MEASUREMENT OF THE ANISOTROPY OF THE ENERGY GAP IN SUPERCONDUCTING INDIUM BY AN ULTRASONIC METHOD

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The temperature dependence of the absorption coefficient for longitudinal ultrasound was measured along the principal crystallographic axes at small sound-field amplitudes regime. The anisotropy of the energy gap $2\Delta(0)$ for the effective interaction zones and the sound absorption coefficients α_n in the normal state are determined on the basis of these measurements. The possible influence of the Fermi-surface topology of indium on the results of the measurements is discussed.

As is well known,[1,2] the ultrasonic method of measurement of the energy gap in superconductors gives a value which is to a certain extent averaged over the effective zone of interaction on the Fermi surface. Nevertheless, if the effective zone of interaction is sufficiently narrow, these data convey important information about the magnitude of the energy-gap anisotropy parameter and can throw light on the relation of the electronic structure of a metal to its superconducting characteristics.

In reality the angular size of the effective zone of interaction is determined by the factor $1/ql$ (q—sound wave vector, l—electron mean free path). It is obvious that the quantity ql must be sufficiently large for the anisotropy to be observable. Thus for the attainable metal purity the requisite ultrasonic frequencies begin at about 50 to 200 MHz.

There is also another limitation for the ultrasonic investigations of electronic absorption in superconductors. Recent experiments on the soft metals Pb,[3] In[4] and Tl[5] indicate that in the superconducting state, in addition to electronic absorption, there is a strong amplitude-dependent absorption due to the interaction of sound with dislocations. Consequently it is necessary to work with very small-amplitude sound fields. Small-amplitude work with the conventional pulse method[4] is extremely difficult because of the small signal-to-noise ratio and the associated substantial experimental errors.

In the present work an attempt to circumvent these difficulties was undertaken by the use of an altered measurement procedure which enabled us to reduce the voltage level on the quartz transducer down to tenths of a volt. Below we describe the results of an experimental investigation of the size of the energy gap in single-crystal indium with different crystallographic orientations. These measurements were performed with sound-field amplitudes for which nonlinear effects were not observed. All measurements were performed at a longitudinal sound frequency of 227 MHz in the temperature interval from 4.2 to 1°K.

EXPERIMENTAL METHOD AND PROPERTIES OF THE SAMPLES

Because of the aforementioned deficiencies of the pulse technique, these investigations on sound absorption in indium were carried out with apparatus which operated in the quasi-continuous mode with synchronous detection, using a high frequency bridge circuit. A block diagram of the arrangement is shown in Fig. 1. The oscillator 1 works in a pulsed-amplitude mode with a ratio of half periods of 1:1. The repetition frequency was 1000 Hz. By changing the setting of the decoupler 2, it is possible to vary within broad limits the amplitude

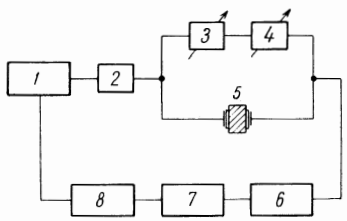

FIG. 1. Block diagram of the apparatus: 1–oscillator, 2–attenuator, 3–variable attenuator, 4–phase shifter, 5–sample, 6–receiver, 7–narrow-band amplifier, 8–synchronous detector. The reference voltage for the synchronous detector 8 is taken from the modulation unit of the oscillator 1.

of the radio-frequency signal without making significant changes in the conditions of operation of the oscillator.

The sample to be studied 5 is placed in one arm of the bridge. In the other arm there is a variable attenuator 3 and a phase shifter 4, which insure smooth and independent control of phase and amplitude. Any change of absorption or velocity of the ultrasound unbalances the bridge and can be determined by reading the attenuator and phase shifter. The receiver 6, tuned to the signal frequency, followed by the narrow band amplifier 7 and the synchronous detector 8, tuned to the repetition frequency, were used as the null indicator. This circuit makes it possible to work at quartz transducer voltages of tenths of a volt and simultaneously enhances the signal to noise ratio by two orders of magnitude in comparison with the pulse method. The voltage level at which amplitude-dependent effects were not observed changed from sample to sample. It fell in the range from 0.2 to 0.6 volts at a frequency of 227 MHz when operating at the ninth harmonic of the transducer.

