(gM_0)^2\eta M_0^2q^2\rho_0^{-2}(\omega^2-s_t^2q^2)^{-2}\sin^2\chi\\
&\quad\times[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi]^2\delta(\omega^2-\omega_2^2),
\end{aligned}\tag{11}$$

where s_l and s_t are the velocities of longitudinal and transverse sound (the remaining components of the tensor $\langle u_i^t u_j^t\rangle$ vanish).

We see that the quantities μ_x, λ_y, H, and u_y fluctuate on a spin wave of frequency $\omega_1(\mathbf{q})$, whereas the quantities μ_y, λ_x, ρ, u_x^t, and u_z^t fluctuate on a spin wave of frequency $\omega_2(q)$. On both spin waves, the correlators of the elastic quantities contain (in comparison with the correlator of the magnetic moment) the small parameter $\zeta^2\eta$, where $\zeta \sim M_0^2\rho_0^{-1}s^{-2}$ (s is the velocity of sound).

We now estimate the contribution of the fluctuations of the magnetic moment and of the fluctuations of elastic quantities to the cross section for scattering of electromagnetic waves on spin waves. As has already been mentioned, the correlators of the fluctuations of elastic quantities are proportional to the small parameter $\zeta^2\eta$; on the other hand, the contribution of magnetic fluctuations to the scattering cross section contains, as was shown earlier,[4] the additional small parameter $(gM_0/\omega)^2$, where ω is the frequency of the incident electromagnetic wave. Therefore in the low-frequency region ($\omega < gM_0\eta^{-1/2}\zeta^{-1}$) the chief contribution to the cross section for scattering of electromagnetic waves on spin waves comes from the interaction of the electromagnetic wave with the fluctuations of the magnetic moment; in the high-frequency region $\omega > gM_0\eta^{-1/2}\zeta^{-1}$, the scattering of electromagnetic waves originates chiefly on the fluctuations of elastic quantities that accompany the spin wave.

To determine the cross section for scattering of electromagnetic waves on spin waves in the low-frequency region ($\omega < gM_0\eta^{-1/2}\zeta^{-1}$), we substitute (9) into the known expression for the differential cross section for scattering of electromagnetic waves on fluctuations of magnetic moment:[4]

$$\begin{aligned}
d\Sigma_m &= g^2c^{-2}\varepsilon\rho_0^2\{\sin^2\vartheta k^2\langle\mu^2\rangle_{\mathbf{q},\Delta\omega}\\
&\quad+2\cos\vartheta k_ik_j'\operatorname{Re}\langle\mu_i\mu_j\rangle_{\mathbf{q},\Delta\omega}\}\frac{do'd\omega'}{4\pi},
\end{aligned}\tag{12}$$

where $\mathbf{k}$ is the wave vector of the incident wave, $\mathbf{k}'$ and ω' are the wave vector and frequency of the scattered wave, $\Delta\omega = \omega - \omega'$, $\mathbf{q} = \mathbf{k} - \mathbf{k}'$, θ is the angle of scattering (the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$), do' is an element of solid angle of the vector $\mathbf{k}'$, and ϵ is the permittivity of the crystal. We get as the result

$$\begin{aligned}
d\Sigma_m &= |N_{\Delta\omega}+1|\left(\frac{gk}{c}\right)^2\varepsilon\hbar(gM_0)^2\left[\beta-\beta'+4(\alpha-\alpha')k^2\sin^2\frac{\vartheta}{2}\right]\\
&\quad\times\{\nu_1\delta(\Delta\omega^2-\omega_1^2)+\nu_2\delta(\Delta\omega^2-\omega_2^2)\}do'd\omega',
\end{aligned}\tag{13}$$

where $\nu_1 = \sin^2\theta + 2\cos\theta k^{-1}k'^{-1}k_xk_x'$; $\nu_2 = \sin^2\theta + 2\cos\theta k^{-2}k_y^2$.

For $\omega > gM_0\eta^{-1/2}\zeta^{-1}$ we must use the expression for the cross section for scattering of electromagnetic waves on elastic fluctuations:[4]

$$d\Sigma_e = d\Sigma_c + d\Sigma_s,$$

$$\begin{aligned}
d\Sigma_c &= \left(\frac{\omega}{c}\right)^4\left\{\left[\frac{\sigma_1}{2}(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2+\sigma_2^2\right\}\rho_0^{-2}\langle\rho^2\rangle_{\mathbf{q},\Delta\omega}\frac{do'd\omega'}{(4\pi)^3},\\
d\Sigma_s &= \left(\frac{\omega}{c}\right)^4\frac{\sigma_1^2}{4}\{2k^2\sin^2\vartheta\langle(u^t)^2\rangle_{\mathbf{q},\Delta\omega}-(1-\cos\vartheta)(k_i+k_i')\\
&\quad\times(k_j+k_j')\langle u_i^tu_j^t\rangle_{\mathbf{q},\Delta\omega}\}\frac{do'd\omega'}{(4\pi)^3},
\end{aligned}\tag{14}$$

where σ_1 and σ_2 are quantities that describe the connection between fluctuations of the permittivity and fluctuations of the deformation tensor u_{ij},

