

PHOTON BREMSSTRAHLUNG IN COLLISION OF HIGH-ENERGY ELECTRONS

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Single bremsstrahlung occurring upon collision of high-energy electrons is considered. The angular distribution and the spectrum of the emitted photons is calculated in the c.m.s. and in the lab., and the radiation of the incoming particle and of the recoil particle is considered in the latter system. The method of the classical currents and the Weizsacker-Williams method are analyzed. It is shown that the latter is applicable to the calculation of the spectrum of the recoil photons emitted by the particle when $\omega > m/2$.

1. Bremsstrahlung of a photon when an electron collides with an electron (positron) is one of the principal electrodynamic processes. Therefore an investigation of this phenomenon, and in particular the spectrum of the emitted photons and their angular distribution, is of considerable interest, particularly for the study of radiative corrections. In addition, this process was investigated recently experimentally both in colliding electron-positron beam experiments^[1] and in collisions of high-energy electrons with immobile electrons^[2].

For a theoretical description of this process we can confine ourselves, with sufficient degree of accuracy, to the lower (e^6) approximation of perturbation theory, with the exception of the region of very soft photons, where many-photon processes become significant (we shall not concern ourselves with this region). In the indicated approximation, the process is represented by eight diagrams (four direct and four exchange (annihilation)). The determination of the differential cross section of the process reduces to the rather laborious calculation of traces and leads to a very cumbersome expression (see^[3], and in the ultrarelativistic limit^[4]). Exact integration of this expression over the emission angles of the final particles, aimed at obtaining the spectrum of the emitted photons (differential cross section with respect to the photon frequency) is a very complicated matter and has not yet been done. However, in limiting cases (nonrelativistic and ultrarelativistic) such a calculation is possible. We shall consider the ultrarelativistic case, when the result for the spectrum can be expanded in inverse powers of the energy. Unfortunately, the earlier results (see^[5,6]) are contradictory.

2. To investigate the photon emission during collisions of electrons (or electrons and positrons) and two-particle electron-positron pair annihilation, a procedure was developed earlier for calculating the integral cross sections with respect to the final states of the particles (in particular, the spectrum of angular distribution of the emitted photons) with the aid of invariant integration over the contribution of individual fermion lines^[7,8]. This procedure turned out to be quite convenient also for an analysis of single photon emission processes. It is possible in this case to obtain an expression for the cross sections for a wide range of reference frames; in the c.m.s. the electrons, naturally, radiate in identical fashion, while in the laboratory system (lab.) it is necessary to distinguish between the emission of the fast (incident) electron and the recoil electron which is at rest prior to the collision.

The bremsstrahlung process in electron collisions can also be described with the aid of approximate methods. With the aid of the Weizsacker-Williams (WW) method it is possible to calculate with logarithmic accuracy the spectrum of the emitted photons in the c.m.s. and in the lab. In the lab. it is necessary to consider separately the radiation of a fast electron and the recoil electron, and for the latter case it is necessary to consider separately the cases $\omega < m/2$ and $\omega > m/2$. As will be shown below the WW method is not applicable when $\omega > m/2$, and leads to incorrect results.

The process under consideration can be investigated with the aid of the method of classical currents, which gives (with logarithmic accuracy) the correct result for $\omega \ll m$.

In this article the bremsstrahlung of the photon will be considered consecutively with the aid of the methods indicated above.

3. An ultrarelativistic particle radiates in the direction of its motion, in a narrow cone with angle $\sim m/E$. This makes the contribution of the interference between the radiation of different particles to be of the order of m^2/ν ($\nu = -(p_1 p_2)$). The contribution of the interference between the direct and exchange (annihilation) diagrams is of the same order. We shall henceforth expand systematically all the quantities in powers of m^2/ν and retain only the higher-order terms of the expansion. It is necessary to consider with this accuracy only the diagrams in which a definite particle radiates (Fig. 1). This question will be considered in greater detail later (Sec. 8).