The samples were prepared from pure indium with an impurity content not greater than 0.0005%. The method of preparation of the samples was similar to that employed earlier.[4] The samples had a cylindrical shape with a diameter of 11 mm and a thickness of 6 mm. This sample thickness permitted us to perform sufficiently detailed measurements down to a temperature of 1°K. Samples were studied for which the sound-wave vector was perpendicular, within 3°, to the planes (100), (001), (110), (011), (111), and (023).

An x-ray method was used to determine the orientation of the samples. However, the slight tetragonality of the indium lattice did not allow us to confidently distinguish in the x-ray photograph the orientations of type (100) and (001) or (110) and (011). For these cases a final judgment was made on the basis of the orientation of the planes of twinning, which for indium are planes of the type (011). An estimate of the magnitude of ql from magneto-acoustical data gives $ql \sim 20$ for the samples used.[1)]

RESULTS OF THE MEASUREMENTS

Figure 2 shows the temperature dependence of α_s/α_n obtained with the small radio-frequency voltages. For the sake of clarity, the curves in the figure are displaced one relative to the other. For

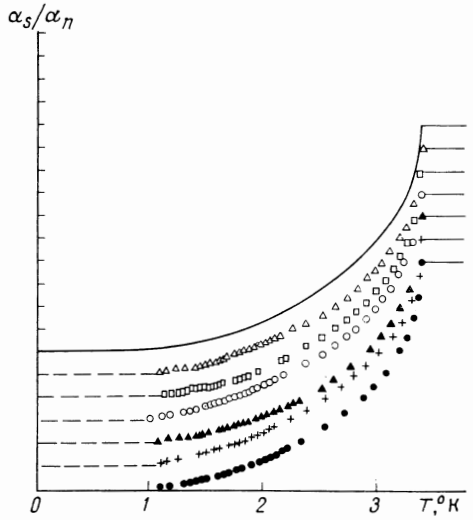

FIG. 2. Dependence of α_s/α_n on the temperature. The scale values on the vertical axis are 0.1. The solid curve is from the BCS theory. Δ — $\mathbf{q} \perp$ (023); □ — $\mathbf{q} \perp$ (111); ○ — $\mathbf{q} \perp$ (011); ▲ — $\mathbf{q} \perp$ (110); + — $\mathbf{q} \perp$ (001); ● — $\mathbf{q} \perp$ (100)

a reliable determination of the size of the energy gap, the measurements were carried out with a sufficient number of points in the range $\tau/\tau_c < 0.6$. It is evident from the figure that the difference between the experimental curves and theory is small. The critical temperature found from the sharp bend of the curves is $T_c = 3.407 \pm 0.002$°K.

The parameter $2\Delta(0)$ was ascertained from the following equation of the BCS theory:

$$\frac{\alpha_s}{\alpha_n} = 2 \cdot \left[\exp \frac{\Delta(T)}{kT} + 1 \right]^{-1}. \qquad (1)$$

This equation determines, in a complex manner, the magnitude of the energy gap, averaged over the effective zone of interaction, generally speaking, near the gap minimum at low temperatures.[1,2] Here $\Delta(T) = \Delta(0)f(T)$ is the temperature-dependent energy gap. The conventional procedure for finding $\Delta(0)$ from the experimental dependence of α_s/α_n on T consists of plotting the straight line $\ln \alpha_s = F(f(T)/T)$ for $T/T_c < 0.5$ and determining $\Delta(0)$ from the slope of the straight line. The function f(T) is assumed to be either equal to unity or is taken from BCS theory, which is more correct. However, this procedure makes the experimental data internally inconsistent.

The point is that the intercept of the straight line $\ln \alpha_s = F(f(T)/T)$ with the axis of the ordinates must be equal to $\ln 2\alpha_n$ according to Eq. (1). The value of α_n determined in this way is either significantly greater than the actual value measured experimentally, if we assume f(T) = 1, or smaller than the experimental value if we take f(T) from the BCS theory. Apparently in the actual case

[1)] gl was estimated from the relation $gl/n \sim 4$, [6] where n is the number of observed geometric-resonance oscillations.

