THEORY OF ANTIFERROMAGNETIC RESONANCE IN METALS

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The role of plasma effects in antiferromagnetic resonance is elucidated. It is shown that in the case of a comparatively weak magnetic field, undamped waves can be propagated in an antiferromagnetic metal; one of these is an auxiliary wave due to spatial dispersion. The frequency dependence of the surface impedance is found, and the character of the singularity is explained. Coupled spin-helicon waves in an antiferromagnetic metal are considered, and the appropriate expressions are obtained for the polarization of the waves and for the reflection coefficient.

1. Consideration of the electrodynamics of ferromagnetic metals, at frequencies close to ferromagnetic resonance, has shown that the frequency dependence of the magnetic susceptibility appreciably changes the character of electromagnetic waves in the metal[1-4]. It is natural to carry over a similar treatment to antiferromagnetic metals. In this, the following things must be kept in mind. First, the energy spectrum and, consequently, the frequency dependence of the magnetic susceptibility depend appreciably on the structure of the ground state of the antiferromagnet. That state can be modified, comparatively easily in fact, by an external magnetic field. This circumstance must be taken into account in any elucidation of the dependence of the high-frequency characteristics of an antiferromagnetic metal (the surface impedance $Z(\omega)$) on the magnetic field. Second, in a uniaxial antiferromagnet with a positive anisotropy constant[1)] the resonance frequencies, even at not too high magnetic fields, are shifted (in comparison with ferromagnets) into a shorter-wave region. In consequence, it is comparatively easy for a situation to arise in which the effect of the magnetic field on the electrical conductivity can be neglected, yet the frequency (temporal) dispersion of this effect must be taken into account.

The role of the magnetic field in the structure of the ground state of a magnet and in the nature of the electrical conductivity is determined by parameters of quite different nature. In the first case, the magnetic field must be compared with the anisotropy field, the exchange field, etc.[6]; in the second, the radius of the electron's orbit must be compared with the length of its free path. Therefore very different situations are possible. We are not presenting all the possible limiting cases but are limiting ourselves to a few examples.

In the case of weak fields (according to both criteria), the role of spatial dispersion in the variation of the surface impedance, near the antiferromagnetic resonance frequencies, is elucidated (Sec. 2). In Sec. 3, the influence of the magnetic field on the electrical conductivity is considered; here it is supposed that the resonance frequencies lie in the range where helicon waves exist (see, for example,[1,3]). In Sec. 4 it is explained how the nature of the ground state is reflected in the properties of electromagnetic waves (here spatial dispersion is neglected). Since the ground state is manifested most significantly in the polarization of the waves, this question receives much attention. The collapse of the magnetic moments in a strong magnetic field[6] is accompanied by an approach to infinity of the relaxation time of the magnetic moments; the Appendix considers the effect of this on the frequency dependence of the absorption in antiferromagnetic metals and dielectrics.

We shall suppose that the constant magnetic field is directed along the preferred axis of the uniaxial antiferromagnet (Fig. 1a), and that an electromagnetic wave is being propagated along this same axis. In the case of a half-space (see below),

[1)] In the present communication we discuss only such antiferromagnets. We remark that an antiferromagnet of the type considered, in a magnetic field $H > H_a = M[2\alpha(\lambda + \eta)]^{1/2}$ (see below), simulates a uniaxial antiferromagnet with negative antisotropy constant, since at $H \gtrsim H_c$ the magnetic moments of the sublattices are almost parallel to the plane perpendicular to the chosen axis. "Escape" of the magnetic moments from this plane is similar to weak ferromagnetism [5].

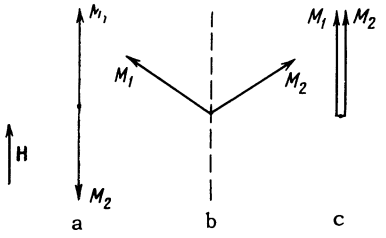

FIG. 1

this means that the axis of the antiferromagnet and the magnetic field are perpendicular to the surface, and that the electromagnetic wave is normally incident on it.

