FLUCTUATIONS IN SHOWERS PRODUCED BY ELECTRONS WITH ENERGIES FROM 45 to 330 MEV

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A cloud chamber with lead plates was used to study the fluctuations in electron-photon showers produced by electrons with energies 45, 130, 230, and 330 MeV. The dependence of the shower energy loss fluctuations on the absorber thickness t was determined for the 330 MeV energy. The half-width of the loss curve ranges from 65% for $t = 10.2$ g/cm^2 to 30% for $t = 49.8$ g/cm^2. The half-width of the loss curve is proportional to $E_0^{-1/2}$. The distributions of the showers with respect to the number of electrons at different depths of cascade development are determined and are found to be satisfactorily described by a combined Poisson distribution (separately for showers with even and odd electron numbers). An analysis of the electron-number fluctuation correlation for different absorber thicknesses has shown that, regardless of the primary energy of the electrons and the absorption depth, the correlations are significantly decreased after the shower passes through a lead layer 6–8 g/cm^2 thick. All the shower characteristics have been obtained for a secondary-electron spectrum cut off at ~ 1 MeV.

1. INTRODUCTION

An essential feature of the cascade process that develops when electrons of high energy pass through matter is the presence of appreciable shower-particle-number fluctuations which are connected with the statistical character of the interaction (pair production and bremsstrahlung), and at which appreciable energy transfer takes place.

The existing cascade theory [1] describes sufficiently accurately (at least for large particle energies with small Z) the electron-photon shower in the mean, and makes it possible to determine such shower characteristics as the average particle number, the particle energy distribution, etc., and also makes it possible to trace qualitatively the character of the variation of the fluctuations in the number of particles with varying thickness of matter [2,3]. However, there is as yet no shower calculation method yielding the probability characteristics of the shower1) such as the fluctuations of the number of particles, the correlations of these fluctuations, etc.

Knowledge of the probability characteristics of showers is essential not only for the construction of a complete cascade theory, but is important in applications. Thus, data on the electron-number fluctuations at different depths of shower development are essential to solve the important methodological problem of efficiency of γ-quantum and electron detectors (γ spectrometers, γ telescopes, "range" telescopes for electrons, etc.). Information on the scatter of the total energy released by the shower particles in a thick layer of matter, or on the fluctuations of the intensity of Cerenkov radiation emitted by shower particles, are used in the construction of total-absorption spectrometers and shower spark detectors, which are widely used presently in accelerator experiments.

Information on the probability characteristics of showers can be obtained either experimentally or by modeling the cascade process by the Monte Carlo method [6]. Recent experimental papers give data on the electron-number fluctuations produced by electrons in copper and in lead [7,8]. On the other hand, data on such shower characteristics as fluctuation correlation and the dependence of the shower energy-loss fluctuations on the thickness of the absorber have not yet been published.

The purpose of the present work was to investigate the fluctuations and correlations of the fluctuations in showers produced in lead by electrons with energies from 45 to 330 MeV.

1)This problem was considered by Furry [4] and by Tyapkin [5].

2. EXPERIMENTAL SETUP

We investigated the electron-photon showers with a cloud chamber with lead plates [9]. The experiments were made with the electron beam of the synchrocyclotron of the Nuclear Problems Laboratory of the Joint Institute for Nuclear Research. The cloud chamber was bombarded with electrons of energy E_0 equal to 330 ± 20, 230 ± 15, 130 ± 10, and 45 ± 5 MeV. The measurement procedure was described by us in a preceding paper[9] devoted to a study of average shower characteristics.

To investigate the shower fluctuations we selected photographs on which the distance between the tracks of the electrons entering the chamber was large enough to be able to separate the showers reliably. At E_0 = 330 MeV, this distance was chosen to be 50 mm, thereby ensuring separation of neighboring showers over the entire length of the chamber. For lower energies, the minimum distance between primary-electron tracks was taken equal to 25 mm, making it possible to separate neighboring showers up to the sixth gap (22.7 g/cm^2 of lead). The number of showers selected in this manner was 426, 254, 303, and 268 respectively for the energies listed above.

The electron tracks[2)] on the obtained shower photographs were processed using the same selection criteria as in the investigation of the average shower characteristics [9]. The cutoff energy E for the spectrum of the secondary electrons was ~ 1 MeV.