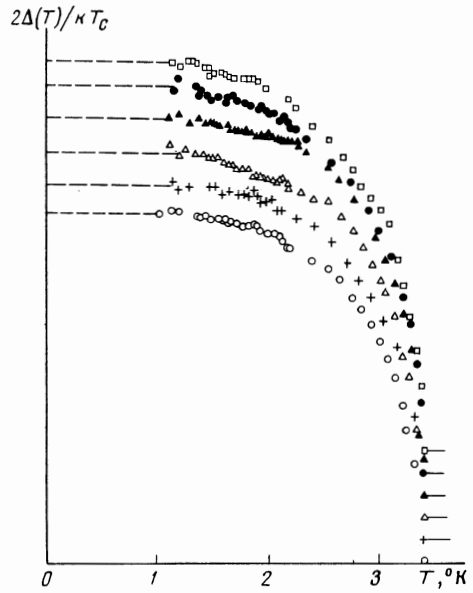

FIG. 3. Dependence of $2\Delta(T)/kT_c$ on the temperature. The curves are shifted along the ordinate axis by an amount of 0.2. The symbols for the experimental points are the same as in Fig. 2.

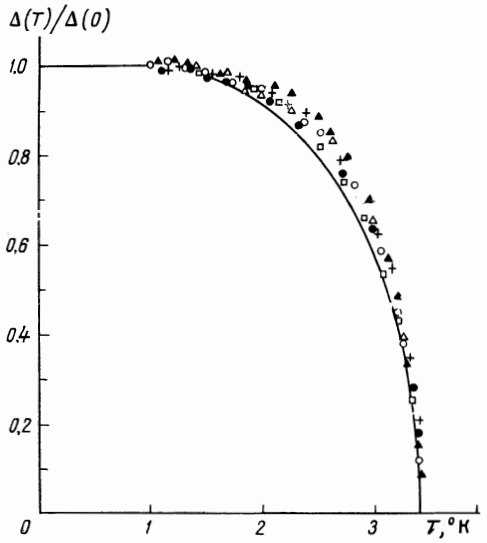

FIG. 4. Dependence of $\Delta(T)/\Delta(0)$ on the temperature. The symbols are the same as in figure 2.

f(T) lies between these two values. Consequently, to determine the size of the gap, Eq. (1) was used in inverted form:

$$\Delta(T) = kT \ln\left(\frac{2\alpha_n}{\alpha_s} - 1\right).$$

The parameter $\Delta(0)$ was obtained by plotting $\Delta(T)$ vs. T and extrapolating to zero temperature. These plots are shown in Fig. 3.

The values of $2\Delta(0)$ and α_n for the samples studied are presented in the table. Also given are the results of a similar investigation from a recent paper,[7] converted to our frequency.

The values of α_n determined in the present work are smaller than those given in [7], although the relationship between them remains. Evidently the reason for this involves the insufficient purity of the samples used ($ql \sim 20$ in comparison with $ql \sim 200$[7]). However the source of the small disagreement is not completely clear. It should be pointed out also that the thicknesses of the samples in the cited reference were too small, 0.8 to 1.6 mm, for sufficiently precise measurements in the low-temperature region.

In Fig. 4 we show the dependence of $\Delta(T)/\Delta(0)$ on T. For temperatures $T/T_c < 0.6$ all of the points lie practically on one curve, which confirms the conclusion of Pokrovskiĭ[8] that the relative anisotropy is independent of the temperature. When $T/T_c > 0.6$, the curves differ significantly. This is due either to the complex character of the averaging in the effective zone in the high-temperature region, or simply to errors in the determination of α_n, since they begin to have a strong effect at T close to T_c.

The results of the ultrasonic measurements of the energy gaps in indium differ from those of other measurements: 3.63 ± 0.1—from experiments on the tunneling effect;[9] 4.1 ± 0.2—from infrared absorption measurements on bulk samples;[10] 3.64 ± 0.2—from measurements of the critical field.[11]

Whereas disagreement with the tunneling data can be attributed to the smoothing out of the anisotropy in films, the reason for disagreement with the remaining data is not clear. Probably the discrepancy is not in the magnitude of the gaps but to the method of interpretation of the experimental data. Nor is the possibility excluded of an influence exerted on the ultrasonic measurements by the amplitude-independent internal friction, mentioned by Mason.[12] We remark however that in the case of