$$\delta\varepsilon_{ij} = \sigma_1u_{ij} + \sigma_2\delta_{ij}u_{nn}$$

(in order of magnitude, $\sigma_1 \sim \sigma_2 \sim 1$). On introducing the notation

$$\begin{aligned}
R_0 &= 2\left\{\left[\frac{1}{2}\sigma_1(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2+\sigma_2^2\right\}(1-\cos\vartheta)\\
&\quad\times\sin^2 2\chi(2f-\beta+\beta')^2,\\
R_1 &= \sigma_1^2[\sin^2\vartheta-2(1-\cos\vartheta)k^{-2}k_y^2]\cos^2\chi(f-\beta+\beta')^2,\\
R_2 &= \sigma_1^2[\sin^2\vartheta-2(1-\cos\vartheta)k^{-2}(k_x\cos\chi-k_z\sin\chi)^2]\\
&\quad\times[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi],
\end{aligned}$$

we find

$$\begin{aligned}
d\Sigma_e &= |N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar(gM_0)^2\eta M_0^2q^2\rho_0^{-2}\{(\omega_1-s_t^2q^2)^{-2}\\
&\quad\times R_1\delta(\Delta\omega^2-\omega_1^2)+[(\omega_2^2-s_l^2q^2)^{-2}R_0+(\omega_2^2-s_t^2q^2)^{-2}R_2]\\
&\quad\times\delta(\Delta\omega^2-\omega_2^2)\}\frac{do'd\omega'}{(4\pi)^2}.
\end{aligned}\tag{15}$$

In this expression (as in all other expressions for a scattering cross section), q and χ are to be replaced by their values

$$q = 2k\sin\frac{\vartheta}{2},\quad \chi = \arccos\left\{\left(2\sin\frac{\vartheta}{2}\right)^{-1}(\cos\theta-\cos\theta')\right\},\tag{16}$$

where $\theta\,(\theta')$ is the angle between the vector $\mathbf{k}\,(\mathbf{k}')$ and the anisotropy axis.

In the intermediate frequency range $\omega \sim gM_0\eta^{-1/2}\zeta^{-1}$, the scattering cross section separates into the sum of three terms:

$$d\Sigma = d\Sigma_m + d\Sigma_e + d\Sigma_{em}.$$

The terms $d\Sigma_m$ and $d\Sigma_e$, describing the scattering of electromagnetic waves on magnetic and on elastic fluctuations, are determined by formulas (13) and (15), whereas the interference term $d\Sigma_{em}$ has the form

$$d\Sigma_{em} = |N_{\Delta\omega}+1|\frac{\Delta\omega}{\omega}\left(\frac{\omega}{c}\right)^4 \varepsilon\hbar (gM_0)^2\rho_0^{-1}k^2\{(\omega_1^2 - s_t^2q^2)^{-1} \times R_1'\delta(\Delta\omega^2-\omega_1^2) + [(\omega_2^2 - s_l^2q^2)^{-1}R_0' + (\omega_2^2 - s_t^2q^2)^{-1}R_2'] \times \delta(\Delta\omega^2-\omega_2^2)\}\frac{do'd\omega'}{4\pi}, \quad (17)$$

where R_0', R_1', and R_2' are functions of the angle variables and are of order of magnitude unity (we shall not present here the complicated expressions for the functions R').

We remark that the expressions given above for the scattering cross section (and also for the correlation functions) do not take account of the attenuation of spin waves. In order to take account of the attenuation of the spin waves, it is sufficient in these expressions to make the substitutions

$$\delta(\Delta\omega^2-\omega_j^2) \to \pi^{-1}2\gamma_j\omega_j\{(\Delta\omega^2-\omega_j^2)^2+(2\gamma_j\omega_j)^2\}^{-1}, \quad (18)$$

where γ_j is the damping decrement of the spin-wave ($j = 1, 2$).

We shall discuss briefly the relation between the frequency and the direction of propagation of the scattered wave. The δ-functions contained in expressions (13), (15), and (17) enable us to find the frequencies of the radiation scattered in a definite direction, if we know the frequency and direction of propagation of the incident wave. By use of (8) and (16) we get for the frequencies ω'

$$\omega' = \omega \pm gM_0\left\{(\beta-\beta')+4(\alpha-\alpha')k^2\sin^2\frac{\vartheta}{2}\right\}^{1/2} \times \left\{2\eta+(\beta+\beta')+8\pi+4(\alpha+\alpha')k^2\sin^2\frac{\vartheta}{2} - 2\pi\sin^{-2}\frac{\vartheta}{2}(\cos\theta-\cos\theta')^2\right\}^{1/2} \quad (19)$$

or

$$\omega' = \omega \pm gM_0\left\{(\beta-\beta')+4(\alpha-\alpha')k^2\sin^2\frac{\vartheta}{2}\right\}^{1/2} \times \left\{2\eta+(\beta+\beta')+4(\alpha+\alpha')k^2\sin^2\frac{\vartheta}{2}\right\}^{1/2}. \quad (19')$$

We see that in the spectrum of the scattered radiation there occur two pairs of sharp maxima (magnon satellites); the two Stokes satellites (and the two anti-Stokes) are separated by a narrow gap with width of order $gM_0\eta^{-1/2}$ (and also if the resolution with respect to frequency or scattering angle is low), then one pair of satellites is superposed on the other.