In the c.m.s. both electrons radiate, naturally, in the same manner. The diagrams of Fig. 1 show the radiation of particle 1. By calculating the contribution of these diagrams, we obtain the radiation cross section of this particle. Particle 2 radiates in exactly the same manner. Thus, the complete expression for the photon spectrum in the c.m.s. is double the contribution of the diagrams of Fig. 1. In the lab. it is necessary to consider separately the radiation of the fast particle and of the recoil particle. We can here, too, confine ourselves to an analysis of the contribution of the diagrams of Fig. 1, by assuming that: 1) particle 2 is at rest (to calculate the contribution to the radiation by the fast particle); 2) particle 1 is at rest (to calculate the contribution to the radiation of the recoil particle). The total expression for the spectrum is the sum of these two contributions.

The exchange-type diagrams (for electron-electron collisions) make the same contribution as the diagrams of the direct type. By virtue of the identity of the electrons, the total contribution of the direct and the exchange diagrams must be divided by two. We can therefore consider only the contribution of the direct diagrams and disregard the identity of the electrons. The contribution of the annihilation diagrams (in the case of electron-positron collisions) can be calculated exactly^[8]. It is of the order of m^2/ν and will therefore be disregarded in the future.

4. The exact expression for the contribution of the diagrams of Fig. 1 to the cross section was obtained in^[7] (formula (40)). We write out here (in covariant form) only the terms which make, with the indicated accuracy, a contribution to the spectrum of the emitted photons. We have

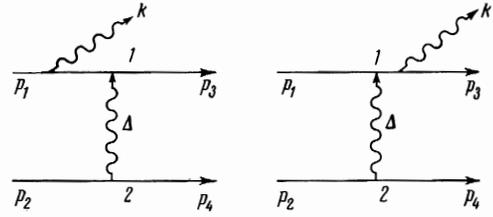


FIG. 1.

$$d\sigma = \frac{2r_0^2 \alpha}{\pi v^3} \int \frac{d\kappa_1 d\kappa_2 d\kappa_3 d\Delta^2}{\Delta^4 g \sin \varphi} \left\{ -\frac{2v^2}{\kappa_3^2} + \frac{1}{\kappa_1 \kappa_3} \left[4v^2 \left(1 - \frac{\kappa_2}{v} \right) \right. \right. \\ \left. \left. + \Delta^2 v^2 \left(\left(1 - \frac{\kappa_2}{v} \right)^2 + 1 \right) \right] - \frac{2v^2}{\kappa_1^2} \left(1 - \frac{\kappa_2}{v} \right)^2 - \frac{\kappa_1}{\kappa_3} - \frac{\kappa_3}{\kappa_1} \right\}, \quad (1)$$

where

$$\kappa_1 = -(kp_1), \quad \kappa_2 = -(kp_2), \quad \kappa_3 = -(kp_3), \\ v = -(p_1 p_2), \quad \Delta = p_2 - p_4 = p_3 + k - p_1; \quad (2)$$

here and henceforth we use the metric $(ab) = (a \cdot b) - a_0 b_0$, $\hbar = c = m = 1$.

In the expression for the cross section (1) we have gone over to the natural variables of the problem:

$$\int \frac{d^3 p_3 d^3 p_4 d^3 k}{E_3 E_4 \omega} \delta(p_1 + p_2 - p_3 - p_4 - k) \\ = \frac{4\pi}{v^2 - 1} \int \frac{d\kappa_1 d\kappa_2 d\kappa_3 d\Delta^2}{g \sin \varphi}, \quad (3)$$

where

$$g \sin \varphi = 2 \left(\frac{v^2}{v^2 - 1} \right)^{1/2} \sqrt{S}, \quad S = -[Q\kappa_3^2 - 2P\kappa_3 + R]; \\ Q = \left(1 - \frac{\kappa_2}{v} \right)^2 + \frac{2\kappa_1 - 1}{v^2}, \quad (4)$$

