PONDEROMOTIVE EFFECTS OF ELECTROMAGNETIC RADIATION

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An additional mechanical friction, which is commensurate with the friction produced by a high-vacuum gas, arises during mechanical vibrations of a body absorbing or reflecting electromagnetic radiation. It is shown that a similar but appreciably larger effect should be observable in an optical Fabry-Perot resonator. The sign of the effect may be reversed at different settings of the resonator mirror. When the optical power fed to the resonator is sufficiently large, vibrational instability of the mirrors may set in. A possible use of the derived relations in astrophysical estimates is indicated.

THE pondermotive action of electromagnetic radiation is extensively used in experimental techniques, especially for the absolute measurements of radiation power [1,2] and for satellite orientation [3]. Optical methods of measuring small mechanical displacements are among the best from the point of view of sensitivity. Optical levers that record mechanical displacements of $\sim 1 \times 10^{-12}$ cm have already been constructed [4], and it can be assumed that the attained stability of gas lasers will make measurements of displacements of $\sim 1 \times 10^{-13}$ cm feasible.[5] In high-precision physical experiments with test bodies (such as the experiments of Einstein and de Haas, Eotvos-Dieke, Millikan, etc.) it is important to know how strongly the test body is influenced by the optical system used to register the small displacements. An important factor is that, in principle, a very high degree of sensitivity can be attained in such experiments under conditions of weightlessness and when the friction between the test body and the laboratory is very low [6]. We consider in this paper two peculiarities of the ponderomotive action of electromagnetic radiation, which may turn out to be essential in experiments with trial bodies.

The light-radiation pressure experienced by a highly absorbing body depends on the velocity of its motion relative to the source:

$$F_{\text{em}} = \frac{W}{c}\left(1 + \frac{v}{c}\right) = \frac{W}{c} + \frac{Wv}{c^2}, \tag{1}$$

where W is the power incident on the stationary body, v its velocity relative to the source and in the radiation direction, and c the speed of light. Since the quantity $\Delta F_{\text{em}} = Wv/c^2$ depends on the velocity of the body relative to the source and is opposite in direction to v, we can write for the oscillatory motion of the body (in the source direction) $\Delta F_{\text{em}} = H_{\text{em}} v$, where

$$H_{\text{em}} = W/c^2 \tag{2}$$

as the meaning of an additional friction coefficient ("friction of light"). If the body is absolutely reflecting, then H_{em} is twice as large.

To explain the energy relations in this effect, let us imagine that a well reflecting plate, on which a power $W_0 = Nh\nu_0$ is incident, vibrates in the direction of the source, and that its velocity is v during one half-cycle and $-v$ during the other. Then the frequencies of the reflected photons during the respective half cycles are $\nu_0(1 + v/c)(1 - v/c)^{-1}$ and $\nu_0(1 - v/c)(1 + v/c)^{-1}$, and the power of the light reflected from the plate, averaged over the cycle, whose value at $v \ll c$ is

$$W_{\text{ref}} = \frac{1}{2}\left[Nh\nu_0\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right)^{-1} + Nh\nu_0\left(1 - \frac{v}{c}\right)\left(1 + \frac{v}{c}\right)^{-1}\right] \cong W_0 + \frac{2W_0 v^2}{c^2}, \tag{3}$$

will be larger than W_0. The additional power $2W_0 v^2/c^2$, which must be supplied by the extraneous source that causes the mechanical displacement of the plate, can be represented as a product of the friction force F_{fr} by the velocity v:

$$\Delta W = \frac{2W_0 v^2}{c^2} = F_{\text{fr}} v = H_{\text{em}} v^2, \tag{4}$$

where $H_{\text{em}} = 2W_0/c^2$, in accordance with (2).

The magnitude of this effect is small. For comparison we indicate that the friction produced by a rarefied gas H_{gas} has the same order of magnitude as H_{em} only in high vacuum. For a sphere of radius a we have [7]

$$H_{gas} = \frac{16}{3\sqrt{2}} \pi^{1/2} a^2 \mu^{1/2} (\varkappa T)^{1/2} n, \tag{5}$$

where n is the concentration of the gas molecules, μ the molecule mass, T the temperature, and κ Boltzmann's constant. In a hydrogen atmosphere at $p = 10^{-11}$ mm Hg, T = 100°K, and a = 1 cm, we have $H_{gas} = 3.5 \times 10^{-13}$ g/sec. At W = 10 W we have $H_{em} = 2 \times 10^{-13}$ g/sec for a highly reflecting surface. Thus, the optical level will add to the trial body a small supplementary damping H_{em}, which is comparable with gas friction only in high vacuum[1)].