We remark that because of the factor $[N_{\Delta\omega}+1]$ that occurs in the expression for the scattering cross section, at low temperatures ($T \ll \hbar|\Delta\omega|$) scattering occurs only with diminution of frequency; if $T \gtrsim \hbar|\Delta\omega|$, the intensities of the Stokes and anti-Stokes satellites are of the same order of magnitude.

Spin waves in a crystal with anisotropy of the easy-plane type. In an antiferromagnet with anisotropy of the easy-plane type, two branches of spin waves can be propagated, with frequencies $\omega = \omega_s(q)$ and $\omega = Vq$, where

$$\omega_s^2(q) = \omega_0^2 + V^2q^2, \quad \omega_0 = gM_0|\beta-\beta'|^{1/2}(2\eta)^{1/2},$$
$$V = gM_0(2\eta)^{1/2}(\alpha-\alpha')^{1/2}. \quad (20)$$

We shall give the expressions for the nonvanishing components of the tensors $\langle\mu_i\mu_j\rangle$ and $\langle\lambda_i\lambda_j\rangle$:

$$\langle\mu_z^2\rangle_{q,\omega} = \langle\lambda_\perp^2\rangle_{q,\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = 2\pi\hbar|N_\omega+1|\rho_0^{-2}\omega^2\eta^{-1}\delta(\omega - V^2q^2),$$
$$\langle\mu_\perp^2\rangle_{q,\omega} = \langle\lambda_z^2\rangle_{q,\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = 2\pi\hbar|N_\omega+1|\rho_0^{-2}\omega^2\eta^{-1}\delta(\omega^2-\omega_s^2), \quad (21)$$

where $r_\perp = (\lambda^0)^{-1}\mathbf{r}[\mathbf{n}\times\lambda^0]$. For the correlator of the fluctuations of the magnetic field we have

$$\langle H_iH_j\rangle_{q,\omega} = 4(2\pi)^3\hbar|N_\omega+1|\omega^2\eta^{-1}q^{-2}q_iq_j \times\{\cos^2\chi\,\delta(\omega^2-V^2q^2)+\sin^2\chi\sin^2\varphi_0\,\delta(\omega^2-\omega_s^2)\}, \quad (22)$$

where φ_0 is the angle between the vector $\boldsymbol{\lambda}^0$ and the x axis (the y axis, as before, is chosen perpendicular to the $(\mathbf{n}, \mathbf{q})$ plane).

The correlators of the crystal density and of the displacement vector are determined by the formulas

$$\langle\rho^2\rangle_{q,\omega} = 8\pi\hbar|N_\omega+1|(gM_0)^2\eta M_0^2q^4(\omega^2-s_l^2q^2)^{-2}\sin^2\chi \times\{\cos^2\chi\cos^2\varphi_0(2f-\beta+\beta')^2\delta(\omega^2-\omega_s^2) + \sin^2\chi\sin^2 2\varphi_0 f^2\delta(\omega^2-V^2q^2)\}$$
$$\langle u_i^tu_j^t\rangle_{q,\omega} = 8\pi\hbar|N_\omega+1|(gM_0)^2\eta M_0^2q^2\rho_0^{-2}(\omega^2-s_t^2q^2)^{-2} \times\{\varkappa_i\varkappa_j\delta(\omega^2-\omega_s^2)+\sin^2\chi\varkappa_i'\varkappa_j'\delta(\omega^2-V^2q^2)\},$$
$$\varkappa_y = (f-\beta+\beta')\cos\chi\sin\varphi_0, \qquad \varkappa_y' = f\cos 2\varphi_0,$$
$$\varkappa_z = -\operatorname{tg}\chi\varkappa_x = \sin\chi\cos\varphi_0[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi],$$
$$\varkappa_z' = -\operatorname{tg}\chi\varkappa_x' = {}^1/_2 f\sin 2\chi\sin 2\varphi_0. \quad (23)$$

We see that on both spin waves, the correlators of the elastic quantities contain (by comparison with the correlators of the magnetic moment) the small parameter $\zeta_1^2 \sim \zeta^2\omega^{-2}(gM_0\eta)^2$. In the case of a wave with frequency $\omega_s(q)$, the parameter ζ_1^2 is of order $\zeta^2\eta$; but in the case of a wave with a linear dispersion law, the parameter ζ_1^2 is appreciably larger, and equal in order of magnitude to $\zeta^2_{\min}\{\eta\beta_\perp^{-1};\ (aq)^{-2}\}$, where $\beta_\perp$ is a constant that describes the anisotropy of different directions lying in the easy plane, and a is the lattice constant.