The magnetic-susceptibility tensor was calculated in[6]; there it was shown that at fields

$$H < H_a = M[2a(\lambda + \eta)]^{1/2},$$

where M is the magnetic moment of a sublattice and λ and η are the anisotropy constants (we remark that $\lambda, \eta \sim 1$), and where α is the constant of exchange interaction between the sublattices ($\alpha = \Theta/\mu_0 M \gg 1$, Θ = a quantity of the order of the Curie-Néel temperature of the antiferromagnet, μ_0 = the Bohr magneton), it is convenient to define the magnetic susceptibility through quantities $\mu_\pm$:

$$b_\pm = \mu_\pm h_\pm, \quad b_3 = h_3,$$

where $b_\pm = b_1 \pm ib_2$ and $h_\pm = h_1 \pm ih_2$ are, respectively, the high-frequency components of the magnetic induction and of the magnetic field. Axis 3 is directed along the constant magnetic field. If we neglect dissipative terms, the expression for the circular components $\mu_\pm$ can be written in the form

$$\mu_\pm = \frac{\omega \mp gH + (\Omega^2 + 4\pi\Omega_1^2)^{1/2}}{\omega \mp gH + \Omega} \frac{\omega - \omega_a^\pm}{\omega - \omega_p^\pm}, \qquad (1)$$

where

$$\omega_a^\pm = [\Omega^2 + 4\pi\Omega_1^2]^{1/2} \pm gH, \quad \omega_p^\pm = \Omega \pm gH,$$
$$\Omega = gH_a, \quad \Omega_1 = \sqrt{2(\lambda + \eta)}\, gM$$

(g is the gyromagnetic ratio, and $\hbar g = \mu_0$).

For $H > H_a$, turnover of the spins occurs (Fig. 1b), and the magnetic susceptibility takes the form

$$\mu_{ik} = \begin{pmatrix} \mu_1 & i\mu_{12} & 0 \\ -i\mu_{12} & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}; \quad \mu_1 = 1 + \frac{4\pi\omega_1^2 \cos^2\theta}{\omega_0^2 - \omega^2} \qquad (2)$$

$$\mu_2 = 1 + \frac{4\pi}{\alpha} \frac{\omega_0^2}{\omega_0^2 - \omega^2}, \quad \mu_3 = 1 + \frac{4\pi}{\alpha} \frac{\nu^2 + i\nu\omega}{\nu^2 + \omega^2},$$

$$\mu_{12} = \frac{8\pi g M \omega \cos\theta}{\omega_0^2 - \omega^2}. \qquad (3)$$

The characteristic frequencies that follow from the formula (3) can be expressed in the following form:

$$\omega_0^2 = g^2M^2[4\alpha^2\cos^2\theta - 2(\lambda + \eta)\alpha\sin^2\theta], \quad \omega_1^2 = 4\alpha g^2M^2;$$
$$\cos\theta = H/2\alpha M \approx H/H_e \quad (H_e = 2\alpha M). \qquad (4)$$

In the expression for the longitudinal component of the magnetic susceptibility, we have kept the dissipative term $\nu = 2\alpha\gamma\sin^2\theta$, where γ is the relaxation constant (of the dimensions of frequency) that occurs in the Landau-Lifshitz equation[6]. The reciprocal of ν is called the magnetic relaxation time and is denoted by τ_M ($\tau_M = \nu^{-1}$).

When $H \approx H_e = 2\alpha M$, the magnetic moments of the sublattices set themselves parallel to one another; and when $H > H_e$, the magnetic structure becomes insensitive to the size of the magnetic field.

To construct the electrodynamics of an antiferromagnetic metal, it is still necessary to give the relation between the current density and the electric field—the electrical conductivity tensor σ_{ik}.

2. We consider first the case of comparatively weak magnetic fields ($r_H \gg l$, where r_H is the radius of the electron orbit in the magnetic field and where l is the length of the free path). In this case we can neglect altogether the effect of the magnetic field on the motion of the electrons and can set

$$\sigma_{ik} = \sigma(\omega)\delta_{ik}, \quad \sigma(\omega) = \sigma_0/(1 - i\omega\tau),$$

where τ is the time of free travel. When $\omega\tau \gg 1$,

$$\sigma(\omega) \approx i\sigma_0/\omega\tau.$$

In the case of a quadratic, isotropic dispersion law, the static electrical conductivity σ_0 is equal to $Ne^2\tau/m^*$ (N = density of electrons, m^* = effective-mass coefficient). The inequality $\omega\tau \gg 1$, for sufficiently pure specimens at a low temperature ($l \gtrsim 10^{-3}$ cm), is satisfied when $\omega > 10^{11}$ sec^{-1}; that is, already in the centimeter range.