Orientation	α_n, dB/cm	$2\Delta(0)/kT_c$	α_n, dB/cm[7]	$2\Delta(0)/kT_c$[7]
$q \perp (100)$	36±2.0	3.45±0.1	49±5	3.2±0.2
$q \perp (001)$	42±2.5	3.15±0.1	53±5	3.1±0.2
$q \perp (110)$	75±3.5	3.35±0.1	81±7	3.1±0.2
$q \perp (011)$	77±3.5	3.10±0.1	—	—
$q \perp (111)$	68±3.5	3.45±0.1	—	—
$q \perp (023)$	60±3.5	3.25±0.15	—	—

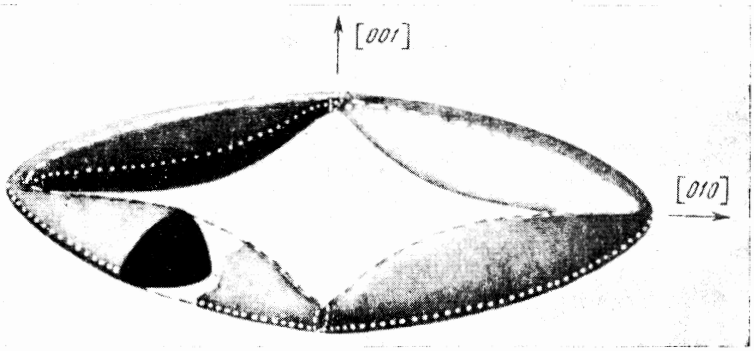

FIG. 5. The structure of the Fermi surface of indium according to [14,15]. The dotted line is the effective zone of interaction for $\mathbf{q} \perp$ (001); the dashed line is for $\mathbf{q} \perp$ (010).

$\mathbf{q} \perp$ (100) and $\mathbf{q} \perp$ (111) the measurements were carried out on two samples and the results coincided. This implies a weak influence of lattice dislocations on the results of the measurements. As was stated in [7], it seems unlikely that this effect can lead to an alteration of the relative anisotropy.

DISCUSSION OF THE RESULTS

It is hardly possible to carry out fully an analysis of the experimental data from the point of view of their relation to the topology of the Fermi surface, since there is no such model at the present time. Nevertheless indium is a suitable metal for making reasonable judgments about the data.

The table shows striking discrepancies in the sizes of the gaps in generally equivalent orientations of type (100) and (001), as well as (110) and (011). It is possible to attempt to understand the nature of these discrepancies by using the concept of an effective zone of interaction on the Fermi surface.

According to the nearly free electron model, the Fermi surface of indium consists of the second hole zone and the third electron zone (see [13]). The electron arms, designated generally by α and β differ somewhat from each other as a consequence of the tetragonal lattice. However the experimental data and a more thorough theoretical analysis[14, 15] show that the arms α are generally absent but the arms β are quite reliably observed experimentally. It is easy to show that, for the indicated pairs of orientations of the second hole zone, the regions of interaction differ from each other only slightly in size and shape. Consequently we can propose that the values of the gaps there also are the same and that for this reason the second hole zone cannot be responsible for the observed anisotropy.

Owing to the absence of the arms α, the situation changes radically if it is applied to the third electron zone. The third zone is sketched in Fig. 5 according to the experimental data.[14] The effective regions of interaction are schematically marked on it for the directions of the sound-wave vectors $\mathbf{q} \perp$ (010) and $\mathbf{q} \perp$ (001). It is evident that the regions differ considerably from each other in their position on the Fermi surface. It is possible to make a similar analysis also for the orientations $\mathbf{q} \perp$ (110) and $\mathbf{q} \perp$ (011). In general it is possible to state the proposition that the energy gap reaches a minimum value $2\Delta(0) \sim 3.0\ kT_c$ in the third electron zone near the outer contour of the central third-zone section, perpendicular to the (001) axis.

From the above arguments it is clear that indium is a suitable metal for determining the influence of the Fermi-surface topology on the superconducting properties. Consequently tunnel-effect measurements of the anisotropy would be extremely desirable for clarifying the picture.

In conclusion we note that our measurement scheme enabled us to determine the dispersion of the velocity of the ultrasound sufficiently accurately in the superconducting state. The accuracy of the measurements of relative changes of velocity can reach 2 to 3×10^{-5} for comparatively large signal-to-noise ratios, and are limited primarily by the frequency drift of the oscillator. In spite of the fact that Balkareĭ[16] predicted some dispersion, no changes of velocity of the ultrasound in the superconducting state, exceeding the accuracy of the experiment, were observed.

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