3. FLUCTUATION OF ENERGY LOSSES

When a shower passes through matter, the development of the cascade process is accompanied by a rapid loss of shower energy. The loss of energy $U(E_0, t)$ of a shower traversing a distance t (measured in radiation units) through matter is connected with the number of electrons in the shower $N(E_0, E, t)$ by the relation

$$U(E_0, t) = \varepsilon \int_0^t dt' \frac{N(E_0, E, t')\overline{\sec\theta(t')}}{p(E_0, E, t')}, \qquad (1)$$

Here ϵ is the critical energy, $\overline{\sec\theta(t)}$ is the secant of the angle of electron scattering in the shower at the depth p, averaged over the tracks[9], and the function $p(E_0, E, t)$ takes into account the introduction of the cutoff energy of the secondary-electrons spectrum.

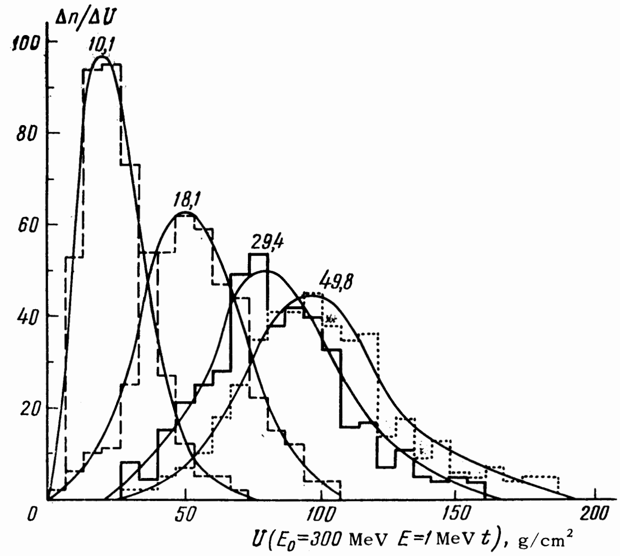

FIG. 1. Distributions of the shower numbers with respect to the quantity $U(E_0, E, t)$. For clarity, we have drawn through the experimentally obtained histograms curves which are tagged by the corresponding values of the absorber thickness t, in g/cm^2.

For many problems of practical importance (Cerenkov total-absorption spectrometers, shower spark chambers, etc.) interest is attached not to the total loss (1) (the determination of which is a difficult problem, since the shower contains many low-energy electrons [9]), but that part of the loss $U(E_0, E, t)$ which is released in the matter by the electrons having an energy higher than the cutoff threshold E. In this case relation (1) simplifies to

$$U(E_0, E, t) = \varepsilon \int_0^t N(E_0, E, t')\overline{\sec\theta(t')}\,dt'. \qquad (2)$$

The values of U were calculated by us for energies E_0 equal to 330 and 45 MeV. For each shower, the integral (2) was calculated by summing the product of the number of electrons in the gaps between the lead plates by the half sum of the thicknesses of the plates forming the gaps. The values of $\overline{\sec\theta(t)}$ were determined by measuring the three-dimensional angle θ of the tracks relative to the shower axis.

Figure 1 shows the obtained distributions of the shower numbers with respect to the quantity $U(E_0, E, t)$ for 330 MeV energy. As seen from this figure, the distributions become more symmetrical with increasing depth of shower development, and the relative fluctuations of the energy loss decrease. Thus, at t = 10.2 g/cm^2 the half-width of the distribution amounts to 65% and at t = 49.8 g/cm^2 it amounts to 30%. For E_0 = 45 MeV, the half-width of the loss distribution at t = 22.7 g/cm^2 (which corresponds to absorption of 90% of

2)We make no distinction here between electrons and positrons in the shower.