When the electrical conductivity is imaginary, it is more convenient to describe the metal by its dielectric permittivity

$$\varepsilon(\omega) = -\omega_L^2/\omega^2, \qquad (5)$$

where $\omega_L^2 = 4\pi Ne^2/m^*$ is the square of the plasma (Langmuir) frequency of the electrons. For an arbitrary dispersion law, the expression for the permittivity is actually not changed, but the square of the plasma frequency is related in a more complicated way to the microscopic characteristics of the conduction electrons[7,8].

In weak magnetic fields, it is natural to go over to circularly polarized waves ($E_1 \pm iE_2$ etc.). Then the dispersion equation, which relates the frequency ω to the wave vector k of a plane monochromatic wave, splits up and takes the very simple form

$$k^2 = \omega^2 c^{-2} \varepsilon(\omega) \mu_\pm(\omega), \qquad (6)$$

where $\mu_\pm$ is defined by Eqs. (1). Figure 2 depicts the dependence of the index of refraction $n^2 = c^2k^2/\omega^2$ on frequency according to Eq. (6). There are regions of transparency for both polarizations. We recall that in an ordinary metal at $\omega\tau \gg 1$, waves cannot propagate at all[9].

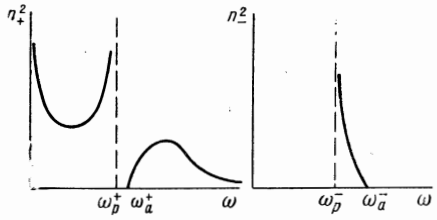

FIG. 2

At a frequency close to the resonance frequency (for example, when $\omega \approx \Omega + gH$ for "plus" polarization), account must be taken of the spatial dispersion of the magnetic susceptibility tensor. If we restrict ourselves to the leading terms in the expansion of the magnetic susceptibility in powers of the square of the wave vector, and if we replace the frequency by $\omega_p^\pm$ everywhere except in the resonance denominator, we get an equation for the index of refraction that is useful for both polarizations:

$$n^2 = \varepsilon / (\beta_\pm n^2 - \xi). \qquad (7)$$

Here

$$\beta_\pm = \frac{\Theta}{\hbar\omega_a^\pm} \frac{a^2(\omega_p^\pm)^2}{c^2}, \qquad \xi = \frac{\omega - \omega_p^\pm}{\omega_a^\pm},$$

a is the lattice constant. The relation of the constant Θ to the exchange integral can be found in[7].

It is convenient to rewrite Eq. (7), by solving for ξ (see Fig. 3),

$$\xi = |\varepsilon| / n^2 + \beta_\pm n^2. \qquad (8)$$

From this it is evident that in an antiferromagnetic metal (when $\omega\tau \gg 1$), auxiliary waves can always propagate when $\omega > \omega_g^\pm = \omega_p^\pm(1 + 2\sqrt{|\epsilon|\beta_\pm})$. The limiting frequency $\omega_g^\pm$ should under these conditions be regarded as a resonance frequency, since just at $\omega = \omega_g^\pm$ there is observed a singularity of the surface impedance $Z_\pm$. As was shown in[3], near this frequency the surface impedance is very sensitive to the nature of the behavior of the magnetic moment at the boundary. This is due to the fact that when $\omega = \omega_g^\pm$, the field in the metal forms a standing wave with wave vector $k = \omega_g^\pm c^{-1} n_{min}^\pm$, where $n_{min}^\pm = (|\epsilon|/\beta_\pm)^{1/4}$ is the value of the index of refraction at $\omega = \omega_g^\pm$ (see Eq. (8)). The surface impedance under these conditions is determined by the phase of the standing wave; the phase in turn is determined by the relation between the magnetic moment and its derivative along the normal at the boundary. If we start from the assumption that at the boundary the magnetic moment induced by the external magnetic field vanishes, then it is easy to obtain the following expression for the impedance ($Z_\pm = ie_\pm(0)/h_\pm(0)$):

$$Z_\pm = \frac{1}{\beta_\pm} \frac{1}{n_1 n_2 (n_1 + n_2)}. \qquad (9)$$

Here n_1 and n_2 are the roots of Eq. (7); the signs of the roots must be so chosen that with allowance for dissipation, the field would attenuate with penetration into the metal (as $z \to \infty$). When $\omega > \omega_g^\pm$, so that n_1 and n_2 are real numbers, $n_1 < 0$, whereas $n_2 > 0$. This choice of signs is connected with the fact that the left branch of the curve in Fig. 3 describes anomalous dispersion, whereas the right describes normal dispersion.

