# INFLUENCE OF INTERELECTRON SCATTERING ON THE GALVANOMAGNETIC EFFECTS IN STRONG ELECTRIC FIELDS

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The influence of interelectron scattering on the galvanomagnetic effects in strong electric fields is investigated under conditions of inelastic scattering on optical phonons at low temperatures $T \ll \omega_0$ ($\omega_0$ is the optical phonon frequency). A displaced Maxwellian distribution is employed for describing the electron distribution. The temperature $T^*$ and drift velocity u of the distribution are determined from the energy and momentum balance equations. Inelasticity of scattering results in a large anisotropy of the distribution, i.e., $mu^2 \gg T^*$. It is shown that "disruption" of dissipative effects occurring for $H/E = 2c/v_0$ ($v_0$ is the electron velocity for an energy $\omega = \epsilon_0$) takes place irrespective of the efficiency of the interelectron collisions. However the singularity of the galvanomagnetic characteristics for this critical value of the ratio H/E is weakened by interelectron collisions. The effect of interelectron collisions is particularly pronounced under conditions of open-circuited Hall contacts.

## 1. INTRODUCTION

GALVANOMAGNETIC effects in strong electron fields under conditions of inelastic scattering of electrons by optical phonons have been studied in [1] and [2]. At low temperatures $T \ll \omega_0$, where $\omega_0$ is the frequency of the optical phonons, it is convenient to divide momentum space into two regions—an active region in which $\epsilon(\mathbf{p}) < \omega_0$ and a passive region in which $\epsilon(\mathbf{p}) > \omega_0$. In the active region the frequency of scattering by the lattice is determined by the spontaneous emission time of the phonon $\tau^+$, and in the passive region—by the induced absorption time of the phonon $\tau^-$. In the temperature interval under consideration, these times differ appreciably:

$$\tau^+ \approx N_0\tau^-, \quad N_0 = e^{-\omega_0/T} \ll 1. \tag{1.1}$$

There appear therefore two characteristic fields $E^\pm$ given by the relations

$$eE^\pm\tau^\pm = p_0, \quad p_0 = \sqrt{2m\omega_0}. \tag{1.2}$$

In the range of fields

$$E^- \ll E \ll E^+ \tag{1.3}$$

the electron has time between two acts of phonon absorption to cover the entire section of the trajectory in the passive region; however, having reached its boundary, i.e., having attained an energy $\epsilon = \omega_0$, it emits a phonon and stops almost instantaneously. Owing to the fact that in the passive region the dynamics of the electron is not "reduced" by collisions with the lattice, the nature of the trajectories in the passive region has a decisive effect on the electron distribution and the galvanomagnetic effects. Therefore the gauss-ampere characteristic (the dependence of the current j on the magnetic field H for a fixed electric field E) has singularities at two values of H at which the topology of the electron trajectories in the passive region changes.[2]

In this paper we study the effect of interelectron collisions on these singularities of the gauss-ampere characteristics. To this end, we consider the opposite limiting case when the interelectron collisions fully maxwellize the distribution both in the passive and in the active region; the essential criteria for this have been discussed in [3].

Restricting ourselves for simplicity to an isotropic model, we have the following Maxwell distribution (normalized to unity):

$$f(\mathbf{p}) = (2\pi mT^*)^{-3/2} \exp\left\{-\frac{(\mathbf{p} - m\mathbf{u})^2}{2mT^*}\right\}. \tag{1.4}$$

The parameters of this distribution—the electron temperature $T^*$ and the drift velocity u—are obtained from the equations of the energy and momentum balance.[4]

## 2. BALANCE EQUATIONS

The balance equations of the energy and momentum are of the following form:

$$e\mathbf{E}\mathbf{u} = Q, \tag{2.1a}$$

$$e\mathbf{E} + \frac{e}{c}[\mathbf{u}\mathbf{H}] = \mathbf{N}. \tag{2.1b}$$*

We have here on the left-hand side the energy and momentum acquired by the electron system from the field in unit time, and on the left the energy and momentum lost in unit time in collisions with the lattice (calculated per electron). It is convenient to calculate the quantities on the right-hand side as follows. One introduces the "powers" of energy and momentum loss by a test electron with momentum $\mathbf{p}$ [5]:

$$Q(\mathbf{p}) = \int d\mathbf{p}'\, W(\mathbf{p}, \mathbf{p}')\,[\varepsilon(\mathbf{p}) - \varepsilon(\mathbf{p}')], \tag{2.2a}$$

$$\mathbf{N}(\mathbf{p}) = \int d\mathbf{p}' W(\mathbf{p}, \mathbf{p}')[\mathbf{p} - \mathbf{p}']. \tag{2.2b}$$