The quantity $W_N(E_0, E = 1\text{ MeV}, t)$

E_0, MeV	N	t, Г/см² 3.4	6.8	10.15	13.6	18.1	22.7	29.4	38.6	49.8
45	0	0.100±0.027	0.314±0.028	0.580±0.030	0.728±0.042	0.911±0.017	0.963±0.012	0.985±0.007		
	1	0.780±0.030	0.500±0.030	0.276±0.027	0.194±0.024	0.045±0.013	0.026±0.01	0.011±0.007		
	2	0,067±0.016	0.127±0.020	0.116±0.020	0.071±0.016	0.022±0.009	0.011±0.006	0.004±0.004		
	3	0,049±0.013	0.045±0.013	0.019±0.008	0.007±0.005	0.007±0.005	0	0		
	4	0	0.015±0.007	0,004±0.004	0	0.011±0.007	0	0		
	5	0	0	0.004±0.004	0	0,004±0.004	0	0		
130	0	0.066±0.014	0.132±0.019	0.181±0.022	0.277±0.030	0.400±0.035				
	1	0,693±0.027	0.465±0.029	0.380±0.028	0.351±0,032	0.373±0.035				
	2	0.158±0.021	0.238±0.025	0.266±0.026	0.282±0.030	0.186±0.028				
	3	0.066±0.014	0.136±0.020	0.140±0.012	0.073±0.017	0.031±0.013				
	4	0.017±0.008	0,023±0.009	0.027±0,010	0.013±0.008	0.005±0.005				
	5	0	0.003±0.003	0	0.004±0.004	0.005±0.005				
	6	0	0	0,003±0,003	0	0				
	7	0	0.003±0.003	0.003±0,003	0	0	0			
230	0	0.047±0.013	0.067±0.016	0.107±0.020	0.131±0,022	0,220±0.031	0.325±0.043			
	1	0.621±0.030	0.353±0.030	0.250±0.027	0.237±0,028	0.317±0.035	0.291±0.042			
	2	0.226±0.026	0.286±0.028	0,282±0.028	0.360±0.031	0,277±0.034	0.222±0.038			
	3	0,090±0.018	0.180±0.024	0.234±0.027	0.165±0.024	0.113±0.024	0,12 ±0.03			
	4	0.012±0.007	0.055±0.014	0.091±0.018	0.081±0.018	0.062±0.018	0,034±0.017			
	5	0.004±0,004	0,055±0.014	0.028±0.010	0.026±0.010	0.011±0.007	0.008±0.008			
	6	0	0.004±0.004	0.008±0.005	0	0	0			
330	0	0.019±0.006	0.034±0,009	0,068±0.012	0.064±0,012	0.115±0.016	0.190±0.020	0.324±0.023	0.480±0.024	0.605±0.024
	1	0,670±0.023	0.363±0.023	0.198±0.020	0.191±0.019	0.182±0.019	0.230±0.02	0.253±0.021	0.280±0.022	0.242±0.021
	2	0.136±0.017	0.21 ±0.02	0.214±0.02	0.220±0.020	0.270±0.020	0,210±0.020	0,205±0.020	0.160±0.018	0.103±0.015
	3	0.143±0.018	0.246±0.021	0.262±0.021	0.264±0.022	0.220±0.020	0,160±0.018	0.137±0.017	0.031±0.008	0.040±0.010
	4	0.019±0.006	0.078±0.013	0.113±0.015	0.127±0.016	0.139±0.018	0.106±0.015	0.059±0.011	0.036±0,009	0.009±0.005
	5	0.012±0.005	0.040±0.010	0.101±0.015	0,090±0.014	0.083±0.013	0.064±0.012	0.010±0.005	0.013±0.006	0
	6	0.002±0.002	0,016±0.007	0.031±0.008	0.019±0.006	0,026±0.007	0.019±0.006	0.007±0.005	0.007±0.005	0
	7	0	0,014±0.006	0.009±0.005	0,021±0.007	0.019±0.006	0.014±0.006	0.007±0.005	0	0
	8	0	0	0.002±0,002	0.005±0.003	0	0.005±0,003	0.005±0.003	0	0
	9	0	0	0,004±0.002	0.002±0.002	0	0	0	0	0

the shower energy, the same as for $E_0 = 330$ MeV at $t = 49.8$ g/cm^2), turned out to be 80%. Comparison of this quantity with that obtained at $E_0 = 330$ MeV shows that the distribution width is proportional to $E_0^{-1/2}$.

4. ELECTRON-NUMBER FLUCTUATIONS

The probabilities $W_N(E_0, E, t)$ of recording N electrons at a specified shower depth t were determined by counting the number of tracks in the gaps between the plates for each shower. The obtained values of the probabilities are listed in the table.