On substituting the values of n_1 and n_2 in formula (9), we get the explicit dependence of the surface impedance on the frequency near resonance:

a) for $\omega > \omega_g^\pm$,

$$R_\pm = \frac{\sqrt{2|\varepsilon|}}{\sqrt{\xi^2 - 4|\varepsilon|\beta_\pm}} \{[\xi + \sqrt{\xi^2 - 4|\varepsilon|\beta_\pm}]^{1/2} + [\xi - \sqrt{\xi^2 - 4|\varepsilon|\beta_\pm}]^{1/2}\}, \qquad X_\pm = 0;$$

b) for $\omega < \omega_g^\pm$,

$$R_\pm = 0, \qquad X_\pm = -(|\varepsilon|)^{-1/2}[2\sqrt{|\varepsilon|\beta_\pm} - \xi]^{-1/2}. \qquad (11)$$

It is seen from the last formulas that on approach to resonance from the higher-frequency side, the real part of the impedance has a root-type singularity ($R \sim 1/\sqrt{\omega - \omega_g}$); whereas on approach from the lower-frequency side, the imaginary part has one ($X \sim 1/\sqrt{\omega - \omega_g}$).

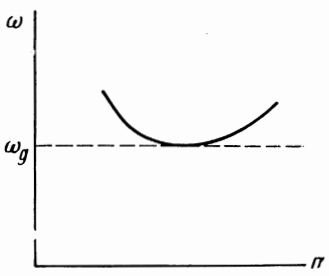

FIG. 3

It is natural that allowance for dissipative processes should somewhat blur this picture. In par-

ticular, the root-type singularity can be detected only in case the total line width $\Delta\omega$ is smaller than the shift of resonance due to spatial dispersion, $\omega_g - \omega_p$. When spatial dispersion is taken into account, the complex nature of the dielectric permittivity leads to an additional widening $\Delta\omega_\tau$ of the resonance line; it is therefore natural to compare the size of $\Delta\omega_\tau$ with $\omega_g - \omega_p^\pm = 2\sqrt{|\epsilon|\beta_\pm}\omega_p^\pm$.

By expanding $\sigma(\omega)$ in powers of $1/\omega\tau$ and keeping the first term, it is easy to show that $\Delta\omega_\tau \ll |\omega_g - \omega_p|$. In fact, on putting

$$\varepsilon(\omega) = -\omega_L^2\omega^{-2}(1 - i/\omega\tau),$$

we find from the dispersion equation (6)

$$\Delta\xi_g = 2\sqrt{|\varepsilon|\beta_\pm}, \quad \Delta\xi_\tau = \sqrt{|\varepsilon|\beta_\pm}/\omega\tau.$$

Thus the skin effect in certain cases gives a possibility of observing not only spatial dispersion of the magnetic susceptibility, but also auxiliary undamped waves due to the spatial dispersion. The existence of auxiliary waves is directly connected with the negative sign of the permittivity. We recall that in the optical range, an auxiliary wave can be propagated only if an exciton possesses a negative effective mass (see, for example,[10]) (under our conditions, when $\beta < 0$). But a spin wave ("magnetic exciton") has a positive effective mass. Therefore undamped auxiliary waves can exist only when the dielectric permittivity is negative. Furthermore, in metals (when $1/\tau \ll \omega \ll \omega_L$), $|\epsilon| \gg 1$. This appreciably increases the index of refraction and facilitates observation of spatial dispersion.

We remark that observation of spatial dispersion in magnets is favored by the circumstance that the coefficient associated with k^2 is connected with the exchange energy ($\sim\Theta/\hbar$), while the resonance and other frequencies in $\mu_{ik}(\omega)$ are connected with the anisotropy energy. Because of this, in dimensionless variables the coefficient associated with n^2 in the denominator contains an additional large factor $\Theta/\hbar\omega_p$. For observation of spatial dispersion it is of course necessary that the antiferromagnetic resonance line shall be narrow. The treatment carried out here shows only that the broadening of the antiferromagnetic resonance line by spatial dispersion is appreciably smaller than the shift of the resonance frequency.

3. We now consider the opposite limiting case. Let $\omega\tau \ll 1$, but $l \gg r_H$. This is possible if

$$1/\omega_H \ll \tau \ll 1/\omega, \qquad (12)$$

where ω_H is the Larmor precession frequency of the electrons. When estimates are made specific, it is necessary to remember that in what follows, still one more condition must be satisfied: $\omega\tau_M \gg 1$, where τ_M is the magnetic relaxation time, determined by the width of the magnetic resonance line ($\Delta\omega \sim 1/\tau_M$). We suppose further that normal skin effect prevails[2)]; that is, $kl \ll 1$.