Then the right-hand parts of (2.1) are calculated as averages of the quantities (2.2) over the distribution (1.4):

$$Q = \int d\mathbf{p}\, f(\mathbf{p}) Q(\mathbf{p}), \quad \mathbf{N} = \int d\mathbf{p}\, f(\mathbf{p})\mathbf{N}(\mathbf{p}). \tag{2.3}$$

It is convenient to represent W in the form of a sum of components $W^+$ and $W^-$ which correspond to emission and absorption of a phonon. Then the quantities (2.2), and as a result also (2.3), will also split up into the sum of the corresponding terms. At the same time the energy difference in (2.2a) which is equal to $\pm\omega_0$ is taken out from under the integral. The resultant quantities

$$\frac{1}{\tau_0^{\pm}(\mathbf{p})} = \int d\mathbf{p}'\, W^{\pm}(\mathbf{p}, \mathbf{p}') \tag{2.4}$$

are the lifetimes of the state $\mathbf{p}$ with respect to emission and absorption of a phonon.[5] Because of the isotropy of the model it follows from symmetry considerations that

$$\tau_0^{\pm}(\mathbf{p}) = \tau_0^{\pm}(\varepsilon), \quad \mathbf{N}^{\pm}(\mathbf{p}) = \mathbf{p}/\tau^{\pm}(\varepsilon), \tag{2.5}$$

where $\tau^{\pm}$ are the corresponding relaxation times of the momentum of the test particle.

Relations (2.5) make it possible to integrate in (2.3) over the angles; as a result of this one finds for the right-hand parts of (2.1) the final expressions:

$$Q = Q^+ - Q^-, \tag{2.6a}$$

$$\mathbf{N} = \mathbf{N}^+ + \mathbf{N}^-, \quad \mathbf{N}^{\pm} \| \mathbf{u}, \tag{2.6b}$$

where

$$Q^{\pm} = \int_{\omega_0,0}^{\infty} d\varepsilon\, g(\varepsilon)\, f_0(\varepsilon)\, \frac{\omega_0}{\tau_0^{\pm}(\varepsilon)},$$

$$N^{\pm} = \int_{\omega_0,0} d\varepsilon\, g(\varepsilon)\, \frac{1}{3} f_1(\varepsilon)\, \frac{p(\varepsilon)}{\tau^{\pm}(\varepsilon)}. \tag{2.7}$$

Here the integration is carried out from $\epsilon = \omega_0$ for phonon emission and from $\epsilon = 0$ for absorption; $g(\epsilon)$ is the density of states.

The functions $f_0$ and $f_1$ are the coefficients of the expansion of the distribution (1.4) in Legendre polynomials of the angle between $\mathbf{p}$ and $\mathbf{u}$ which appear on integration over the angles. They are calculated by elementary integration and for what follows below it is convenient to write them by introducing the following dimensionless parameters:

$$y = \frac{\varepsilon}{\omega_0}, \quad b = \frac{\omega_0}{T^*}, \quad a = \frac{u}{v_0} = \left[\frac{mu^2/2}{\omega_0}\right]^{1/2}. \tag{2.8}$$

We then have

$$f_0(\varepsilon) = C \exp\{-b(y + a^2)\}\, S(2ab\sqrt{y}), \tag{2.9a}$$

$$^1/_3 f_1(\varepsilon) = C \exp\{-b(y + a^2)\} S'(2ab\sqrt{y}), \tag{2.9b}$$

where

$$C = \frac{1}{p_0^3}\left(\frac{b}{\pi}\right)^{3/2}, \quad S(x) = \frac{1}{x}\operatorname{sh} x \tag{2.10}$$

and the prime denotes differentiation over the inner argument.