We shall now consider the scattering of electromagnetic waves on spin waves. In the case of a spin wave with a linear dispersion law, the chief contribution to the scattering cross section comes from the interaction of the electromagnetic wave with the fluctuations of density and of the displacement vector that accompany the spin wave. On substituting (23) into (14) and introducing the notation

$$D_l' = 2\left\{\left[\frac{\sigma_1}{2}(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2+\sigma_2^2\right\}\sin^2\chi\sin^2 2\varphi_0/2,$$

$$D_t' = \sigma_1^2\left\{\sin^2\vartheta\varkappa'^2-\sin^2\frac{\vartheta}{2}k^{-1}k'^{-1}(k_i+k_i')(k_j+k_j')\varkappa_i'\varkappa_j'\right\}$$

$$\times\sin^2\chi,$$

we get

$$d\Sigma = |N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar(gM_0)^2\eta M_0^2q^{-2}\rho_0^{-2}$$

$$\times\{(V^2-s_l^2)^{-2}D_l'+(V^2-s_t^2)^{-2}D_t'\}\delta(\Delta\omega^2-V^2q^2)$$

$$\times\frac{do'd\omega'}{(4\pi)^2}. \tag{24}$$

In the case of a spin wave with frequency $\omega_s(q)$, the fluctuations of elastic quantities that accompany the spin wave make the principal contribution to the scattering cross section in the high-frequency region $\omega > gM_0\eta^{-1/2}\zeta^{-1}$. On substituting (23) into (14), we find for the value of $d\Sigma$ in this range

$$d\Sigma_e = |N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar(gM_0)^2\eta M_0^2q^2\rho_0^{-2}\cdot\{(\omega_s^2-s_l^2q^2)^{-2}D_l$$

$$+(\omega_s^2-s_t^2q^2)^{-2}D_t\}\delta(\Delta\omega^2-\omega_s^2)\frac{do'd\omega'}{(4\pi)^2} \tag{25}$$

where

$$D_l = \frac{1}{2}\left\{\left[\frac{\sigma_1}{2}(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2+\sigma_2^2\right\}$$

$$\times\sin^2 2\chi\cos^2\varphi_0(2f-\beta+\beta')^2,$$

$$D_t = \sigma_1^2\left\{\sin^2\vartheta\varkappa^2-\sin^2\frac{\vartheta}{2}k^{-1}k'^{-1}(k_i+k_i')(k_j+k_j')\varkappa_i\varkappa_j\right\}.$$

In the low-frequency region $\omega < gM_0\eta^{-1/2}\zeta^{-1}$, the scattering of an electromagnetic wave on a spin wave with frequency $\omega_s(q)$ is determined chiefly by the interaction of the electromagnetic wave with the fluctuations of magnetic moment. On using (21) and (12), we get for the value of $d\Sigma$ in this range

$$d\Sigma = |N_{\Delta\omega}+1|\left(\frac{gk}{c}\right)^2\varepsilon\hbar(gM_0)^2\Big[\,|\beta-\beta'|$$

$$+4(\alpha-\alpha')k^2\sin^2\frac{\vartheta}{2}\Big]\nu\delta(\Delta\omega^2-\omega_s^2)\,do'd\omega', \tag{26}$$

where

$$\nu = \sin^2\vartheta+2\cos\vartheta k^{-1}k'^{-1}(k_xk_x'\sin^2\varphi_0+k_y^2\cos^2\varphi_0).$$

We shall discuss briefly the relation between the frequency and the direction of the scattered wave. The δ-functions contained in expressions (24), (25), and (26) enable us to find the frequencies of the radiation scattered in a definite direction, if we know the frequency and the direction of propagation of the incident wave. By use of (20) and (16), we get for the frequencies ω'

$$\omega' = \omega\pm gM_0(2\eta)^{1/2}\left\{|\beta-\beta'|+4(\alpha-\alpha')k^2\sin^2\frac{\vartheta}{2}\right\}^{1/2} \tag{27}$$

or

$$\omega' = \omega\pm 2\frac{V}{c}\omega\sqrt{\varepsilon}\sin\frac{\vartheta}{2}. \tag{27'}$$

We see that in the spectrum of the scattered radiation there appear two pairs of sharp maxima (magnetic satellites).

<u>Sound waves.</u> The correlators of the fluctuations of the crystal density and of the components of the displacement vector on sound waves are determined by the formulas

$$\langle\rho^2\rangle_{q,\omega} = 2\pi\hbar|N_\omega+1|\rho_0q^2\delta(\omega^2-s_l^2q^2), \tag{28}$$
$$\langle u_i^tu_j^t\rangle_{q,\omega} = 2\pi\hbar|N_\omega+1|\rho_0^{-1}(\delta_{ij}-q^{-2}q_iq_j)\delta(\omega^2-s_t^2q^2).$$

The correlator of the fluctuations of the magnetic moment of unit mass has the form

$$\langle\mu_i\mu_j\rangle = 8\pi\hbar|N_\omega+1|(gM_0)^2\omega^2\mu_0^2q^2\rho_0^{-1}\{R_{ij}^l\delta(\omega^2-s_l^2q^2)$$
$$+R_{ij}^t\delta(\omega^2-s_t^2q^2)\}, \tag{29}$$

where the nonvanishing components of the tensor R are determined in the case of anisotropy of the easy-axis type by the formulas

$$R_{yy}^l = (\omega^2-\omega_2^2)^{-2}(f-{}^1/_2\beta+{}^1/_2\beta')^2\sin^2 2\chi,$$
$$R_{xx}^t = (\omega^2-\omega_1^2)^{-2}(f-\beta+\beta')^2\cos^2\chi,$$
$$R_{yy}^t = (\omega^2-\omega_2^2)^{-2}[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi]^2,$$