When this condition and conditions (12) are satisfied, the form of the effective permittivity is appreciably dependent on the energy spectrum of the electrons. If $N_1 \neq N_2$ (N_1 = electron density, N_2 = "hole" density) and the Fermi surface is closed, then

$$\varepsilon_{\text{eff}}^{\pm} = \pm\frac{4\pi\sigma_{12}}{\omega}, \quad \sigma_{12} = \frac{(N_1 - N_2)ec}{H}, \qquad (13)$$

where σ_{12} is the Hall component of the electrical conductivity, and where the signs correspond to waves with right and left circular polarization[11,12]. On substituting the expression (13) in the dispersion equation (6), we find the relation between the index of refraction n and the frequency ω (see Fig. 3). The impedance in this case also is described by formula (9), where now n_1 and n_2 are the roots of the dispersion equation

$$n^2 = \pm|\varepsilon|/(\beta_\pm n^2 - \xi). \qquad (14)$$

We notice that undamped auxiliary waves occur for only one of the two polarizations: for left circular polarization if $\sigma_{12} > 0$, for right if $\sigma_{12} < 0$. The change of the law of dispersion for a polarization for which no auxiliary wave occurs is shown in Fig. 4. In this case the surface impedance has no singularity at all. The possibility of such a (macroscopic) treatment, however, is limited to those frequencies at which the index of refraction is not too large.

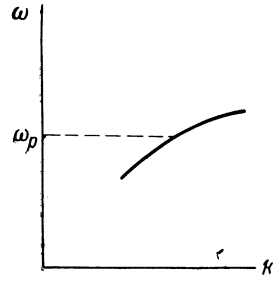

FIG. 4

4. The transition to large magnetic fields is accompanied by a rearrangement of the ground

[2)] In the previous case ($\omega\tau \gg 1$), there was no necessity to be more precise about the nature of the skin effect, because when $\omega\tau \gg 1$ the nature of the skin effect determines only the dissipative part of the effective dielectric permittivity, which we are omitting.

state of the antiferromagnet. For $H > H_a$ the equilibrium structure is that pictured in Fig. 1b; the magnetic susceptibility tensor is determined by the expressions (2) and (3). The change of the magnetic susceptibility, naturally, expresses itself in the dispersion law of the waves that propagate in the metal and in the character of the reflection of waves from a metallic half-space. We investigate the role of the structure of the ground state of an antiferromagnet, omitting comparatively fine effects that are due to allowance for spatial dispersion.

Using a standard method, we obtain

$$n_\pm^2 = \pm\frac{4\pi\sigma_{12}}{\omega}(\sqrt{\mu_1\mu_2}\pm\mu_{12}), \qquad (15)$$

$$Z_\pm = \left(\frac{\omega}{4\pi\sigma_{12}}\right)^{1/2}[\mu_{12}\pm\sqrt{\mu_1\mu_2}]^{1/2}. \qquad (16)$$

To the indices of refraction n_+ and n_- correspond different polarizations of the waves that propagate in the metal:

$$E_\pm = E_1 \pm i\sqrt{\mu_2/\mu_1}\,E_2, \qquad H_\pm = H_1 \pm i\sqrt{\mu_2/\mu_1}\,H_2. \quad (17)$$

At those frequencies at which $\mu_2/\mu_1 > 0$, these are elliptically polarized waves; at those for which $\mu_2/\mu_1 < 0$, plane-polarized. It is easy to see that the last condition, $\mu_2/\mu_1 < 0$, is satisfied in the narrow frequency interval

$$\omega_a\left(1+\frac{2\pi}{\alpha}\right) < \omega < \omega_a\left(1+\frac{2\pi}{\alpha}\right) + 4\pi gM(\lambda+\eta)\sin^2\theta.$$

When $\mu_2/\mu_1 < 0$, the principal directions are not orthogonal to each other; and when $\mu_1 = 0$ or $\mu_2 = 0$, they fuse (a similar phenomenon was described in[12]).