Explicit expressions for the times $\tau_0^{\pm}$ and $\tau^{\pm}$ have been obtained in [6]. We shall write them in the following form

$$\frac{1}{\tau_0^{\pm}(\varepsilon)} = \frac{1}{\tau_0}\left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right)\varphi^{\pm}(\sqrt{y}), \tag{2.11a}$$

$$\frac{1}{\tau_0^{\pm}(\varepsilon)} = \frac{1}{\tau_0}\left(N_0 + \frac{1}{2} \pm \frac{1}{2}\right)\psi^{\pm}(\sqrt{y}). \tag{2.11b}$$

Here $\tau_0$ is a constant of the order of the spontaneous emission time of a phonon; the functions $\varphi^{\pm}$ and $\psi^{\pm}$ are different for deformation (DO) and polarization (PO) interaction of electrons with optical phonons:

$$\begin{aligned} &DO: \quad \varphi^{\pm}(\sqrt{y}) = \psi^{\pm}(\sqrt{y}) = (y \mp 1)^{1/2}; \\ &PO: \quad \varphi^{+}(\sqrt{y}) = y^{-1/2}\operatorname{Arch} y^{1/2}, \\ &\varphi^{-}(\sqrt{y}) = y^{-1/2}\operatorname{Arsh} y^{1/2}, \\ &\psi^{\pm}(\sqrt{y}) = (2y)^{-1}[(y \mp 1)^{1/2} \pm \varphi^{\pm}(\sqrt{y})]. \end{aligned} \tag{2.12}$$

Expressions having no meaning in the real region should be considered to vanish.

The values of Q and N have been calculated both for DO [7] and for PO,[8] however only for small anisotropy of the distribution when $^1/_2\, mu^2/T^* = a^2 b \ll 1$. We note in this connection that such a distribution is for inelastic scattering only valid for weak heating $T^* - T \ll T$ which corresponds to $E \ll E^-$. The range of fields (1.3)

*$[\mathbf{u}\mathbf{H}] = \mathbf{u} \times \mathbf{H}$.

of interest to us corresponds, as can be shown, to $a \approx 1$ and $b \gg 1$, i.e., $a^2 b \gg 1$. This enables one to calculate the integrals (2.7) by the method of steepest descents with respect to the parameter b. In (2.9a) one can replace sinh x by $\frac{1}{2}\exp x$, and in (2.9b) one can differentiate with respect to x only the rapidly changing exponent. We then have under the integral sign, upon integrating with respect to y,

$$\exp\{-b(\sqrt{y}-a)^2\}, \tag{2.13}$$

with the saddle point at $y = a^2$ and with a characteristic length of the change $\Delta y \approx b^{-1/2} \ll 1$, whereas the remaining multipliers in the integrand change over a length $\Delta y \approx 1$. For $N^-$ and $Q^-$ the integration interval from $y = 0$ to $y = \infty$ always contains a saddlepoint; for $N^+$ and $Q^+$ the saddlepoint falls in the integration interval from $y = 1$ to $y = \infty$ only when $a > 1$. Therefore when $a < 1$ only the neighborhood of the point $y = 1$ contributes to $N^+$ and $Q^+$; as can be seen from (2.12), in this neighborhood $\varphi^+$ and $\psi^+$ are the same for DO and PO.

Allowing for all these remarks, we find

$$Q = \frac{\omega_0}{\tau_0}\Phi(a,b), \quad N = \frac{p_0}{\tau_0}\Psi(a,b), \tag{2.14}$$

where

$$\Phi(a,b) = \Phi^+(a,b) - N_0\varphi^-(a),$$
$$\Psi(a,b) = \Psi^+(a,b) + N_0 a\psi^-(a). \tag{2.15}$$

Here the terms connected with the emission of phonons have in accordance with the location of the saddle point different forms:

$$\Phi^+(a,b) = \Psi^+(a,b) = \chi(a,b)$$
$$\equiv \tfrac{1}{4}a^{-1}(1-a)^{-3/2}b^{-1}\exp\{-b(1-a^2)\}, \quad a<1; \tag{2.16}$$

$$\Phi^+(a,b) = \varphi^+(a), \quad \Psi^+(a,b) = a\psi^+(a), \quad a>1. \tag{2.17}$$