As seen from Fig. 2, the distinct nature of the probabilities for recording showers with an odd number of electrons becomes clearly pronounced in the region up to the maximum of the cascade curve, and to a lesser degree in the region of the maximum. This "even-odd" effect was first noted by Wilson [10] and related to the fact that an electron pair is produced upon conversion of a γ quantum. In the case of showers produced by γ quanta, this effect is manifest in the fact that showers with even number of electrons predominate. As the shower develops and its energy is subdivided, ionization losses and other interactions of γ quanta and electrons with matter, which lead to the vanishing of the "even-odd" effect, become significant besides pair production and bremsstrahlung. The presence of the "even-odd" effect is one of the main difficulties in the theoretical description of a shower at low development depths.

To describe the experimentally obtained functions $W_N(E_0, E, t)$, we have attempted to use the Poisson distribution (Fig. 2). The discrepancy between this distribution and the obtained probabilities is particularly large at small thicknesses. In the region of the maximum of the cascade curve, the Poisson distribution agrees satisfactorily with the experimental data. Beyond the maximum, however, the discrepancy again increases. Thus, the Poisson distribution cannot be used to describe the fluctuations of the number of particles in the shower in an interval t of any appreciable magnitude.

The use of the Furry distribution [4] for the description of the shower fluctuations is even less justified, especially at low depths [7]. This distribution was obtained for a very simplified scheme of the cascade process. Furry's scheme disregards the difference between the electrons and

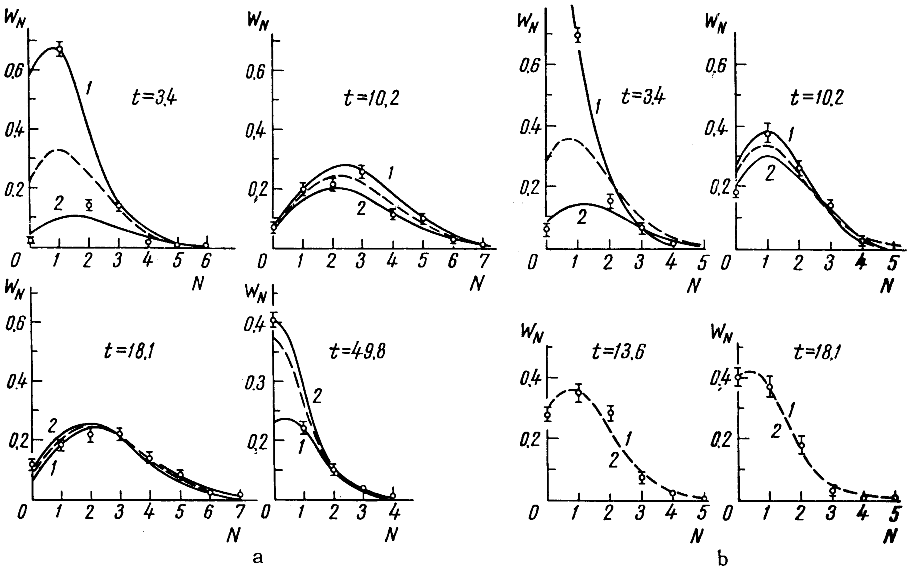

FIG. 2. Probabilities $W_N(E_0, E = 1$ MeV, t) for different absorber thicknesses (in g/cm²): a – $E_0 = 330$ MeV, b – $E_0 = 130$ Mev. For clarity in comparing the experimental data with the approximating distributions, the figure shows the generalized distributions [Eq. (4)], in which the factorial of the integer N is replaced by the generalized factorial – the gamma function $\Gamma(N + 1)$. The curves 1 show the distributions (4) corresponding to odd N, and the curves 2 correspond to even N. The dashed curves are the generalized Poisson distributions.

the photons, and neglects the ionization loss. The Furry distribution is defined only for N > 0 and does not reflect the "even-odd" effect.