$$P = \frac{\kappa_1}{v^2} [v(v - \kappa_2) + \kappa_1 - 1] \\ - \frac{\Delta^2}{2v^2} [\kappa_2(\kappa_1 + \kappa_2) + (v + 1)(\kappa_1 - \kappa_2)], \\ R = \frac{\kappa_1^2}{v^2} [v^2 - 1] + \frac{\Delta^2}{v^2} (\kappa_1 + \kappa_2)(\kappa_2 - v\kappa_1) \\ + \frac{\Delta^4}{4v^2} (\kappa_1 + \kappa_2)^2. \quad (5)$$

We have left out from cross section (1) all the terms containing Δ^2 in the numerator with the exception of $v^2 \Delta^2$, $v \kappa_2 \Delta^2$, and $\kappa_2^2 \Delta^2$. It can be shown that the omitted terms make a contribution $\sim 1/\nu$ to the spectrum. As already noted, terms

of this kind will be systematically discarded. We note here also that the formulated approach is not applicable near the hard end of the spectrum of the emitted photons, where $E - \omega \sim 1$ and where this neglect is no longer valid.

We proceed to integrate the cross section (1). In [7] we used initially integration with respect to κ_1 ($\kappa_1 + \kappa_2 = 2\epsilon\omega = \text{const}$), and then with respect to κ_3 and Δ^2 . However, this order of integration turns out to be inconvenient, if we wish to obtain an expression that is applicable in different reference frames. The point is that we are interested in the cross section at a fixed frequency ω of the emitted photon. But $\omega = (\kappa_1 + \kappa_2)/2\epsilon$ in the c.m.s., $\omega = \kappa_2$ for the radiation of the fast particle, and $\omega = \kappa_1$ for the radiation of the recoil particle. Therefore we shall first integrate with respect to κ_2 , then with respect to Δ^2 , and finally with respect to the required quantity κ_1 or κ_2 ¹⁾.

The limits of integration with respect to κ_3 are determined by the zeroes of the expression $g \sin \varphi$ (4) (see [9]). Carrying out this integration we obtain for I_n :

$$I_n = \int \frac{\kappa_3^n d\kappa_3}{\sqrt{S}} \quad I_1 = \frac{\pi P}{Q^{1/2}}, \quad I_0 = \frac{\pi}{Q^{1/2}},$$

$$I_{-1} = \frac{\pi}{R^{1/2}}, \quad I_{-2} = \frac{\pi P}{R^{3/2}}, \quad (6)$$

where the quantities Q , P , and R are given by formulas (5).

5. We now proceed to integration with respect to Δ^2 . The region of integration over the variables Δ^2 , κ_1 , and κ_2 are determined from the condition $P^2 - QR \geq 0$. This condition can be written in the form

$$(2\nu\kappa_1\kappa_2 - \kappa_1^2 - \kappa_2^2) \{ \Delta^2(\nu + 1 - \kappa_1 - \kappa_2) \times [\Delta^2 - 2(\nu - \kappa_2 - 1)] + 2\kappa_1^2 \} \geq 0, \quad (7)$$

hence

$$\Delta_{min}^2 = (\nu - \kappa_2 - 1) \times \left\{ 1 \mp \left[1 - \frac{2\kappa_1^2}{(1 + \nu - \kappa_1 - \kappa_2)(\nu - \kappa_2 - 1)^2} \right]^{1/2} \right\}, \quad (8)$$

$$\kappa_{2,1}(\nu - \sqrt{\nu^2 - 1}) \leq \kappa_{1,2} \leq \kappa_{2,1}(\nu + \sqrt{\nu^2 - 1}). \quad (9)$$

For fixed κ_2 this region is shown in Fig. 2. The boundary of the limit is the third-order curve (7), and the branch shown here lies in the physical region of the variables. The region of variation

