RECONSTRUCTION OF THE PROCESS OF ELASTIC SCATTERING IN COHERENT LIGHT

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The process of elastic scattering of particles from a force center can be reconstructed by optical means with the aid of a hologram. It is shown that the light field behind the hologram is modulated by the scattering amplitude and is an exact copy of the scattered quantum-mechanical probability wave.

THE two-step photography method proposed by Gabor [1] and perfected by Leith and Upatnieks [2] will undoubtedly find extensive use in nuclear physics, plasma physics, x-ray structure analysis, neutron diffraction, defectoscopy, etc.

In this paper we consider the quantum-mechanical probelm of scattering in a potential field and show that information concerning the character of the interaction forces, contained in the scattering amplitude $f(\theta, \varphi)$, can be recorded on a hologram. When the hologram is illuminated by a laser, an electromagnetic light wave is produced, and has in the wave zone the form

$$E \sim \frac{e^{ik_1r}}{r} f(\theta, \varphi). \tag{1}$$

This expression is similar to the scattered quantum-mechanical probability wave at large distances outside the range of the forces. By virtue of this, the process of elastic scattering can be reproduced in coherent light.

A practical realization of scheme undoubtedly entails considerable difficulties. However, its realization makes possible, first, repeated a posteriori processing of the information. Second, the differential scattering cross section can be measured with high precision by using sensitive photocells.

This will permit, in particular, an investigation of the differential cross section of elastic forward scattering at high energies. Further, inasmuch as the electromagnetic wave (1) has the same phase as the scattering amplitude, we can expect that the use of the hologram will make a correct phase-shift analysis of the experimental data feasible.

Assume that particles of mass m emitted from a point source at the origin are scattered by some target. In the stationary formulation, the scattering of particles with energy E by a potential $V(\mathbf{r})$ is determined by the Schrödinger equation, which can be transformed into the following integral equation (see [3]) ($k_0 = \sqrt{2mE/\hbar^2}$ is the wave number)

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + \psi_{\text{scat}}(\mathbf{r}),$$

$$\psi_0(\mathbf{r}) = A_0\frac{e^{ik_0r}}{r}, \qquad U(\mathbf{r}) = \frac{2m}{\hbar^2}V(\mathbf{r}),$$

$$\psi_{\text{scat}}(\mathbf{r}) = -\frac{1}{4\pi}\int \frac{\exp(ik_0|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'. \tag{2}$$

At large distances outside the range of the forces, the scattered wave is spherical and is modulated by the scattering amplitude, which determines the differential cross section of the process.

Let us place behind the target, on a spherical surface S of radius R, a photographic film whose dimensions should be not smaller than those of the first Fresnel zone. The primary and scattered waves interfere and form a diffraction pattern on the surface S. As a result, a positive print of the obtained photograph (hologram) will contain a record of the information carried by the scattered wave.

The photography process records, in the form of film density, the quantity

$$|\psi(R, \theta, \varphi)|^2 = |\psi_0|^2 + \psi_0^+\psi_{\text{scat}} + \psi_0\psi_{\text{scat}}^+ + |\psi_{\text{scat}}|^2. \tag{3}$$

If the intensity of the particles incident on the target is sufficiently large, the last term in (3) can be neglected. In real conditions the source is not a point. In this case, to obtain a sharp diffraction pattern it is necessary that the space between the source and the surface S not exceed the coherence volume V. This volume should correspond to one "cell" in phase space (see also [4]):

$$\Delta p_x \Delta p_y \Delta p_z \Delta V/(2\pi\hbar)^3 \sim 1.$$

If these requirements are too stringent, then it is necessary to obtain the hologram by a technique similar to that of holography of self-luminous objects [5].

Let us consider the reconstruction of the scattering process further. We now remove the source and the target, and illuminate the hologram, which is placed on a surface of radius R, by a source of visible coherent monochromatic linearly-polarized light of frequency $\omega_1 = ck_1$. The source is at the center of curvature of the hologram, and the components of the electric field of the wave are

$$E_r = 0, \quad E_\varphi = A_\varphi e^{ik_1 r}/r, \quad E_\theta = 0.$$

The light wave striking the hologram is diffracted. As will be shown later, the diffracted electromagnetic wave has a structure similar to that of the probability wave (2).