To describe the shower fluctuations, we used a distribution describing the shower with the aid of two independent Poisson distributions, which determines the values of $W_N(E_0, E, t)$ for either even or odd values of N only. For odd N

$$W_N \sim e^{-\alpha_1}\alpha_1^N / N! \tag{3}$$

and analogously for even N. Normalizing this distribution we obtain

$$W_N = \begin{cases} a\alpha_1^N/\sinh\alpha_1 N!, & N - \text{ odd} \\ (1-a)\alpha_2^N/\cosh\alpha_2 N!, & N - \text{ even} \end{cases} \tag{4}$$

The value of α_1 is connected with the mean value ($\bar{N}_1$) of the number of electrons N in showers with an odd number of electrons by the relation

$$\bar{N}_1 = \alpha_1 \coth \alpha_1. \tag{5}$$

Similarly for "even" showers

$$\bar{N}_2 = \alpha_2 \tanh \alpha_2. \tag{6}$$

The mixing parameter a in the distribution (4) is determined experimentally.

Figure 2 shows the "even" and "odd" distributions (4), corresponding to the experimentally obtained values of W_N. As seen from this figure, the distribution (4) describes satisfactorily the fluctuations in the number of electrons in the shower at all depths of its development. An equally good agreement was obtained also at energies E_0 equal to 220 and 45 MeV. As shown by our calculations, the distribution (4) also describes satisfactorily

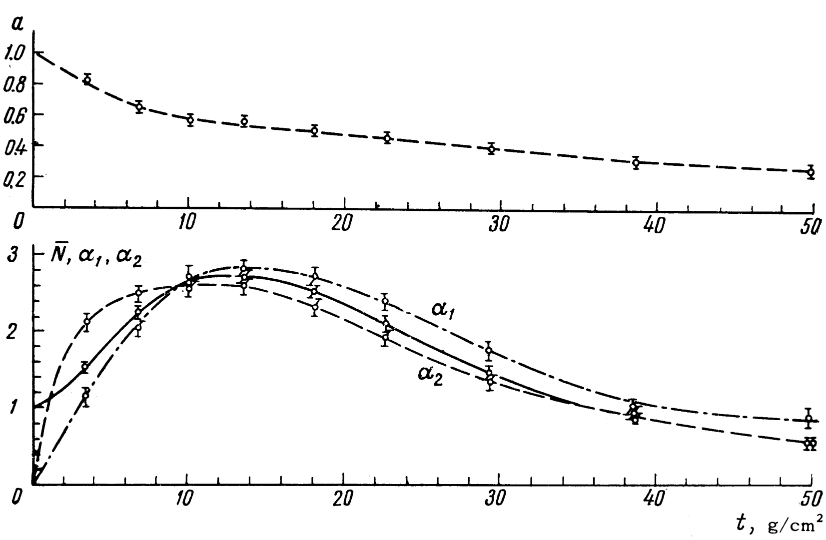

FIG. 3. Dependence of the parameter a and of the quantities α_1, α_2, and $\bar{N}$ on the absorber thickness at $E_0 = 330$ MeV. The curves are drawn through the experimental points.