and in the case of anisotropy of the easy-plane type by the formulas

$$R_{\perp\perp}^l = (\omega^2-\omega_s^2)^{-2}(f-{}^1/_2\beta+{}^1/_2\beta')^2\sin^2 2\chi\cos^2\varphi_0,$$
$$R_{z\perp}^l = R_{\perp z}^l = -(\omega^2-\omega_s^2)^{-1}(\omega^2-V^2q^2)^{-1}f(f-{}^1/_2\beta+{}^1/_2\beta')$$
$$\times\sin^2\chi\sin 2\chi\cos\varphi_0\sin 2\varphi_0,$$
$$R_{zz}^l = (\omega^2-V^2q^2)^{-2}f^2\sin^4\chi\sin^2 2\varphi_0,$$
$$R_{\perp\perp}^t = (\omega^2-\omega_s^2)^{-2}\{(f-\beta+\beta')^2\sin^2\varphi_0\cos^2\chi$$
$$+[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi]^2\cos^2\varphi_0\},$$
$$R_{z\perp}^t = R_{\perp z}^t = (\omega^2-\omega_s^2)^{-1}(\omega^2-V^2q^2)^{-1}f\sin 2\chi\sin$$
$$\times\varphi_0\{{}^1/_2(f-\beta+\beta')\cos 2\varphi_0$$
$$+[f\sin^2\chi-(f-\beta+\beta')\cos^2\chi]\cos^2\varphi_0\},$$
$$R_{zz}^t = (\omega^2-V^2q^2)^{-2}f^2\sin^2\chi(\cos^2 2\varphi_0+\cos^2\chi\sin^2 2\varphi_0).$$

We see that the relative fluctuations of the magnetic moment on sound waves are in order of magnitude smaller by a factor $\eta^{1/2}$ than the relative fluctuations of the nonmagnetic quantities, $\delta\mu/\mu_0 \sim f\eta^{-1/2}\,\delta\rho/\rho_0$.

By substituting (28) and (29) into (12) and (14), it is easy to satisfy oneself that the scattering of electromagnetic waves on sound is caused chiefly by interaction of the electromagnetic wave with fluctuations of the density and of the displacement vector. Therefore the scattering cross section of electromagnetic waves on sound in antiferromagnets is determined by the same formula as in the case of ordinary (not magnetically ordered) crystals,

$$d\Sigma = |N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6 \varepsilon\hbar\rho_0^{-1}\left\{\left(\left[\frac{\sigma_1}{2}(1+\cos\vartheta)+\sigma_2\cos\vartheta\right]^2 + \sigma_2^2\right)(1-\cos\vartheta)\,\delta(\Delta\omega^2 - s_t^2q^2) + \frac{1}{4}\sigma_1^2\sin^2\vartheta\,\delta(\Delta\omega^2 - s_t^2q^2)\right\}\frac{do'\,d\omega'}{(4\pi)^2} \quad (30)$$

In closing this section, we remark that in the high-frequency range, the sound satellites surpass the magnon satellites in brightness. The intensities of the magnon satellites distant from the principal line by frequency ω_1, ω_2, or ω_S are comparable with the intensities of sound satellites at $\omega^2 \sim (gM_0)^2(\eta\xi)^{-1}$; the intensities of the satellites with $\Delta\omega = \pm 2Vk\sin(\theta/2)$ are comparable with the intensities of sound satellites at $\omega^2 \sim \eta^{-1}\xi c^2 a^{-2}$ (a is the lattice constant). In the lower-frequency range, the magnon satellites surpass the sound satellites in brightness.

3. FLUCTUATIONS AND SCATTERING OF ELECTROMAGNETIC WAVES IN THE VICINITY OF MAGNETOACOUSTIC RESONANCE

As was shown in the previous section, far from magnetoacoustic resonance, on one set of branches of the oscillations of an antiferromagnet—the spin waves—there are large fluctuations of the magnetic moment and magnetic field and small fluctuations of the density and the displacement vector; on the other hand, on the other branches of the oscillations—longitudinal and transverse sound—there are large fluctuations of the displacement vector and comparatively small fluctuations of the magnetic quantities. The situation is otherwise near magnetoacoustic resonance, when the frequency of one of the spin waves is close to the frequency of one of the sound waves. In the vicinity of resonance, instead of sound and spin waves, there are propagated in the antiferromagnet coupled magnetoelastic waves, on which there are large fluctuations both of the magnetic and of the nonmagnetic quantities. Because of this, on approach to a resonance point there is a change of character of the distribution of the radiation scattered in the antiferromagnet (but not of its total intensity).

Longitudinal magnetoacoustic resonance. The dispersion equation of coupled spin and longitudinal-sound waves has, near resonance, the form

$$(\omega^2-\omega_+^{l2})^2(\omega^2-\omega_-^{l2})^2+(\omega_+^{l2}-\omega_-^{l2})^2\quad(2\gamma\omega)^2=0,$$

where in the case of anisotropy of the easy-axis type

$$\omega_\pm^{l2} = {}^1/_2(\omega_2^2+s_l^2q^2) \pm {}^1/_2\{(\omega_2^2-s_l^2q^2)^2 + 4\eta(gM_0)^2\rho_0^{-1}M_0^2q^2(2f-\beta+\beta')^2\sin^2 2\chi\}^{1/2} \quad (31)$$

and in the case of anisotropy of the easy-plane type

$$\omega_\pm^{l2} = {}^1/_2(\omega_s^2+s_l^2q^2) \pm {}^1/_2\{(\omega_s^2-s_l^2q^2)^2 + 4\eta(gM_0)^2\rho_0^{-1}M_0^2q^2(2f-\beta+\beta')^2\sin^2 2\chi\cos^2\varphi_0\}^{1/2}, \quad (31')$$

in both cases, $\gamma = (2\tau_S)^{-1} + \eta\tau_m^{-1}$. This equation has two solutions, corresponding to the two branches of the magnetoelastic waves with frequencies ω_+^l and ω_-^l and with damping decrement γ.