The electric and magnetic fields of the light wave behind the hologram can be determined from the electrodynamic Huygens principle in the Kirchhoff approximation.[6,7] The electric field intensity of the light wave in the wave zone ($k_1 r \to \infty$) is given by

$$E_\varphi(\mathbf{r}) = -\frac{ik_1}{2\pi}\int_S \frac{\exp(ik_1|\mathbf{r}-\mathbf{R}|)}{|\mathbf{r}-\mathbf{R}|}\alpha(\theta,\varphi)A_\varphi\frac{e^{ik_1 R}}{R}dS, \quad (4)$$

where

$$\alpha(\theta,\varphi) = \gamma|\psi(R,\theta,\varphi)|^2 \quad (5)$$

is the transparency coefficient of the hologram and γ is a quantity proportional to the exposure time.[8]

Substituting in (4) the value of $\alpha(\theta, \varphi)$ from (5) and (3), we get

$$\begin{aligned} E_\varphi(\mathbf{r}) = \gamma A_\varphi\Big\{ & |\psi_0|^2\frac{e^{ik_1 r}}{r} - \frac{\psi_0^+}{4\pi} \\ & \times \int \frac{\exp(ik_1|\mathbf{r}-\mathbf{r}'k_0/k_1|)}{|\mathbf{r}-\mathbf{r}'k_0/k_1|}U(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}' \\ & - \frac{\psi_0}{4\pi}\int\frac{\exp(ik_1|\mathbf{r}+\mathbf{r}'k_0/k_1|)}{|\mathbf{r}+\mathbf{r}'k_0/k_1|}U^+(\mathbf{r}')\psi^+(\mathbf{r}')d\mathbf{r}'\Big\}. \end{aligned} \quad (6)$$

In the integration we took account of the fact that when $k_1 r \gg 1$, $k_0 R \gg 1$, and $r' \sin\theta \ll R$ (θ is the angle between the vectors $\mathbf{r}$ and $\mathbf{r}'$) we have

$$-\frac{ik_1}{2\pi}\int_S \frac{\exp(ik_1|\mathbf{r}-\mathbf{R}|)}{|\mathbf{r}-\mathbf{R}|}\frac{\exp(\pm ik_0|\mathbf{R}-\mathbf{r}'|)}{|\mathbf{R}-\mathbf{r}'|}dS = \frac{\exp(ik_1|\mathbf{r}\mp\mathbf{r}'k_0/k_1|)}{|\mathbf{r}\mp\mathbf{r}'k_0/k_1|} \quad (7)$$

The integral (7) can be calculated by expanding the spherical waves in Legendre polynomials and integrating over the angles. The resultant series is evaluated with the aid of a Watson-Sommerfeld transformation.[9]

We note that relation (7), and consequently also (6), is satisfied with great accuracy if the same hologram (5) one uses during the reconstruction, but transferred to a spherical surface of radius Rk_0/k_1. This method makes it possible to avoid phase distortions during the reconstruction process.

An analysis of expression (6) explains how the scattering process is reconstructed in coherent light. The meaning of the first term of (6) is obvious. The second term in (6) is an electromagnetic wave having the same structure as the scattered probability wave (2) propagating from a target having dimensions larger than initial by a factor $k_0/k_1 = \lambda_1/\lambda_0$. This wave behaves exactly as the scattered probability wave, making it possible to reconstruct the scattering process from the obtained hologram after the target and the incident particles are no longer there. At large distances, this wave is spherical and is spatially-modulated by the scattering amplitude:

$$f(\theta,\varphi) = -\frac{1}{4\pi}\int \exp\left\{-ik_0\frac{\mathbf{r}\mathbf{r}'}{r}\right\}\frac{2m}{\hbar^2}V(\mathbf{r}')\psi(\mathbf{r}')d\mathbf{r}'. \quad (8)$$

Thus, the differential cross section can be measuring by using photocells as the recording elements. The third term of (6) also carries information concerning the scattering process, and describes the wave scattered from a target that is located symmetrically with respect to the initial target.

The electromagnetic waves represented by the second and third terms of (6) can be separated by any one of the known methods.

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