INFRARED SINGULARITIES OF GREEN'S FUNCTION IN AN ARBITRARY GAUGE

V. I. ZHURAVLEV and L. D. SOLOV'EV[1)]

Joint Institute for Nuclear Research

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All the singular terms in the infrared region are obtained in all orders in e for the Green's function of a charged particle with arbitrary gage.

THE explicit form of the terms that are singular in the infrared region were obtained previously[1] for the Green's function of a charged particle with the Feynman gauge. Since these formulas are exact (they are not connected with the perturbation-theory expansions), it is desirable to obtain them in as general a form as possible, i.e., for an arbitrary gauge of the electromagnetic potentials. Let us consider a gauge in which the free propagation function of the photon is

$$\frac{1}{k^2-\lambda^2}\left(g_{mn}-\frac{d(k^2)-1}{k^2-\lambda^2}k_m k_n\right), \tag{1}$$

where λ is the small photon mass, which should be made to approach zero faster than all physical variables. This function corresponds to a potential $A_n^F(x)+\partial_n\Lambda(x)$ (A_n^F corresponds to the Feynman gauge), where the function $\Lambda(x)$ does not satisfy the free equation when $d(0)\neq 1$.[2] In this case we should deal in fact with scalar photons, for which $k^2\neq 0$. This means that when expanding in terms of the complete system of asymptotic states it is necessary to go off the mass shall $k^2=0$. A method for so doing can be readily established by examining the diagrams with propagation function (1). To find the jump of such a diagram on the cut, it is necessary to make the substitution

$$1/(k^2-\lambda^2)^2\rightarrow -2\pi i\delta'(k^2-\lambda^2). \tag{2}$$

Thus, the transition from the Feynman gauge (1) corresponds to the following substitutions of the polarization vectors and phase volumes of the photons[3]:

$$\varepsilon\rightarrow\tilde{\varepsilon}=\varepsilon+(\sqrt{d}-1)\frac{\varepsilon k}{k^2-\lambda^2}k,$$

$$\int d^4k\frac{\delta(k^2-\lambda^2)}{k^2-\lambda^2}\rightarrow -\int d^4k\,\delta'(k^2-\lambda^2). \tag{3}$$

Having made this remark, let us find the singularities of the Green's functions in the infrared region. For a scalar particle (we shall discuss a particle with spin 1/2 in the end) we have

$$G(p^2)=\int\limits_{m^2}^{(m+\delta)^2}\frac{g(r^2)\,dr^2}{r^2-p^2-i0}, \tag{4}$$

$$g(p^2)=(2\pi)^3\sum_N\delta(p-p_N)\langle 0|\Phi|N\rangle\langle N|\Phi^+|0\rangle, \tag{5}$$

where Φ is the operator of the scalar field, δ is arbitrarily small, and we have omitted from (4) a term equal to a constant when $p^2=m^2$. We shall henceforth omit all expressions that lead to such terms in the Green's functions.

It can be shown[1] that in this case

$$\langle 0|\Phi|N\rangle=\langle 0|\Phi|r,k_1,\ldots,k_n\rangle=\prod_{i=1}^{n}e\frac{(2r+k_i)\tilde{\varepsilon}_i}{2rk_i+k_i^2}\cdot Z, \tag{6}$$

where r is the momentum of the charged particles, k_i the momenta of the soft photons, and $Z=\langle 0|\Phi|r\rangle$. Substituting (6) in (5) and summing over all the photons, we get

$$g(p^2)=\frac{Z^2}{(2\pi)^4}\int d^4x\int\frac{d\mathbf{r}}{2r^0}e^{-i(p-r)x+F}, \tag{7}$$

where

$$F=-\frac{e^2}{(2\pi)^3}\int d^4k\,\theta(k^0)\,\delta(k^2-\lambda^2)\left[\frac{(2r+k)^2}{(2rk+k^2)^2}+\frac{d(k^2)-1}{k^2-\lambda^2}\right]e^{ikx}=-\frac{e^2}{(2\pi)^3}\int\frac{d\mathbf{k}}{2k^0}\left[\frac{m^2}{(rk)^2}+\frac{1}{rk}+\frac{d(0)-1}{2}\frac{\partial}{\partial k^0}\frac{1}{k^0}\right]e^{ikx}. \tag{8}$$

We have assumed here that $d'(0)<\infty$. Calculating the integrals (7), we obtain a generalized power-law function of $x=p^2m^{-2}-1$:

$$g(p^2)=\frac{Z^2}{m^2}\left(\frac{m}{\lambda}\right)^{\beta}e^{\alpha/\pi-C\beta}\frac{x_+^{\beta-1}}{\Gamma(\beta)}\left[1-\left(\frac{1}{2}+\frac{\beta}{4}\right)x\right], \tag{9}$$

where α is the fine-structure, C is Euler's con-

[1)]Institute of High-energy Physics (Serpukhov).

stant, Γ the gamma function, and

$$\beta = \alpha(d(0)-3)/2\pi. \tag{10}$$

Substituting (9) in the spectral representation (4), we obtain the infrared asymptotic expression for the Green's function:

$$G(p^2) = \frac{Z_1}{m^2}(-x)^{\beta-1}\left[1-\left(\frac{1}{2}+\frac{\beta}{4}\right)x\right] + \text{const}, \tag{11}$$

$$Z_1 = Z^2\left(\frac{m}{\lambda}\right)^{\beta} e^{\alpha/\pi - C\beta}\Gamma(1-\beta). \tag{12}$$

This expression differs from the corresponding expression in the Feynman gauge[1] (d = 1) in that $-\alpha/\pi$ has been replaced by β and that the factor $\exp\{\alpha(d(0)-1)/2\pi\}$ has been added in Z_1. Such a substitution is valid also for the Green's function of a particle with spin 1/2.

In the case d(0) = 3 the Green's function has a pole in the infrared region, but the residue at this pole differs in general from unity when Z = 1, and depends on the manner in which λ approaches zero in the second factor of (1). If $k^2 - \lambda^2$ in this factor is replaced by $k^2 - a\lambda^2$, then $e^{\alpha/\pi}$ in (9) and (12) is replaced by

$$\exp\left\{\frac{\alpha}{\pi}\left[1+\frac{d(0)-1}{4}\left(1-\frac{a\ln a}{a-1}\right)\right]\right\}. \tag{13}$$

Consequently, when d(0) = 3 the residue at the pole is equal to unity if a ln d = 3a − 3 (a ≈ 17).

The infrared singularities of the electronic Green's function in a gauge with $d_l = 0$ were investigated independently also by Braun[4], who reached the conclusion that in this case the Green's function has a pole singularity with unity residue at the pole. We have seen that the latter conclusion is in general incorrect.

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[1] L. D. Solov'ev, JETP **48**, 1740 (1965), Soviet Phys. JETP **21**, 1166 (1965).

[2] V. I. Ogievetskiĭ and I. V. Polubarinov, JETP **40**, 926 (1961), Soviet Phys. JETP **13**, 647 (1961).

[3] V. D. Mur and V. D. Skarzhinskiĭ, JETP **40**, 1076 (1961), Soviet Phys. JETP **13**, 759 (1961).

[4] M. Braun, JETP **51**, 606 (1966), Soviet Phys. JETP **24**, 404 (1967).

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