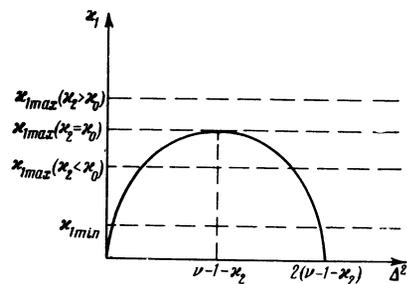


FIG. 2.

of κ_1 depends on the relation between κ_2 and the quantity

$$\kappa_0 = \frac{\nu - 1}{\nu + 1 + \sqrt{\nu^2 - 1}} \approx \frac{1}{2} \quad (\nu \gg 1); \quad (10)$$

if $\kappa_2 > \kappa_0$, then the straight line $\kappa_1 = \kappa_1 \text{max}$ passes above the limiting curve, and when $\kappa_2 < \kappa_0$ it crosses the limiting curve (Fig. 2).

In integrating with respect to Δ^2 we shall systematically expand the obtained result in powers of κ_1/ν . Rigorous analysis shows that the higher-order terms of this expansion make a contribution $\sim 1/\nu$ to the spectrum. We discarded terms of the type $1/\nu\kappa_1\kappa_2$, which make a contribution $\sim 1/\nu$ to the spectrum, and terms of the type $[\nu^2\kappa_1(1 - \kappa_2/\nu)]^{-1}$, which make a contribution $\sim 1/\nu$ in the c.m.s. and in the lab.

As a result of integration we have

$$d\sigma = d\sigma_l + d\sigma_{nl}, \quad (11)$$

$$d\sigma_l = \frac{r_0^2 \alpha}{\nu^3} \frac{d\kappa_1}{\kappa_1^2} d\kappa_2 L \left\{ 2\nu(\nu - \kappa_2) + \kappa_2^2 + \frac{2}{\kappa_1} \kappa_2(\kappa_2 - \nu) + \frac{\kappa_2^2}{\kappa_1^2} \left(1 - \frac{\kappa_2}{\nu} \right) \right\}, \quad (12)$$

$$d\sigma_{nl} = \frac{r_0^2 \alpha}{\nu^3} \frac{d\kappa_1}{\kappa_1^2} d\kappa_2 \left\{ \frac{1}{\kappa_1^2} \left(1 - \frac{\kappa_2}{\nu} \right) \times [-3\kappa_1^2 \nu^2 + \kappa_1 \kappa_2 \nu (8 + \kappa_1) - 4\kappa_2^2] - \nu^2 \right\}, \quad (13)$$

$$L = 2 \ln \left[2\nu \left(\frac{\nu}{\kappa_1 + \kappa_2} - 1 \right) \right]. \quad (14)$$

6. We obtained an expression for the bremsstrahlung differential cross section with respect to κ_1 and κ_2 . This expression is suitable both for the c.m.s. and for the lab. for radiation of the fast particle and the recoil particle. It gives the angular distribution of the radiation for all these cases. To obtain the radiation spectrum it is necessary to integrate the cross section (11) at a fixed fre-

¹⁾A similar approach was used in the authors' paper [9].

quency ω of the radiated photon. In the c.m.s. $\omega = (\kappa_1 + \kappa_2)/2\epsilon$, with $\omega = \kappa_2$, for the fast particle and $\omega = \kappa_1$ for the recoil particle. The region of applicability of the variables κ_1 and κ_2 follows from formulas (8) and (9) and is defined by the inequalities

$$\begin{aligned} \kappa_{2,1} &\geq \kappa_{1,2}(\nu - \sqrt{\nu^2 - 1}), \\ \kappa_{2,1} &\leq \kappa_{1,2}(\nu + \sqrt{\nu^2 - 1}), \quad \kappa_1 + \kappa_2 \leq \nu - 1. \end{aligned} \quad (15)$$

This region is shown in Fig. 3. The corner points have coordinates κ_0 and κ_{\max} , where κ_{\max} gives the maximum frequency of the emitted photon in the appropriate reference frame:

$$\kappa_{\max} = (\nu - 1) / [\nu + 1 - \sqrt{\nu^2 - 1}].$$

We must now consider the different systems separately.

