PARAMETERS OF PHENOMENOLOGICAL SUPERFLUIDITY THEORY AND THE λ-POINT SHIFT

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Some new empirical expressions for the parameters of the phenomenological theory of superfluidity are derived. Their temperature dependence differs from that assumed previously and as a result the formula for the λ-point shift in narrow gaps and for rotating helium must be modified.

THE phenomenological theory of superfluidity was constructed by Ginzburg and Pitaevskiĭ [1]. They used the following expansion for the free energy per unit volume of helium II with the density of the superfluid component $\rho_s = m|\psi|^2$ (m—mass of a helium atom, ψ—wave function):

$$F_s = F_n - \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \ldots \qquad (1)$$

It is well known that the feasibility of such an expansion is not obvious and that microscopic theory (not developed to a sufficient degree at present) can not justify it. Nevertheless, theory based on the assumption of the existence of such an expansion has led to a satisfactory qualitative explanation of the experimental data.

Recent measurements of ρ_s at temperatures near the λ-point [2] demonstrated that in its vicinity $\rho_s \propto (T_\lambda - T)^{2/3}$, in contradiction with [1], where the equilibrium value of ρ_s (in the absence of vortices and bounding surfaces) was defined as $\rho_s = m\alpha/\beta$ and in which it was assumed that $\alpha \propto (T_\lambda - T)$, $\beta \neq f(T)$.

An examination of the problem described above is the purpose of this communication. The contradiction by itself does not attest to the validity of the fundamental propostion of the phenomenological theory of superfluidity. It will be shown that there is a possibility of restoring agreement between theory and experiment and at the same time of preserving the expansion (1) and the basic deductions from it (leaving open the question of rigorously substantiating the theory). For this it is sufficient to make an empirical revaluation of the parameters α and β from the basic results of Clow and Reppy [2] and the precise measurements of specific heat by Fairbank et al. in the neighborhood of the λ-point [3] (in the same way as it was done by Ginzburg and Pitaevskiĭ [1], employing data available in 1958). As will be shown below such a revaluation improves agreement of the theory also with other experimental data, which is quite natural, since the parameters in the new version of the phenomenological theory proposed here have a form more consistent with the general considerations in the works of Josephson [4] and Tyson and Douglass [5].

According to the cited experimental research [2,3] we must satisfy the equations (for $10^{-2}\,{}^\circ\mathrm{K} \geq T_\lambda - T \geq 0$)

$$\frac{\rho_s}{\rho} = \frac{m\alpha}{\rho\beta} = 1.44(T_\lambda - T)^{2/3}, \qquad (2)$$

$$\Delta c = \frac{1}{\rho} T_\lambda \frac{d^2}{dT^2}\left(\frac{\alpha^2}{2\beta}\right) = 5.20\cdot 10^7 \frac{\text{ergs}}{\text{g}\cdot\text{degree}} \qquad (3)$$

where ρ is the density and Δc is the difference of specific heats in the superfluid and normal states and is not dependent on temperature.

The estimates of Ginzburg and Pitaevskiĭ (subscript 1) and the corollaries of Eqs. (2) and (3) (subscript 2) are juxtaposed below:

$$\alpha_1 = 4.5\cdot 10^{-17}(T_\lambda - T)\,\text{ergs} \quad \alpha_2 = 1.11\cdot 10^{-16}(T_\lambda - T)^{4/3}\text{ergs},$$

$$\beta_1 = 4\cdot 10^{-40}\,\text{ergs}\cdot\text{cm}^3,$$

$$\beta_2 = 3.52\cdot 10^{-39}(T_\lambda - T)^{2/3}\,\text{ergs}\cdot\text{cm}^3,$$

$$l_1 = 4.3\cdot 10^{-8}(T_\lambda - T)^{-1/2}\,\text{cm}, \quad l_2 = 2.73\cdot 10^{-8}(T_\lambda - T)^{-2/3}\text{cm},$$

$$v_{m1} = 3.7\cdot 10^3(T_\lambda - T)^{1/2}\,\text{cm}\cdot\text{sec}^{-1},$$

$$v_{m2} = 5.76\cdot 10^3(T_\lambda - T)^{2/3}\,\text{cm}\cdot\text{sec}^{-1}.$$

Here l is a characteristic length and v_m is a characteristic velocity (the maximum velocity compatible with superfluidity).