We give the expression for the correlator of the fluctuations of density near the point of longitudinal resonance ($|\omega_2^2 - q^2s_l^2| < \xi^{1/2}\omega_2^2$ or $|\omega_S^2 - s_l^2q^2| < \xi^{1/2}\omega_S^2$):

$$\langle\rho^2\rangle_{q;\omega} = 2\hbar\omega(N_\omega+1)\rho_0q^2\gamma\{[(\omega^2-\omega_+^{l2})^2 + (2\gamma\omega)^2]^{-1} + [(\omega^2-\omega_-^{l2})^2+(2\gamma\omega)^2]^{-1}\}. \quad (32)$$

If the damping of the waves is small, $\gamma < \xi^{1/2}\omega$, then this expression takes the form

$$\langle\rho^2\rangle_{q,\omega} = \pi\hbar|N_\omega+1|\rho_0q^2\{\delta(\omega^2-\omega_+^{l2})+\delta(\omega^2-\omega_-^{l2})\}. \quad (33)$$

But if the damping is not small ($\omega \gg \gamma > \xi^{1/2}\omega$), then (32) reduces to the first of the relations (28), describing the fluctuations of density on a sound wave far from resonance.

In the case of a crystal with anisotropy of the easy-axis type, near resonance, we have

$$\langle\mu_y^2\rangle_{q,\omega} = \langle\lambda_x^2\rangle_{q,\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = \pi\hbar|N_\omega+1|\rho_0^{-2}\omega^2\eta^{-1} \times\{\delta(\omega^2-\omega_+^{l2})+\delta(\omega^2-\omega_-^{l2})\}; \quad (34)$$

in the case of a crystal with anisotropy of the easy-plane type,

$$\langle\mu_\perp^2\rangle_{q,\omega} = \langle\lambda_z^2\rangle_{q,\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = \pi\hbar|N_\omega+1|\rho_0^{-2}\omega^2\eta^{-1} \times\{\delta(\omega^2-\omega_+^{l2})+\delta(\omega^2-\omega_-^{l2})\}. \quad (34')$$

As for the remaining components of the tensors $\langle\mu_i\mu_j\rangle$ and $\langle\lambda_i\lambda_j\rangle$, and also the components of the correlator of the displacement vector, $\langle u_i^t u_j^t\rangle$, these quantities near a point of longitudinal magnetoacoustic resonance are determined by the same formulas as in the nonresonant range.

Passing to the study of the scattering of electromagnetic waves near a resonance point, we remark first of all that magnetoacoustic resonance can manifest itself in the scattering of electromagnetic waves only in the high-frequency region. In fact, from the condition for resonance it follows that $q > s^{-1} gM_0 \eta^{1/2}$, and therefore the frequency of the incident wave must satisfy the inequality $\omega > gM_0 \eta^{1/2}$ c/s. By using relations (12) and (14) and by taking into account that, in accordance with (33) and (34), near magnetoacoustic resonance $\delta\mu/\mu_0 \sim (\eta\zeta)^{-1/2}\delta\rho/\rho_0$, it is easy to see that in the frequency range $\omega > gM_0\eta^{1/2}$ c/s the principal contribution to the scattering cross section of electromagnetic waves comes from the interaction of these waves with the fluctuations of the elastic quantities; this interaction is characterized by the scattering cross section (14). On substituting (32) into (14) and on taking into account that the fluctuations of the displacement vector near longitudinal resonance are small, we get

$$d\Sigma = \pi^{-1}(N_\omega + 1)\hbar\Delta\omega\gamma\left(\frac{\omega}{c}\right)^6 \varepsilon\rho_0^{-1}(1-\cos\theta)\left\{\left[\frac{\sigma_1}{2}(1+\cos\theta) + \sigma_2\cos\vartheta\right]^2 + \sigma_2^2\right\}\{[(\Delta\omega^2 - \omega_+^{l2})^2 + (2\gamma\Delta\omega)^2]^{-1} + [(\Delta\omega^2 - \omega_-^{l2})^2 + (2\gamma\Delta\omega)^2]^{-1}\}\frac{do'd\omega'}{(4\pi)^2}. \tag{35}$$

If the damping of magnetoelastic waves is small ($\gamma \ll \zeta^{1/2}\Delta\omega$), then this formula takes the form

$$d\Sigma = \frac{1}{2}|N_{\Delta\omega} + 1|\left(\frac{\omega}{c}\right)^6 \varepsilon\hbar\rho_0^{-1}\left\{\left[\frac{\sigma_1}{2}(1+\cos\vartheta) + \sigma_2\cos\vartheta\right]^2 + \sigma_2^2\right\}(1-\cos\vartheta)\cdot\{\delta(\Delta\omega^2 - \omega_+^{l2}) + \delta(\Delta\omega^2 - \omega_-^{l2})\}\frac{do'd\omega'}{(4\pi)^2}. \tag{36}$$

On comparing this formula with the relation (30), we see that each of the longitudinal-sound satellites splits, on approach to the longitudinal-resonance point, into two lines of equal intensity.