## 3. SOLUTION OF THE BALANCE EQUATIONS

Proceeding to the solution of the balance equations, we introduce the characteristic field $E_0$ and dimensionless parameters with the aid of the following relations

$$eE_0\tau_0 = p_0, \quad \omega_H = eH/mc,$$
$$\xi = \frac{E}{E_0}, \quad \zeta = \omega_H\tau_0, \quad \varkappa = \frac{\zeta}{2\xi} = \frac{H}{E}\frac{v_0}{2c}. \tag{3.1}$$

One can readily see that

$$\tau^+ \approx \tau_0, \quad \tau^- \approx N_0^{-1}\tau_0, \quad E^+ \approx E_0, \quad E^- \approx N_0 E_0, \tag{3.2}$$

so that instead of (1.3) we have

$$N_0 \ll \xi \ll 1. \tag{3.3}$$

From (2.1) there follows a system of equations for a and b:

$$\Phi(a,b) = 2a\Psi(a,b), \tag{3.4a}$$
$$\xi^2 = \zeta^2 a^2 + \Psi(a,b)^2. \tag{3.4b}$$

After solving this system one finds the Hall angle $\vartheta$ between E and u:

$$\cos\vartheta = \xi^{-1}\Psi(a,b). \tag{3.5}$$

Equation (3.4a) is obtained after scalar multiplication of (2.1b) by u and use of Eq. (2.1a). To obtain (3.4b) one must transfer in (2.16) the Lorentz term to the right-hand side and square it. Equation (3.5) is obtained from (2.1a).

We note above all that in the range of fields (1.3) the system (3.4) has no solutions for $a - 1 \approx 1$ or $a - 1 \gg 1$, i.e., for u which exceeds $v_0$ appreciably. From (2.15) and (2.17) it follows that for such values of a we have $\Psi \approx 1$ or $\Psi \gg 1$; it follows therefore from (3.4b) that $\xi \approx 1$ or $\xi \gg 1$ in contradiction to (3.3). For this reason we shall assume below that $a < 1$; it is readily seen that in conjunction with $b \gg 1$ this means that the main bulk of electrons is in the passive region in accordance with the results obtained in [2] without account of interelectron scattering. Assuming $a < 1$, we leave out a small interval of values of this parameter $a - 1 \ll 1$. For such a close to $a = 1$ we have, as is seen from (2.15) and (2.12), $\Psi \ll 1$. It follows therefore from (3.4b) that such a are only possible in a small interval of values $\kappa$ on the left of $\kappa = \frac{1}{2}$. As will become clear from the following, this interval is of no particular interest.

Using (2.16), we obtain for $a < 1$, instead of (3.4), the following system:

$$\chi(a,b)(1-2a) = N_0[2a^2\psi^-(a) + \varphi^-(a)], \tag{3.6a}$$

$$\xi^2 - \zeta^2 a^2 - \chi(a,b)^2 = 2N_0\chi(a,b)a\psi^-(a) + N_0^2 a^2\psi^-(a)^2. \tag{3.6b}$$

Let us first consider the lattice at absolute zero, i.e., $T = 0$ or $N_0 = 0$. Then we have from (3.6a) $a = \frac{1}{2}$ or $\chi(a, b) = 0$. One can see from (2.16) that the latter equality is satisfied for $b = \infty$. From (3.6b) we find that $a = \frac{1}{2}\kappa$ corresponds to the root $b = \infty$ and

$$b = 4L(2^{-3/2}\xi^{-1}(1-\varkappa^2)^{-1/2}) \tag{3.7}$$

corresponds to the root $a = \frac{1}{2}$, where $x = L(z)$ is the solution of the equation $x \exp x = z$. According to (3.3) we have $z \gg 1$ and consequently $b \gg 1$ which justifies the assumption made in calculating the integrals. Approximately,

$$L(z) = \ln \frac{z}{\ln \frac{z}{\ln \frac{z}{\dots}}}, \quad z \gg 1 \tag{3.8}$$

We shall restrict ourselves to the first approximation $L(z) = \ln z$, leaving out under the logarithm sign factors of the order of unity compared with larger factors.