Modification of the temperature dependence of α and β leads in particular to a new formula for the λ-point shift in narrow gaps. According to Ginzburg and Pitaevskiĭ [1], the critical value d_c

of the width d of a flat gap is given by $d_c = \pi l$. Therefore for a shift of the λ-point ΔT_λ we have

$$(\Delta T_\lambda)_1 = -\frac{2\cdot 10^{-14}}{d^2}\,{}^\circ\mathrm{K}, \quad (\Delta T_\lambda)_2 = -\frac{2.5\cdot 10^{-11}}{d^{3/2}}\,{}^\circ\mathrm{K} \quad (4)$$

(d is measured in cm).

In the experimentally realizable conditions, with which it is possible to compare formulas (4), the λ-point shift is not measured in flat gaps but in capillaries or in unsaturated films of helium II deposited in a finely porous material. In this connection it is interesting to note that for a film, with the encircling walls of the cylindrical capillary having radius R, the critical thickness $d_c = (R - r)_c$ (r is the inside radius of the film) is determined by the equation $J_0(r)N_0(R) - J_0(R)N_0(r) = 0$. The solutions of this equation have been tabulated, for example, in [6] (J and N are the Bessel and Neumann functions). The dimensionless critical width $\delta_c = (R - r)_c/l$ varies from $\delta_c = \pi$ when $R/r \to 1$ (compare with [1]) up to $\delta_c = j_{01} = 2.40$ when $R/r \to \infty$ (compare with [7]). We had to change the numerical coefficients in formulas (4) accordingly. However, the magnitude of δ_c stays near π up to rather large values of R/r. Therefore, it is possible in practice to use formulas (4) for any unsaturated film if we consider that the change from the value $\delta_c \sim 3$ to the value $\delta_c \sim 2.4$ occurs in a relatively narrow interval of film thickness before complete filling of the pores (in the latter case in the second version of formulas (4) and with the use of cylindrical channels, the factor 2.5 is replaced by 1.7 and d is replaced by R).

The experimental data on the λ-point shift in thin films do not have the same high accuracy as in [2,3]. Nevertheless it is a fairly unambiguous verification of the second variant of formulas (4), as is shown below by a comparison of formulas (4) with the data of different authors. These data are summarized in the article by Brewer et al.[8] in curve 2 of Fig. 2.

$d\cdot 10^8$, cm:	9	9.9	12	14	15	17.5	19	22.5
$(\Delta T_\lambda)_1/(\Delta T_\lambda)_{\mathrm{exp}}$:	3.0	2.8	2.2	1.9	2.1	1.9	2.0	2.2
$(\Delta T_\lambda)_2/(\Delta T_\lambda)_{\mathrm{exp}}$:	1.1	1.1	1.1	0.9	1.0	1.0	1.1	1.3

Without presenting the corresponding numbers, we also remark that the experimental data on the shift of the specific heat maximum [8] also favor the $d^{-3/2}$ law rather than d^{-2}.

Revaluation of the parameters of the phenomenological theory also changes the formula for the λ-point shift for rotating helium II with angular velocity ω_0 in a wide vessel ($R/l \gg 1$) as proposed in [9]. In its new version this formula has the form

$$\Delta T_\lambda = -5.4\cdot 10^{-9}\omega_0^{3/4}.$$

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Note added in proof (2 February 1967). It is well known that the question of the boundary condition at the free surface of a film is still open. We use the condition $\psi(r) = 0$. If we substitute the condition $\partial\psi/\partial r = 0$, then the critical thickness of a cylindrical film will be determined by the equation $J_0(R)N_1(r) - N_0(R)J_1(r) = 0$. Then $\delta_c \to \pi/2$ as $R/r \to 1$ and $\delta_c \to 2.40$ as $R/r \to \infty$. Apparently the experimental facts favor the boundary condition $\psi(r) = 0$.

1 V. L. Ginzburg and L. P. Pitaevskiĭ, JETP 34, 1240 (1958), Soviet Phys. JETP 7, 858 (1958).
2 T. R. Clow and J. D. Reppy, Phys. Rev. Lett. 16, 887 (1966).
3 W. M. Fairbank, Liquid Helium, ed. by G. Careri, Academic Press, New York and London, 1963, p. 293.
4 B. D. Josephson, Phys. Lett. 21, 608 (1966).
5 J. A. Tyson and D. H. Douglass, J., Phys. Rev. Lett. 17, 472 (1966).
6 E. Jahnke, F. Emde and F. Lösch, Special Functions Nauka 1964.
7 Yu. G. Mamaladze and O. D. Cheĭshvili, JETP Letters 2, 123 (1965), transl. p. 76.
8 D. F. Brewer, A. J. Symonds and A. L. Thomson, Phys. Rev. Lett. 15, 182 (1965).
9 L. V. Kiknadze, Yu. G. Mamaladze, and O. D. Cheĭshvili, JETP 48, 1520 (1965), Soviet Phys. JETP 21, 1018 (1965).

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