If, however, $\gamma \gtrsim \zeta^{1/2}\Delta\omega$, then the satellites with frequencies $\omega + \omega_+^l$ and $\omega + \omega_-^l$ (and likewise the satellites with frequencies $\omega - \omega_+^l$ and $\omega - \omega_-^l$) are superposed on one another. The scattering cross section of electromagnetic waves is then determined by the same formula (30) as in the nonresonant range.

Transverse magnetoacoustic resonance. In crystals with anisotropy of the easy-axis type, two transverse magnetoacoustic resonances are possible: resonance between a spin wave with frequency $\omega_1(\mathbf{q})$ and transverse sound polarized perpendicular to the $(\mathbf{n}, \mathbf{q})$ plane, and resonance between a spin wave with frequency $\omega_2(\mathbf{q})$ and transverse sound polarized in the $(\mathbf{n}, \mathbf{q})$ plane. In the first case, the frequencies of coupled magnetoelastic waves are determined by the expression

$$\omega_{1\pm}^{t2} = {}^1/_2(\omega_1^2 + s_t^2q^2) \pm {}^1/_2\{(\omega_1^2 - s_t^2q^2)^2 + \eta(4gM_0)^2\rho_0^{-1}M_0^2q^2\cos^2\chi(f - \beta + \beta')^2\}^{1/2} \tag{37}$$

in the second case, by the expression

$$\omega_{2\pm}^{t2} = {}^1/_2(\omega_2^2 + s_t^2q^2) \pm {}^1/_2\{(\omega_2^2 - s_t^2q^2)^2 + \eta(4gM_0)^2\rho_0^{-1}M_0^2q^2[f\sin^2\chi - (f - \beta + \beta')\cos^2\chi]^2\}^{1/2}. \tag{37'}$$

We give the formulas for the correlators of the fluctuations of the magnetic moment and of the displacement vector near the first of the resonances ($|\omega_1^2 - s_t^2 q^3| < \zeta^{1/2}\omega_1^2$):

$$\langle\mu_x^2\rangle_{\mathbf{q},\omega} = \langle\lambda_y^2\rangle_{\mathbf{q},\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = \pi\hbar|N_\omega + 1|\rho_0^{-2}\omega^2\eta^{-1}\{\delta(\Delta\omega^2 - \omega_{1+}^{t2}) + \delta(\Delta\omega^2 - \omega_{1-}^{t2})\},$$

$$\langle u_y^2\rangle_{\mathbf{q},\omega} = \pi\hbar|N_\omega + 1|\rho_0^{-1}\{\delta(\omega^2 - \omega_{1+}^{t2}) + \delta(\omega^2 - \omega_{1-}^{t2})\}. \tag{38}$$

Near the second resonance ($|\omega_2^2 - s_t^2q^2| < \zeta^{1/2}\omega_2^2$), we have

$$\langle\mu_y^2\rangle_{\mathbf{q},\omega} = \langle\lambda_x^2\rangle_{\mathbf{q},\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2 = \pi\hbar|N_\omega + 1|\rho_0^{-2}\omega^2\eta^{-1}\{\delta(\Delta\omega^2 - \omega_{2+}^{t2}) + \delta(\Delta\omega^2 - \omega_{2-}^{t2})\},$$

$$\langle(u_z^t)^2\rangle_{\mathbf{q},\omega} = \mathrm{tg}^2\chi\langle(u_x^t)^2\rangle_{\mathbf{q},\omega} = -\mathrm{tg}\,\chi\langle u_x^t u_z^t\rangle_{\mathbf{q},\omega} = \pi\hbar|N_\omega + 1|\rho_0^{-1}\sin^2\chi\{\delta(\omega^2 - \omega_{2+}^{t2}) + \delta(\omega^2 - \omega_{2-}^{t2})\}. \tag{38'}$$

(The remaining components of the tensors $\langle\mu_i\mu_j\rangle$, $\langle\lambda_i\lambda_j\rangle$, and $\langle u_i^t u_j^t\rangle$, and also the correlator $\langle\rho^2\rangle$, are determined near the resonance points by the same formulas as in the nonresonant range.)