There are thus at zero lattice temperature two branches of the solution of the balance equations: a dissipative branch for $\kappa < 1$:

$$u = {}^1/_2 v_0, \quad T^* = \frac{\omega_0}{\ln[\xi^{-4}(1-\varkappa^2)^{-2}]}, \quad \vartheta = \arcsin \varkappa; \tag{3.9}$$

and a nondissipative for $\kappa > {}^1/_2$:

$$u = cE/H, \quad T^* = 0, \quad \vartheta = \pi/2. \tag{3.10}$$

The limitation $\kappa > {}^1/_2$ for the nondissipative solution is obtained from $a < 1$. The limitation $\kappa < {}^1/_2$ for the dissipative solution follows from (3.4b); if we substitute here $a = {}^1/_2$, we find $\xi^2 > \zeta^2/4$, i.e., $\kappa < 1$. In the region ${}^1/_2 < \kappa < 1$ there exist solutions connected with both branches.

We now proceed to consider the case $T \neq 0$ when there is scattering in the passive region. Then it follows from (3.6a) that $a < {}^1/_2$, i.e., the nondissipative solution in the interval ${}^1/_2 < \kappa < 1$ vanishes. In other words, the nondissipative solution of the balance equations is unstable with respect to weak scattering in the passive region. It is interesting to calculate the corrections to the solutions of the balance equations for the nondissipative and for the dissipative branch near the point at which the latter joins the nondissipative branch at $\kappa = 1$. Leaving out numerical factors of the order of unity, we have: far from the junction point, where $\kappa - 1 \gg (N_0/\xi)^{2/3}$,

$$a - 1/2\varkappa \approx -(N_0/\xi)^2 \varkappa (\varkappa - 1)^{-2}, \tag{3.11a}$$

$$b = (1 - 1/2\varkappa)^{-2} \ln[N_0^{-1}(\varkappa - 1)], \tag{3.11b}$$

$$\cos\vartheta \approx (N_0/\xi)\varkappa(\varkappa - 1)^{-1}; \tag{3.11c}$$

near the junction point, where $|\kappa - 1| \gg (N_0/\xi)^{2/3}$,

$$a - {}^1/_2 \approx -(N_0/\xi)^{2/3}, \tag{3.12a}$$

$$b = \ln[N_0^{-4/3}\xi^{-8/3}], \tag{3.12b}$$

$$\cos\vartheta \approx (N_0/\xi)^{1/3}. \tag{3.12c}$$

As is seen from a comparison of (3.11) and (3.12), the largest effect of scattering is in the passive region in the neighborhood of the point $\kappa = 1$.

## 4. PHYSICAL MEANING OF THE VARIOUS BRANCHES OF THE SOLUTION. "COOLING" OF THE ELECTRON GAS

Let us compare the solutions (3.9) and (3.10) with the results of the treatment without allowance for interelectron scattering in [2]. For the nondissipative solution the velocity u is the velocity of the magnetic drift; $T^* = 0$ means that all the electrons are in the passive region and do not interact with the phonons. It is readily seen that this solution corresponds to closed trajectories on which the electrons have the same average velocity and do not emit phonons. The dissipative solution corresponds to half a unclosed main trajectory. The velocity u of this solution corresponds to the center of the chord joining the beginning and end of that section of the trajectory, and differs slightly from the average electron velocity on this trajectory corresponding to the center of gravity of the arc. For this solution $T^* \neq 0$, i.e., a portion of the electrons is in the active region transferring energy to the lattice. When the number of solutions changes, the points $\kappa = {}^1/_2$ and $\kappa = 1$ correspond to values of the magnetic field for which the topology of the trajectories changes: with increasing $\kappa$ closed trajectories appear in the passive region for $\kappa = {}^1/_2$, and for $\kappa = 1$ the main trajectory closes. The instability of the nondissipative solution with respect to scattering in the passive region corresponds to the circumstance that the electron removed by such scattering from a closed secondary trajectory does not return to it, whereas that removed from the main trajectory returns to it after the emission of a phonon.