A. The center-of-mass system. The angular distribution of the emitted photons in the direction of particle 1 is characterized by sharp peaks in the direction of motion (the denominator contains high powers of $\kappa_1 = \omega\epsilon(1 - \beta \cos \vartheta_k)$). Therefore the main contribution to the integral with respect to κ_1 at fixed frequency $\omega = (\kappa_1 + \kappa_2)/2\epsilon$ is made by the lower limit of integration with respect to κ_1 ($\kappa_1 \approx \kappa_2/2\nu$), so that the upper limit of integration turns out to be insignificant and the solution does not depend on the ratio of the quantities ω and κ_0 . Carrying out the integration with respect to κ_1 , we easily obtain

$$\begin{aligned} d\sigma_c(1) &= 4r_0^2\alpha \frac{d\omega}{\omega} \frac{\epsilon - \omega}{\epsilon} \\ &\times \left[\frac{\epsilon}{\epsilon - \omega} + \frac{\epsilon - \omega}{\epsilon} - \frac{2}{3} \right] \left[\ln \frac{4\epsilon^2(\epsilon - \omega)}{\omega} - \frac{1}{2} \right] \end{aligned} \quad (16)$$

where ϵ is the energy of the electron in the c.m.s. This result coincides with that obtained by Altarelli and Buccella^[6].

B. Radiation of fast particle in the lab. The angular distribution of the radiated photons is characterized by narrow peaks ($\kappa_1 = \omega E(1 - \beta \cos \vartheta_k)$) in the direction of motion of the fast particle. The situation in this case is analogous to the situation in the c.m.s., and the radiation spectrum of the fast particle is

$$\begin{aligned} d\sigma_{L1} &= 4r_0^2\alpha \frac{d\xi}{\xi} (1 - \xi) \left[(1 - \xi) \right. \\ &\left. + \frac{1}{(1 - \xi)} - \frac{2}{3} \right] \left[\ln \frac{2\nu(1 - \xi)}{\xi} - \frac{1}{2} \right], \end{aligned} \quad (17)$$

where $\xi = \kappa_2/\nu$.

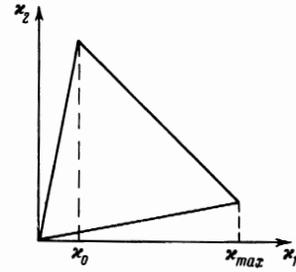


FIG. 3.

C. Radiation of recoil particle in the lab. The angular distribution of the radiated photon ($\kappa_2 = \omega E(1 - \beta \cos \vartheta_k)$, $\kappa_1 = \omega$), as follows from (11)–(13), is quite smooth (almost isotropic). The contribution to the integral is made by the entire region of integration with respect to κ_2 , so that we should carry out separately the integration for $\kappa_1 \leq \kappa_0$ and $\kappa_1 \geq \kappa_0$. As a result we get

$$\begin{aligned} d\sigma_{L2} &= \frac{2}{3} r_0^2\alpha \frac{d\kappa_1}{\kappa_1^2} \left\{ \left[4 - \frac{1}{\kappa_1} + \frac{1}{4\kappa_1^2} \right] \right. \\ &\left. \times \ln(2\nu) - 2 + \frac{2}{\kappa_1} - \frac{5}{8\kappa_1^2} \right\}, \quad \kappa_1 \geq \kappa_0, \end{aligned} \quad (18)$$

