SCATTERING OF PHOTONS IN A HOMOGENEOUS ELECTROMAGNETIC FIELD

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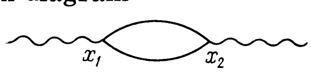
Submitted to JETP editor June 29, 1966; resubmitted October 27, 1966)

J. Exptl. Theoret. Phys. (U.S.S.R.) 52, 467–469 (February, 1967)

The effect of elastic scattering of a photon in a homogeneous electromagnetic field, resulting in a change in photon polarization, is considered and an expression is derived for the probability of such scattering.

A well-known prediction of quantum electrodynamics (cf. e.g.[1]) is the elastic scattering of a photon by the Coulomb field of a nucleus (Delbrück scattering). In this note we consider a similar process of elastic scattering of photons in a homogeneous electromagnetic field. In this case the effect consists only in a change of the photon polarization, whereas the direction of propagation of the photon remains unchanged.

The process under consideration is represented by the Feynman diagram



(1)

and the scattering matrix element has the form[1)]

$$M=\frac{e^2}{2}\int d^4x_1d^4x_2\,\mathrm{Sp}\,\hat{A}_1(x_1)G^c(x_2,x_1)\hat{A}_2(x_2)G^c(x_1,x_2),$$

$$A_{1\mu}(x_1)=e_\mu{}^1\int\frac{d\omega_1}{\sqrt{2\omega_1}}f(\omega)e^{ik_1x_1},\qquad A_{2\nu}(x_2)=\frac{e_\nu{}^2}{\sqrt{2\omega_2}}e^{-ik_2x_2},\tag{2}$$

where $\hat{a}=a_\mu\gamma_\mu$, γ_μ are the Dirac matrices, k_i and ω_i are the photon 4-momenta and frequencies, e_i are the polarization vectors. In (2), $G^c(x,x')$ is the causal electron Green's function in the homogeneous electromagnetic field, which can be represented in the form[2]

$$G^c(x,x')=\Phi(x,x')S^c(x-x').$$

Since $\Phi(x',x)=\Phi^{-1}(x,x')$[2], only the part of the Green's function depending on the coordinate differences survives in (2). In the momentum representation, for $eF/m^2\ll 1$, a condition which is always satisfied, we have

$$S^c(p)=\frac{m-\hat{p}}{m^2+p^2-i\varepsilon}+\frac{e}{4}\left\{\sigma_{\mu\nu}F_{\mu\nu},\ \frac{m-\hat{p}}{(m^2+p^2-i\varepsilon)^2}\right\}+\frac{ie^2\gamma_\mu F_{\mu\nu}p_\nu\sigma_{\alpha\beta}F_{\alpha\beta}}{(p^2+m^2-i\varepsilon)^3}\tag{3}$$

(here m is the electron mass, $\sigma_{\mu\nu}=\frac{1}{2}i[\gamma_\mu,\gamma_\nu]$, $F_{\mu\nu}$ is the electromagnetic field tensor, and $\{a,b\}=ab+ba$.

Only terms which yield a nonzero contribution to the scattering have been retained in (3). In coordinate space the Green's function $S^c(x)$ has the form (up to terms of order $(eF/m^2)^2$

$$S^c(x)=\frac{1}{(4\pi)^2}\int_0^\infty\frac{ds}{s^2}\left[\left(-\frac{\hat{x}}{2s}+m\right)\left(1+\frac{i}{12}e^2sx_\mu F_{\mu\nu}F_{\nu\rho}x_\rho\right.\right.$$
$$\left.-e^2s^2\left(\frac{1}{3}F_{\mu\nu}{}^2+\frac{1}{4}\gamma_5F_{\mu\nu}\tilde{F}_{\mu\nu}\right)\right)-\frac{1}{6}e^2s\gamma_\mu F_{\mu\nu}F_{\nu\rho}x_\rho$$
$$\left.-\frac{i}{4}e^2s\gamma_\mu F_{\mu\nu}x_\nu\sigma_{\alpha\beta}F_{\alpha\beta}-\frac{e}{2}\gamma_\mu F_{\mu\nu}x_\nu+\frac{i}{2}es\sigma_{\alpha\beta}F_{\alpha\beta}\right]$$
$$\times\exp\left(\frac{i}{4s}x_\mu{}^2-im^2s-\varepsilon s\right),\qquad\varepsilon>0;$$

here $\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual field tensor and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$[2]