We note that the nonuniqueness of the solutions of the balance equations also takes place for the case of runaway [9,10] when an extremum of the energy distribution function at the point $\epsilon_s$ corresponds to each value of $T^*$; in this case $\epsilon_s \approx T^*$ and stable solutions for $T^*$ correspond to maxima of the distribution and unstable ones—to minima. The same points $\epsilon_s$ can be defined as the values $\epsilon$ of the test electron for which the energy coming from the field is equal to the energy transferred to the lattice. One can apparently draw the following conclusion: whereas in momentum space there are several regions in which a balance is observed between the action of the field and the scattering, interelectron collisions can maxwellize the distribution near each of these "regions." In the case of runaway these regions are the equal-energy surfaces $\epsilon(p) = \epsilon_s$, whereas in the case considered here it is half the main

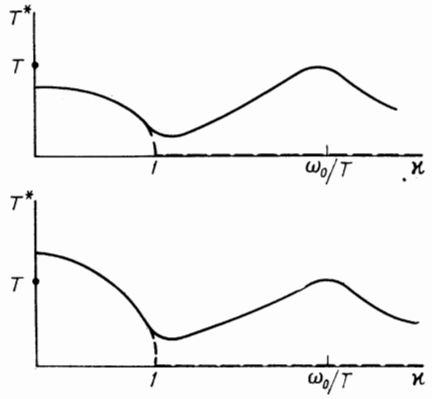


FIG. 1. Dependence of the electron temperature on the magnetic field. The dashed curve shows the dependence at T = 0.

trajectory and the region occupied by closed trajectories.

Let us now discuss the connection between the obtained results with the so-called "cooling effect of the electron gas" observed in [11] and [12] where it was shown that under given conditions one can obtain $T^* < T$ in an electric field (for H = 0). For $\kappa = 0$ it follows from (3.9) that $T^* = \omega_0/\ln \xi^{-4}$, whence one obtains $T^* < T$ for the condition $\xi < N_0^{1/4}$ consistent with (3.3). At the limit of applicability of our results for $\xi \approx N_0$ we have $T^* = T/4$. We note that the conditions for which $T^* < T$ was obtained in [12] are strongly reminiscent of the assumptions under which (3.9) was obtained. The conditions $T \lesssim \omega_0$ and $T \lesssim ms^2$ (for acoustic scattering, s is the speed of sound) obtained in [12] signify in fact [5] that the scattering is sufficiently inelastic, whereas the necessity of expanding in $a^2b$ up to and including the second power [12] means that the effect is connected with an appreciable scattering anisotropy. However, in [12] there is no limitation on the field from below; implicit use is made instead of a limitation on the field from above which is much stronger than that in (1.3).

For $T \neq 0$ the dependence of T* on H/E is nonmonotonic; this follows from (3.9), (3.11b), and (3.12b). The $T^*(\kappa)$ dependence is shown in Fig. 1; the upper part refers to the case $\xi < N_0^{1/4}$, the lower to the case $\xi > N_0^{1/4}$. T* reaches a minimum at the point at which the branches join, i.e., for $\kappa \approx 1$; the value of $T^*_{min}$ is obtained from (3.12b). The maximum is reached for $\kappa \simeq \omega_0/T$; $T^*_{max} \approx T$ if one neglects $\ln(\omega_0/T)$ compared with $\omega_0/T$. In the region $\kappa - 1 \gg (N_0/\xi)^{2/3}$, T* does not depend on E.

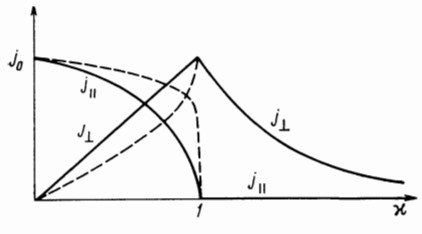


FIG. 2. Gauss-ampere characteristics of the longitudinal and transverse currents (with respect to the field). The dashed curves show the characteristics in the absence of interelectron scattering.