In crystals with anisotropy of the easy-plane type, a single transverse magnetoacoustic resonance is possible. Near a resonance point ($|\omega_s^2 - s_t^2q^2| < \zeta^{1/2}\omega_s^2$), the frequencies of coupled magnetoelastic waves are determined by the formula

$$\begin{gathered}\omega_{s\pm}^{t2} = {}^1/_2(\omega_s^2 + s_t^2q^2) \pm {}^1/_2 \times\{(\omega_s^2 - s_t^2q^2)^2 + \eta(4gM_0)^2\rho_0^{-1}M_0^2q^2\Psi\}^{1/2} \\ \Psi = (f - \beta + \beta')^2\cos^2\chi + f^2\sin^2\chi\cos^2\varphi_0 - (f - {}^1/_2\beta + {}^1/_2\beta')^2\sin^2 2\chi\cos^2\varphi_0.\end{gathered} \tag{39}$$

We give the expressions for the correlators of the fluctuations of the magnetic moment and of the displacement vector in the vicinity of transverse resonance:

$$\langle\mu_\perp^2\rangle_{\mathbf{q};\omega} = \langle\lambda_z^2\rangle_{\mathbf{q},\omega}\left(\frac{\omega}{2gM_0\eta}\right)^2$$
$$= \pi\hbar|N_\omega+1|\rho_0^{-2}\omega^2\eta^{-1}\{\delta(\omega^2-\overset{t2}{\omega}_{s+})+\delta(\omega^2-\overset{t2}{\omega}_{s-})\},$$
$$\langle u_i^t u_j^t\rangle_{\mathbf{q},\omega} = {}^1/_2\pi\hbar|N_\omega+1|\rho_0^{-1}(\delta_{ij}-q^{-2}q_iq_j)\{\delta(\omega^2-\omega_{s+}^{t2})$$
$$+\delta(\omega^2-\omega_{s-}^{t2})+2\delta(\omega^2-s_t^2q^2)\} \qquad (40)$$

(the remaining components of the tensors $\langle\mu_i\mu_j\rangle$ and $\langle\lambda_i\lambda_j\rangle$ are small near resonance).

The differential cross section for scattering of electromagnetic waves near transverse magnetoacoustic resonance can be determined by substituting (38), (38′), and (39) into (14). In the case of anisotropy of the easy-plane type we get

$$d\Sigma = \frac{1}{16}|N_{\Delta\omega}+1|\left(\frac{\omega}{c}\right)^6\varepsilon\hbar\rho_0^{-1}\sigma_1^2\sin^2\vartheta\{\delta(\Delta\omega^2-\omega_{s+}^{t2})$$
$$+\delta(\Delta\omega^2-\omega_{s-}^{t2})+2\delta(\Delta\omega^2-s_t^2q^2)\}\frac{do'\,d\omega'}{(4\pi)^2}. \qquad (41)$$

In the case of anisotropy of the easy-axis type, for $\Delta\omega \sim \omega_1(q) \sim s_t q$ we have

$$d\Sigma = \frac{1}{8}|N_{\Delta\omega}+1|$$
$$\times\left(\frac{\omega}{c}\right)^6\varepsilon\hbar\rho_0^{-1}\sigma_1^2\{[\sin^2\vartheta-2(1-\cos\vartheta)k^{-2}k_y^2]\cdot$$
$$\times[\delta(\Delta\omega^2-\omega_{1+}^{t2})+\delta(\Delta\omega^2-\omega_{1-}^{t2})]$$
$$+4(1-\cos\vartheta)k^{-2}k_y^2\delta(\Delta\omega^2-s_t^2q^2)\}\frac{do'd\omega'}{(4\pi)^2}; \qquad (42)$$

for $\Delta\omega \sim \omega_2(q) \sim s_t q$ we find

$$d\Sigma = \frac{1}{4}|N_{\Delta\omega}+1|$$
$$\times\left(\frac{\omega}{c}\right)^6\varepsilon\hbar\rho_0^{-1}\sigma_1^2\{[\sin^2\vartheta-2(1-\cos\vartheta)k^{-2}k_y^2]$$
$$\times\delta(\Delta\omega^2-s_t^2q^2)+2(1-\cos\vartheta)k^{-2}k_y^2[\delta(\Delta\omega^2-\omega_{2+}^{t2})$$
$$+\delta(\Delta\omega^2-\omega_{2-}^{t2})]\}\frac{do'\,d\omega'}{(4\pi)^2}. \qquad (42')$$

On comparing these formulas with (30), we see that each of the transverse-sound satellites splits into three lines upon approach to a point of transverse resonance.

We remark that the relations (41) and (42) are correct if the damping of the magnetoelastic waves is small ($\gamma < \zeta^{1/2}\Delta\omega$). If $\gamma \gtrsim \zeta^{1/2}\Delta\omega$, then the value of $d\Sigma$ is determined by the same formula (30) as in the nonresonant range.

We shall now discuss briefly the conditions for observation of magnetoacoustic resonance through the angular and spectral distribution of radiation scattered in the antiferromagnet. In order that the phenomenon of magnetoacoustic resonance should manifest itself in the scattering of electromagnetic waves, it is necessary that the angle of scattering ϑ be close to the angle ϑ_0 determined from the equation $\omega(q) = sq$ ($\omega(q) \equiv \omega_1, \omega_2, \omega_s$). The change of character of the distribution of the scattered radiation occurs in a narrow range of angles $\vartheta = \vartheta_0\{1 + O(\zeta^{1/2})\}$ and of frequency changes

$$|\Delta\omega| = 2\frac{\omega s}{c}\sqrt{\varepsilon}\sin\frac{\vartheta}{2}\{1+O(\zeta^{1/2})\};$$

detection of this effect therefore requires a resolution in scattering angle of order $\vartheta_0\zeta^{1/2}$, and in frequency of order $|\Delta\omega|\zeta^{1/2}$.

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