The matrix element is

$$M=\frac{e^2}{2}\frac{e_\mu{}^1e_\nu{}^2}{\sqrt{4\omega_1\omega_2}}f(\omega_2)\Sigma_{\mu\nu}(k_2)\delta^{(3)}(\mathbf{k}_1-\mathbf{k}_2),\tag{4}$$

with

$$\Sigma_{\mu\nu}(k)=\int d^4p\,\mathrm{Sp}\,\gamma_\mu S^c(p)\gamma_\nu S^c(p-k).\tag{5}$$

The tensor $\Sigma_{\mu\nu}(k)$ contains divergent contributions which violate gauge invariance. Only the gauge-invariant part $\Sigma'_{\mu\nu}(k)=\Sigma_{\mu\nu}(k)-\Sigma_{\mu\nu}(0)$ is physically meaningful. A computation yields

$$\Sigma_{\mu\nu}'(k)=\frac{4ie^2\pi^2}{15m^4}[F_{\mu\rho}k_\rho F_{\nu\sigma}k_\sigma+\delta_{\mu\nu}(F_{\rho\sigma}k_\sigma)^2].\tag{6}$$

The probability for the process under consideration turns out to be

$$w=\frac{4\alpha^4\pi^2}{225m^8\omega^2}|f(\omega)|^2(e_\mu{}^1F_{\mu\rho}k_\rho\cdot e_\nu{}^2F_{\nu\sigma}k_\sigma+e_\mu{}^1e_\mu{}^2\cdot(F_{\rho\sigma}k_\sigma)^2)^2,\tag{7}$$

where $\alpha=e^2/4\pi$. Averaging over polarizations results in

1)For strictly monochromatic waves this expression becomes infinite. Therefore we assume that in the initial state we have a superposition of photons, which is always justified in real situations.

$$w = \frac{2\alpha^4\pi^2}{45\omega^2 m^8} |f(\omega)|^2 [(F_{\mu\nu}k_\nu)^2]^2. \qquad (8)$$

Owing to the conservation law $\mathbf{k}_1 = \mathbf{k}_2$, the photon propagation direction does not change upon passing through a uniform electromagnetic field. The only observable effect consists in a change of polarization. In the radiation gauge

$$e_\mu F_{\mu\nu} k_\nu = -\omega \mathbf{e}(\mathbf{E} + \mathbf{H}\times\mathbf{n}), \quad \mathbf{n} = \mathbf{k}/\omega.$$

Let $\mathbf{E} \uparrow\uparrow \mathbf{H}$, $\mathbf{k} \perp \mathbf{H}$. If initially the photon is linearly polarized $\mathbf{e}_1 \uparrow\uparrow \mathbf{E}$, the probability for a change of polarization to $\mathbf{e}_2 \uparrow\uparrow \mathbf{H}\times\mathbf{n}$ $(\mathbf{e}_2 \perp \mathbf{e}_1)$ is

$$w = \frac{4\alpha^4\pi^2\omega^2}{225 m^8} |f(\omega)|^2 (\mathbf{EH})^2. \qquad (9)$$

If either $\mathbf{E}$ or $\mathbf{H}$ vanishes there is no charge in polarization.

If the initial state consists of a superposition of photons with various $\mathbf{k}$, the scattering probability has the form

$$w = \frac{64\alpha^4\pi^5}{225 m^8} |f(\mathbf{k})|^2 (e_\mu^1 F_{\mu\rho} k_\rho \cdot e_\nu^2 F_{\nu\sigma} k_\sigma + e_\mu^1 e_\mu^2 (F_{\rho\sigma} k_\sigma)^2)^2. \qquad (10)$$

Equations (7)—(10) are valid for arbitrary (both small and large) frequencies of the photons (compare with Delbrück scattering,[1]). The polarization change increases with the frequency of the photons.

I am grateful to P. I. Fomin for a discussion.

[1] A. I. Akhiezer and V. B. Berestetskiĭ, Kvantovaya élektrodinamika (Quantum Electrodynamics), 2nd Ed. Fizmatgiz, Moscow, 1959.

[2] J. Schwinger, Phys. Rev. 82, 664 (1951).

Translated by M. E. Mayer
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