## 5. GALVANOMAGNETIC EFFECTS AND GENERATION OF OPTICAL PHONONS

With the aid of (3.9) and (3.10) one can readily calculate the components of the current j parallel and perpendicular to the field E for T = 0. Retaining for $\frac{1}{2} < \kappa < 1$ only the stable dissipative branch, we have

$$j_{\parallel} = \begin{cases} j_0(1-\kappa^2)^{1/2}, & \kappa < 1 \\ 0, & \kappa > 1, \end{cases} \tag{5.1}$$

$$j_{\perp} = \begin{cases} j_0\kappa, & \kappa < 1 \\ j_0\kappa^{-1}, & \kappa > 1, \end{cases} \tag{5.2}$$

where $j_0 = \frac{1}{2} env_0$ is the current in the absence of a magnetic field, and n is the electron concentration.

The corresponding graphs are shown in Fig. 2 where the dashed lines indicate for comparison the currents calculated without allowance for interelectron scattering. It is seen that the main effect—the disruption of dissipative processes for $\kappa = 1$—occurs independently of the frequency of interelectron scattering. However, the singularity at $\kappa = 1$ becomes weaker: 1) the finite discontinuity of the function $j_{\parallel}(\kappa)$ is replaced by an infinite discontinuity of the derivative $\partial j_{\parallel}(\kappa)/\partial\kappa$; 2) the infinite discontinuity of the derivative $\partial j_{\perp}(\kappa)/\partial\kappa$ is replaced by a finite discontinuity.

Allowance for the scattering in the passive region changes the gauss-ampere characteristics appreciably only near $\kappa = 1$ and for $\kappa > 1$. The corrections are easily found with the aid of (3.11) and (3.12). The singularity is smoothed out over the interval

$$\Delta\kappa \approx (N_0/\xi)^{2/3} \approx (E^-/E)^{2/3} \approx (\omega_H\tau^-)^{-2/3} \tag{5.3}$$

On increasing $\kappa$ to $\kappa = 1$ the current $j_{\parallel}$ does not decrease to zero but to a value of the order of $j_0(\omega_H\tau^-)^{-1/3}$; on further increase of $\kappa$ to $(\kappa - 1) \approx 1$ it decreases to a value of the order of $j_0(\omega_H\tau^-)^{-1}$.

The not very sharp difference between the gauss-ampere characteristics with no interelectron scattering and when it predominates is enhanced if one changes the boundary conditions of

the experiment turning from the regime of a given field which was considered above to one of given current (corresponding to closed or open Hall constants—infinite sample and platelet). In this case one fixes in the experiment not the total **E** but only $E_{||}$ along **j** and one measures the total current $j^2 = j_{||}^2 + j_\perp^2$. The dependence of **j** on $\kappa_{||} = Hv_0/2E_{||}c$ is therefore of interest. This dependence is given by the relations

$$j^2 = j_{||}^2(\varkappa) + j_\perp^2(\varkappa), \quad \varkappa = \varkappa_{||}\cos\vartheta(\varkappa), \qquad \operatorname{tg}\vartheta(\varkappa) = j_\perp(\varkappa)/j_{||}(\varkappa). \tag{5.4}$$

Since $j_\perp$ is a continuous function of $\kappa$, and therefore also of $\kappa_{||}$, then at that value of $\kappa_{||}$ which corresponds to $\kappa = 1$ the total current decreases sharply.

Using (5.1) and (5.2), as well as (3.1),[1] we find, respectively, when interelectron scattering predominates:

$$\varkappa_{||}^2 = \varkappa^2/(1-\varkappa^2), \tag{5.5}$$

and with no interelectron scattering:

$$\varkappa_{||}^2 = 1 + \frac{\arcsin\varkappa}{\varkappa}\left[\frac{\arcsin\varkappa}{\varkappa} - 2\sqrt{1-\varkappa^2}\right]. \tag{5.6}$$