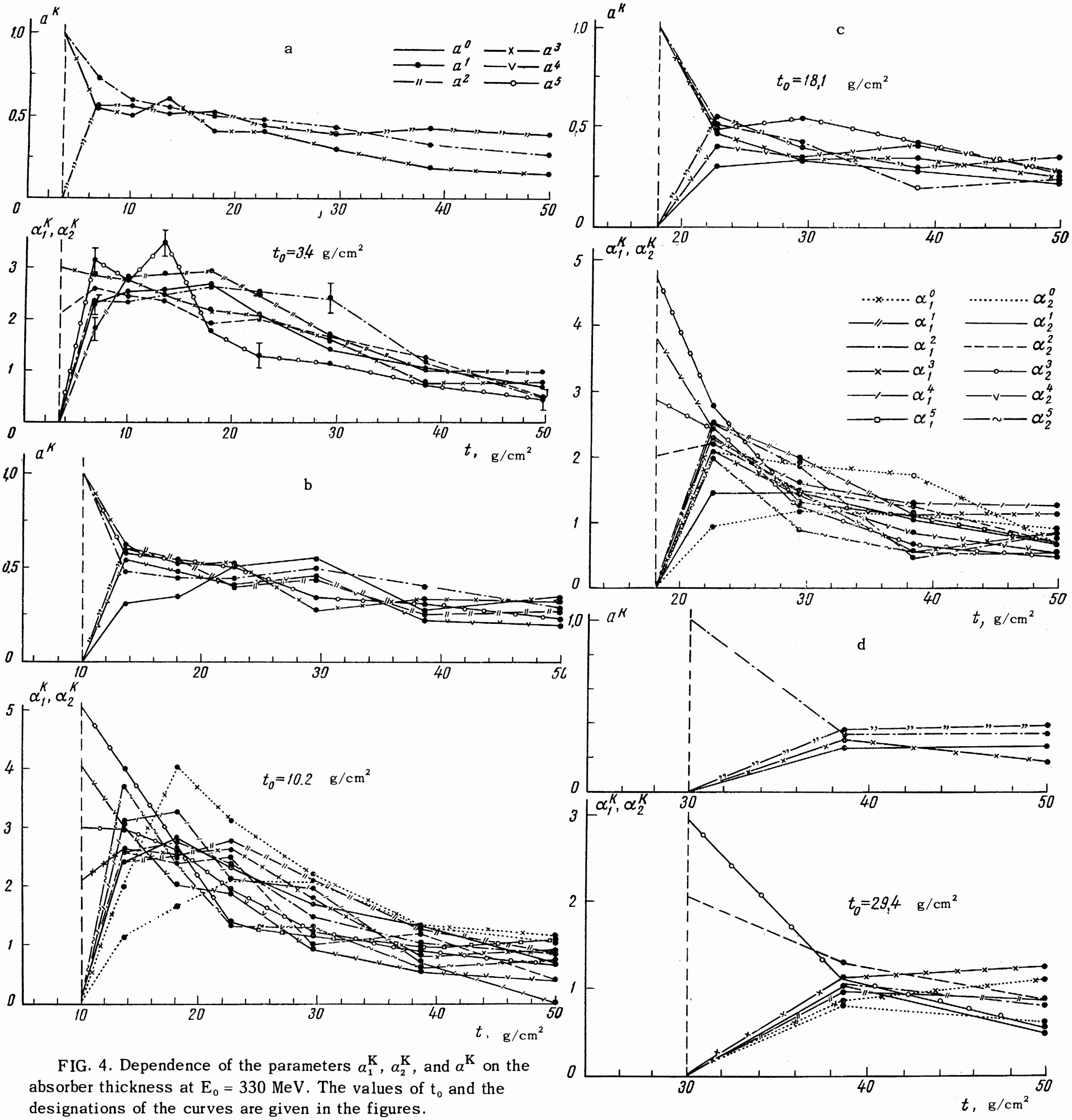

FIG. 4. Dependence of the parameters α_1^K, α_2^K, and a^K on the absorber thickness at $E_0 = 330$ MeV. The values of t_0 and the designations of the curves are given in the figures.

the electron-number fluctuations obtained in lead in investigations of showers in a propane chamber [7].

Figure 3 shows the dependence of α_1 and α_2 on the thickness t at $E_0 = 330$ MeV as obtained in the present investigation. Together with the obtained dependence of the parameter a on t (Fig. 3), the functions $\alpha_1(t)$ and $\alpha_2(t)$ describe the fluctuation of the number of electrons in a shower at given thicknesses t.