$$\begin{aligned} d\sigma_{L2} &= \frac{1}{3} r_0^2\alpha \frac{d\kappa_1}{\kappa_1} \left\{ 16(1 - \kappa_1 + \kappa_1^2) \ln \left(\frac{\nu}{\kappa_1} \right) \right. \\ &- 2(1 - 2\kappa_1) \ln(1 - 2\kappa_1) \left[\frac{1}{4\kappa_1^3} \right. \\ &\left. \left. - \frac{1}{2\kappa_1^2} + \frac{3}{\kappa_1} - 2 + 4\kappa_1 \right] - \frac{1}{\kappa_1^2} + \frac{3}{\kappa_1} - 4 + 4\kappa_1 - 8\kappa_1^2 \right\}, \\ &\kappa_1 \leq \kappa_0; \end{aligned} \quad (19)$$

we see that when $\kappa_1 = \kappa_0$ these two cross sections coincide.

The complete expression for the spectrum in the lab. is the sum of these two cross sections $d\sigma_{L1} + d\sigma_{L2}$ and is given by formulas (17)–(19).

7. Let us analyze the obtained results. We note first that the cross section (17) for the radiation of the fast particle in the lab. coincides with the cross section (16) for the radiation of the particle in the c.m.s. to within a relativistic energy recalculation in the argument of the logarithm. Moreover, we can fix the variable $\eta = \kappa_2 + \alpha\kappa_1$ ($0 \leq \alpha \leq 1$), which means that the frequency is fixed in the system where both particles move, with $\alpha = 1$ in the c.m.s. and $\alpha = 0$ in the lab. Integrating with respect to κ_k , we obtain formula (17), where now $\xi = \eta/\nu$. This means that although the photon radiation spectrum is not a relativistically invariant quantity, it becomes nonetheless possible to recalculate the spectrum from

system to system for fast particles, by virtue of the radiation of the photons in narrow cones.

Let us now compare our results with those found earlier. In the lab. radiation spectrum obtained by Garibyan^[5], he includes correctly the contribution to the radiation of the fast particle, but the contribution to the radiation of the recoil particle is completely missing, and there are terms whose origin can not be established. Altarelli and Buccella^[6] compared their result in the c.m.s. with the result of Garibyan in the lab. We have seen that, accurate to relativistic energy recalculation, the radiation cross section of one particle in the c.m.s. coincides with the radiation cross section of the fast particle in the lab., but in the c.m.s. a similar contribution to the spectrum is made by the other particle, so that the total expression for the spectrum contains an additional factor 2, which consequently was not left out by Garibyan, as was assumed by these authors. No recalculations are possible for the radiation spectrum of the recoil particle (inasmuch as the radiation is not directed into narrow cones).

8. The discarded interference terms can be analyzed by two methods.

The expression for the photon radiation cross section in the approximation of the classical currents (see^[10], formula (16)) contains contributions from all diagrams, including all the interference terms. It is therefore possible to calculate directly the contributions of these terms when $\omega \ll 1$. This is particularly simply done in the c.m.s., where the lower limit of integration with respect to Δ^2 is the same. In the lab. these limits are different, but we can use the lowest ones for estimates. Then calculation shows that all the interference terms are of the order of $1/\nu$.

In the case of arbitrary frequencies, the discarded terms can be estimated with the aid of the Schwartz inequality, as was done for the c.m.s. in^[6]. In the lab. the analysis is similar and, for example, we obtain for the interference terms between the radiation of the different particles

$$d\sigma_{int} \lesssim \frac{ar_0^2}{\nu} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2}; \quad (20)$$

we have discarded terms of this kind.