The nature of the polarization of the waves that propagate in the material depends considerably on the symmetry of the tensors ϵ_{ik} and μ_{ik}. If the effective dielectric permittivity possesses no gyrotropy, the dispersion equation and the surface impedance are described as follows:

$$n_\pm^2 = \frac{1}{\gamma_\pm}\varepsilon(\mu_1-\gamma_\pm\mu_{12}), \quad Z_\pm = \gamma_\pm\left[\frac{\mu_1+\gamma_\pm\mu_{12}}{\varepsilon}\right]^{1/2}, \quad (18)$$

where

$$\gamma_\pm = \frac{i}{2\mu_{12}}[\mu_1-\mu_2\pm((\mu_1-\mu_2)^2+4\mu_{12}^2)^{1/2}].$$

Formulas (18) are correct for an antiferromagnetic dielectric. In this case the dielectric constant is of order unity and is positive. Formula (18) can also be applied, however, to an antiferromagnetic metal, by setting $\epsilon_{eff} = -\omega_L^2/\omega^2$. The choice between formulas (18) and formulas (15) and (16) is tied up with the satisfaction of the corresponding conditions. Formulas (18) are applicable if $\omega\tau \gg 1$ but $l \ll r_H$.

For isotropic dielectric permittivity,

$$E_\pm = E_1 - \gamma_\pm^{-1}E_2, \qquad H_\pm = H_1 + \gamma_\pm H_2, \qquad (19)$$

that is, in this case the electromagnetic waves are always elliptically polarized.

Knowledge of the surface impedance, of course, permits us to find the amplitudes E′ of the waves reflected from a half-space. The general formulas are very cumbersome. We write out only limiting cases. On a metal, even near resonance, $|Z| \ll 1$, and

$$E_1' = -E_1\left[1-\frac{\gamma^2-1}{2\gamma}(Z_+-Z_-)\right]+iE_2(Z_++Z_-),$$

$$E_2' = -E_2\left[1+\frac{\gamma^2-1}{2\gamma}(Z_+-Z_-)\right]-iE_1(Z_++Z_-). \quad (20)$$

For a dielectric near resonance, $|Z| \gg 1$ and

$$E_1' = E_1\left(1+\frac{\gamma^2-1}{2\gamma}\frac{Z_--Z_+}{Z_+Z_-}\right)-iE_2\frac{Z_-+Z_+}{Z_+Z_-},$$

$$E_2' = E_2\left(1-\frac{\gamma^2-1}{2\gamma}\frac{Z_--Z_+}{Z_+Z_-}\right)+iE_1\frac{Z_-+Z_+}{Z_+Z_-}. \quad (21)$$

The last formulas enable us to calculate all the optical properties of the reflected wave—the rotation of the plane of polarization, or the ellipticity and amplitude.

APPENDIX

It was shown in[6] that collapse of the magnetic moments of the sublattices, which sets in at $H = H_e$, is accompanied by increase to infinity of the relaxation time τ_M, which enters into the expression for the longitudinal component of the magnetic susceptibility (see formula (3)). In fact, according to formula (8′) of[6]

$$\nu = \nu_0(1-H^2/H_e^2), \quad H < H_e, \quad \nu_0 = \gamma H_e/M.$$

Observation of this effect can be carried out through the volume absorption of the energy of the magnetic field. As a characteristic of this absorption we may take the absorption coefficient κ of an electromagnetic wave:

$$\varkappa = \frac{4\pi\varepsilon}{\alpha}\frac{\nu\omega}{\omega^2+\nu^2}. \qquad (22)$$

We consider propagation of an electromagnetic wave through an antiferromagnetic dielectric in a direction perpendicular to the static magnetic field.

Of interest is the dependence of κ on the size of the magnetic field near $H = H_e$ (at fixed frequency). If we construct the curve $\kappa = \kappa(H)$, the form of this curve depends appreciably on the relation between the frequency ω and the value of ν_0. If $\omega < \nu_0$, the function $\kappa = \kappa(H)$ has a maximum; but if $\omega > \nu_0$, κ

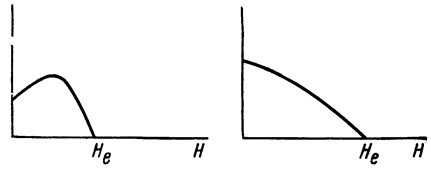

FIG. 5

approaches zero monotonically as H approaches H_e (see Fig. 5).

The infinity in τ_M can also be observed through the change of the nature of the reflection. It is easy to show that at $H = H_e$ the imaginary part X of the impedance should decrease abruptly (for real dielectric permittivity, X vanishes). Since the effective dielectric permittivity of a metal is negative, the infinity in τ_M leads to an abrupt decrease of the real part R of the impedance.

We take this occasion to thank A. Ya. Blank for helpful discussions.

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