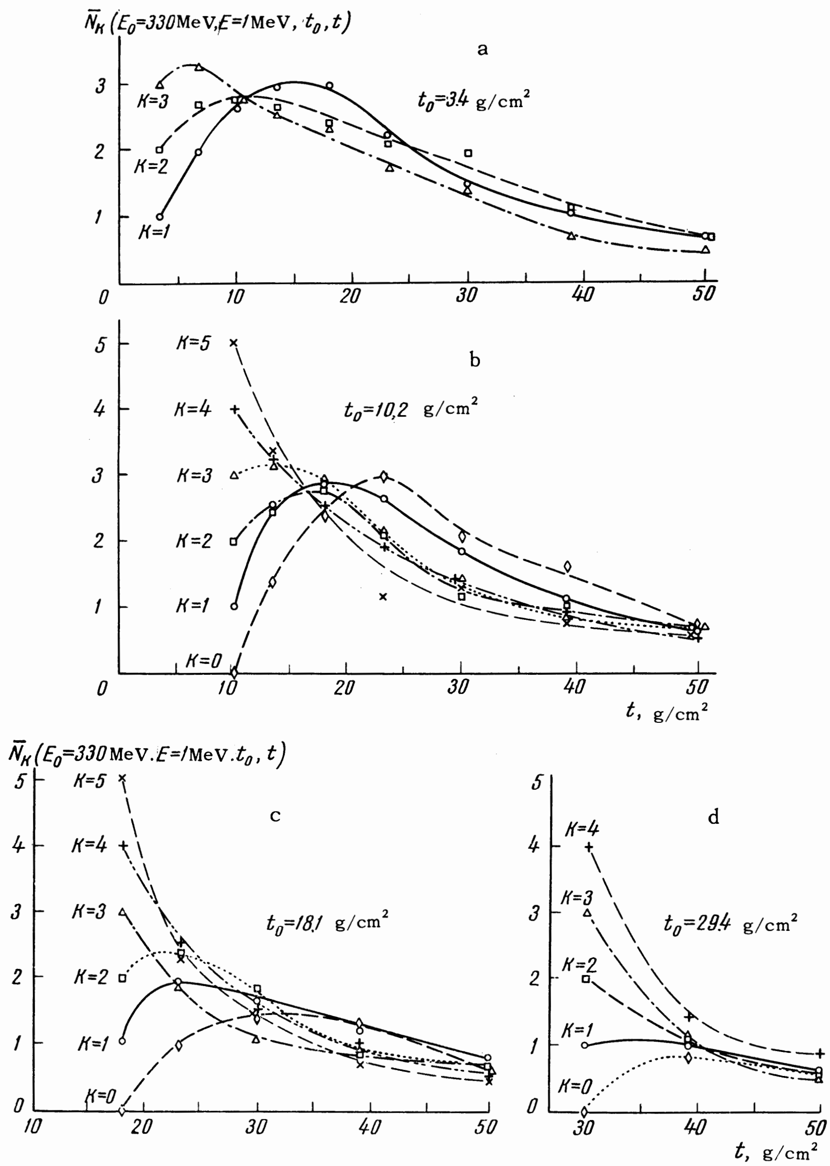

FIG. 5. Average number of electrons $\bar{N}_K(E_0, t_0, t)$ vs. absorber thickness t for different electron numbers K at the depth t_0.

5. FLUCTUATION CORRELATIONS

The fluctuations of the number of electrons in the shower at a given depth are not independent, and are correlated with the fluctuations of the number of electrons at lower depths. Knowledge of these correlations is essential for the construction of multilayer electron and γ counters, shower spark chambers, etc.

It is convenient to characterize the fluctuation correlations in showers by means of the function $W_{KN}(E_0, E, t_0, t)$, defined as the probability of registering N electrons at a depth t, if K electrons were registered at depth t_0. The values of $W_{KN}(E_0, E, t_0, t)$ were obtained by us for four primary-electron energies. Just like the probabilities $W_N(E_0, E, t)$, they are well described by the distributions (4). The values of the parameters α_1^K, α_2^K, and a^K, characterizing these distributions and obtained at $E_0 = 330$ MeV, are shown in Fig. 4. Plots of α_1^K, α_2^K, and a^K against t have a similar character.

As seen from Fig. 4, the behavior of showers having different values of K differs appreciably only at distances 6—8 g/cm^2 from the point t_0. Even in the case of large fluctuations, the fluctuation correlation in the showers turns out to be significant only at a distance of approximately one radiation unit. A similar decrease in the correlation takes place for all the investigated energies E_0.

Additional information concerning the correlation of fluctuations in showers can be obtained by comparing the fluctuations at the point t_0 and the average number of electrons at the depth, i.e., comparison of the cascade curves $\bar{N}_k(E_0, E, t_0, t)$

for showers having at the depth t_0 a different number of electrons K. As seen from Fig. 5, the difference in the values of $\overline{N}_K$ becomes appreciably smoothed out at a distance of one radiation unit from t_0. However, the effect of appreciable fluctuations on the subsequent development of the shower can be traced also at large distances: showers with small number of K are shifted toward large thicknesses t. This effect is clearly seen at small values of t_0 but when t_0 shifts into the region beyond the maximum of the cascade curve, the effect gradually disappears.

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