9. Let us consider the radiation of the photon when electrons collide, using the WW method. The formulation of the problem in the lab. is given in the book by Akhiezer and Berestetskii^[5], which gives also the result for the fast-particle radiation

$$d\sigma_{L1} = 4r_0^2 \alpha \frac{d\omega}{\omega} \frac{E-\omega}{E} \left[\frac{E}{E-\omega} + \frac{E-\omega}{E} - \frac{2}{3} \right] \ln \frac{2E(E-\omega)}{\omega}, \quad (21)$$

which coincides with the logarithmic part of the cross section (17). For the radiation of the recoil particle it is necessary to consider separately the cases $\omega > 1/2$ and $\omega < 1/2$, the region of variation of the frequency of the virtual proton being

$$\begin{aligned} \omega &\leq \omega_1 \leq \infty, & \omega > 1/2, \\ \omega &\leq \omega_1 \leq \omega/(1-2\omega), & \omega < 1/2. \end{aligned} \quad (22)$$

Carrying out the integration in the expression

$$\begin{aligned} d\sigma_{L2} = 2ar_0^2 d\omega \int \frac{d\omega_1}{\omega_1^3} \ln \left(\frac{E}{\omega_1} \right) \left[\frac{\omega_1}{\omega} + \frac{\omega}{\omega_1} \right. \\ \left. + \left(\frac{1}{\omega} - \frac{1}{\omega_1} \right)^2 - 2 \left(\frac{1}{\omega} - \frac{1}{\omega_1} \right) \right], \end{aligned} \quad (23)$$

we obtain

$$d\sigma_{L2} = \frac{2ar_0^2}{3} \frac{d\omega}{\omega^2} \left\{ 4 - \frac{1}{\omega} + \frac{1}{4\omega^2} \right\} \ln \frac{E}{\omega}, \quad \omega \geq \frac{1}{2}, \quad (24)$$

$$d\sigma_{L2} = \frac{16}{3} ar_0^2 \frac{d\omega}{\omega} \{ 1 - \omega + \omega^2 \} \ln \frac{E}{\omega}, \quad \omega \leq \frac{1}{2}; \quad (25)$$

the two expressions coincide for $\omega = 1/2$.

Comparing the result with (18) and (19), we see that formula (25) coincides with the principal logarithmic term of the cross section (19) (we note that the integral (23) gives also the second logarithmic term in (19), but its retention constitutes an exaggeration of the accuracy, since this term is nowhere logarithmically large; its retention leads to an incorrect behavior of the cross section as $\omega \rightarrow 0$). However, formula (24) does not coincide with the logarithmic part of the cross section (18) ($\ln(E/\omega)$ in the approximate formula and $\ln(2E/m)$ in the exact formula), so that here the WW method gives an incorrect result. This is connected with the fact that an essential factor for the applicability of the WW method is the closeness of the pole in the momentum transfer Δ^2 . In the case of the fast-particle radiation

$$\Delta_{min}^2 \approx \omega^2/4E^2(E-\omega)^2, \quad (26)$$

so that the pole is actually close; but for the radiation of the recoil particle

$$\Delta_{min}^2 \approx \omega^2/E(E-\omega) \quad (27)$$

and for large ω the pole in Δ^2 lies already sufficiently far from the region of integration, which makes the WW method inapplicable.

In the c.m.s. the WW method is valid and leads to the logarithmic term in formula (16).

10. The bremsstrahlung of the photon can be considered also with the aid of the classical-currents approximation (see^[10], formula (16)). In^[10]

the spectrum was obtained in the c.m.s. In the lab. we consider separately the radiation of the fast particle and the recoil particle, with Δ_{\min}^2 given by formulas (26) and (27). Then we readily obtain for $\omega \ll 1$ (the classical-current method is applicable for photon energies much lower than the characteristic energies of the problem, while in the lab. this is the electron mass)

$$d\sigma_{L1} = \frac{16}{3} r_0^2 \alpha \frac{d\omega}{\omega} \ln\left(\frac{2E^2}{\omega}\right), \quad d\sigma_{L2} = \frac{16}{3} r_0^2 \alpha \frac{d\omega}{\omega} \ln\left(\frac{E}{\omega}\right). \quad (28)$$

These results follow also from (17) and (19) when $\omega \ll 1$. We see that the cross sections for the radiation of soft photons by the fast particle and by the recoil particle differ only by a factor $2E$ in the argument of the logarithm.

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