We now consider the conditions under which another ponderomotive effect turns out to be sufficiently appreciable and can be observed under ordinary laboratory conditions. An optical Fabry-Perot resonator can be excited at the fundamental mode by a sufficiently monochromatic light source. The Q of such a resonator is [a]

$$Q_{opt} \cong 2\pi l/f\lambda, \tag{6}$$

where l is the distance between mirrors, λ the wavelength, and $(1 - f)$ the reflection coefficient. Relation (6) is valid when $f \ll 1$. Q_{opt} can become quite large (up to $Q_{opt} \sim 10^{10}$ [8]), since modern multilayer dielectric coatings have $f \approx 1 \times 10^{-2}$.

The light pressure experienced by the mirrors of a resonator tuned to resonance is $F_{mir} = 2W/f$, where W is the power fed to the resonator. The force F_{mir} changes greatly when one of the mirrors is slightly displaced, owing to the large value of Q_{opt}. This means that the presence of light pressure introduces additional mechanical rigidity K_{em} to the rigidity K_{mech} of the mirror mounting, and K_{em} can be either positive or negative, depending on the wing of the resonance curve on which the external-source wavelength falls.

Calculation of the maximum value of $K_{em} = \partial F_{mir}/\partial l$ under the same assumptions as Q_{opt} (relation (6)) yields the following expression for $(K_{em})_{max}$:

$$(K_{em})_{max} \cong \pm 4\pi W/cf^2\lambda. \tag{7}$$

The sign of $(K_{em})_{max}$ depends on whether the right or left slope of the resonance curve has been chosen. K_{em} is quite large and can greatly alter, in the case of small mechanical oscillations, the natural frequency of the mirror oscillation. When W = 300 mW, $\lambda = 6 \times 10^{-5}$ cm, and $f = 1 \times 10^{-2}$ we get $(K_{em})_{max} = \pm 2 \times 10^5$ dyne/cm.

An important factor in the subsequent reasoning is that when one of the mirrors is displaced, the light pressure on it changes not instantaneously but only after a time $\tau = l/cf$ equal to the time necessary for the oscillations in the resonator to settle (the time constant of the resonator). This means a similar delay in the effect of the rigidity K_{em} in the presence of mechanical vibrations of the mirror. Therefore the equation for the mechanical vibrations of the mirror (regarded as an oscillator with one degree of freedom) should be

$$m\ddot{x} + H_{mech}\dot{x} + [K_{mech} \pm K_{em}|_\tau] x = 0, \tag{8}$$

where m is the mirror mass, K_{mech} the rigidity of the mounting, and H_{mech} the friction in this mounting.

It is known that delay of positive rigidity leads to regeneration and of negative rigidity to degeneration [9]. The value of this additional friction, if $\tau \ll \sqrt{m/K_{mech}}$, is

$$(H_{em})_{max} = \pm (K_{em})_{max}\tau = \pm \frac{W}{c^2} \frac{4\pi l}{f^3\lambda}. \tag{9}$$

We have substituted $(K_{em})_{max}$ in (9). Thus, an effect similar to the "friction of light" W/c^2 but larger by a factor $4\pi l/f^3\lambda$, should take place in a Fabry-Perot resonator. It is important here that the sign of H_{em} can be either positive or negative. If $|-H_{em}| > H_{max}$, then the mirror oscillations will build up to a stationary value limited by the nonlinear $K_{em}(l)$ dependence. In this case the light flux emerging from the resonator will be modulated.

It is interesting to note that the presence of such a coupling between two vibrating systems so far apart in frequency (optical and mechanical) is connected formally with the presence of the nonlinear term $K_{em}|_\tau$, which contains a quantity W proportional to the square of the field amplitude in the optical resonator. A similar coupling, which has been investigated experimentally, takes place in radio devices used to register small mechanical vibrations [9]. We present estimates for conditions under which growing oscillations of the mirrors take place. If $l = 10^2$ cm, $\lambda = 6 \times 10^{-5}$ cm, $f = 1 \times 10^{-2}$, and W = 300 mW, then $(H_{em})_{max} = \pm 6 \times 10^{-2}$ g/sec. When m = 10 g and $\omega_0 = \sqrt{K_{mech}/m} = 2 \times 10^2$ sec^{-1} we need $Q_{mech} = 10^5$ to get $|H_{em}|_{max} = H_{mech}$.

In conclusion we note that in astrophysical estimates it is apparently meaningful to take into consideration the value of H_{em} in the case of a

1) We note that the effect under consideration, which should take place for vibrations of either a reflecting or an absorbing body, is similar to the Poynting-Robertson effect [10,11] (the loss of angular momentum by a solar satellite or planet moving on a circular orbit and absorbing solar radiation).

body vibrating in the direction of the source. Owing to this effect, the elliptical orbits of small asteroids should gradually become circular. For an asteroid of small mass 1 g and of area 1 cm^2, rotating in an elliptic orbit around the sun at a distance on the order of the earth-sun distance, the time constant is $\tau_0 = m/H_{em} = mc^2/2W \cong 2 \times 10^7$ years. During that time, the elliptic orbit should become essentially circular.

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