With increasing $\kappa$ the quantity $\kappa_{||}$ increases, $\kappa = \infty$ and $\kappa_{||} = (1+\pi^2/4)^{1/2}$ corresponding to the value $\kappa = 1$. In other words, in the regime of a given current the interelectron scattering increases strongly the value of the magnetic field at which the disruption of dissipative processes occurs. The reason for this can be understood if one compares the behavior of $\tan\vartheta(\kappa)$ for $\kappa \to 1$ in both cases. With interelectron scattering $\tan\vartheta \to \infty$ for $\kappa \to 1$, whereas with no interelectron scattering $\tan\vartheta \to \pi/2$. Therefore in the first case on approaching $\kappa = 1$ the Hall field $E_\perp$, and with it the total field E, increase without limit, decreasing thereby $\kappa$ (the Hall field acts as a feedback). In the second case E is bounded and $\kappa = 1$ can be reached, albeit with a slightly larger, by a factor of $(1+\pi^2/4)^{1/2}$, magnetic field.

For open Hall contacts the nature of the dependence of T* on the magnetic field also changes. Under these conditions the critical point $\kappa = 1$ corresponds to $\kappa_{||} = \infty$. The minimum in the dependence of T* on $\kappa_{||}$ is therefore not reached and the interval of monotonic decrease from $\kappa = 0$ to $\kappa = 1$ extends over the entire axis from $\kappa_{||} = 0$ to $\kappa_{||} = \infty$.

The disruption of the dissipative processes can be observed on the generation of optical phonons. The number of phonons with momentum **q** generated in unit time is

$$dI(\mathbf{q}) = n\,d\mathbf{q}\int\limits_{\varepsilon>\omega_0} f(\mathbf{p})W^+(\mathbf{p},\mathbf{p}-\mathbf{q})\,d\mathbf{p}. \tag{5.7}$$

Utilizing the distribution (1.4), and omitting constants and pre-exponential factors, we find the spectral density of the generated phonons

$$dI(\mathbf{q}) = \exp\left\{-b\left[\frac{1}{4}\sin^2\alpha + \frac{1}{4}\left(\frac{q}{p_0}+\frac{p_0}{q}\right)^2 - \frac{1}{2}\cos\alpha\left(\frac{q}{p_0}+\frac{p_0}{q}\right)\right]\right\} d\mathbf{q}, \tag{5.8}$$

where $\alpha$ is the angle between **q** and **u**. For fixed q this function has a maximum in $\alpha$ for $\alpha = 0$, and for fixed $\alpha$ a maximum in q for $q = p_0$. This means that the basic characteristics of the phonon emission are the same as without interelectron scattering. On going away from the point of the maximum the spectral density decreases exponentially; the characteristic widths of the peak are

$$\Delta q \approx p_0 b^{-1/2} \ll p_0, \quad \Delta\alpha \approx b^{-1/2} \ll 1, \tag{5.9}$$

i.e., the phonon emission is monochromatic to a high degree.

The monochromaticity in the absence of interelectron scattering can be estimated from Fig. 1 of[1] in the following way: $\Delta q \approx \Delta^+ p + \Delta^- p$ and $\Delta\alpha \approx \Delta^- p/p_0$ where $\Delta^+ p$ is the penetration depth of electrons into the active region, and $\Delta^- p$ is the radius of the sphere into which they return after emission of a phonon. From energy conservation and (2.19)[1] we have:

$$v_0\Delta^+ p = \frac{1}{2m}(\Delta^- p)^2, \quad \Delta^+ p \approx p_0\left(\frac{E}{E_0}\right)^{2/3}. \tag{5.10}$$

As a result we find

$$\Delta q \approx p_0\xi^{1/3} \ll p_0, \quad \Delta\alpha \approx \xi^{1/3} \ll 1. \tag{5.11}$$

Comparing (5.11) and (5.9), in which b has been substituted in accordance with (3.9), we see that consistent with the weakening of all singularities the interelectron scattering decreases the monochromaticity. The total number of generated phonons can be found in the following obvious way:

$$I = \int dI(\mathbf{q}) = \frac{nQ}{\omega_0} = \frac{E}{\omega_0} j_{||}. \tag{5.12}$$

In conclusion we note that qualitatively all the results obtained above are also valid in the case when the scattering in the passive region is of a different nature, when it is connected with impurities and acoustic phonons; the meaning of $\tau^-$ changes accordingly, but it is essential that, as before, $\tau^- \gg \tau^+$.

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