RAPID RECOMBINATION OF PLASMA JETS

L. I. GUDZENKO, S. S. FILIPPOV and L. A. SHELEPIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 5, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 1115-1119 (October, 1966)

Rapid cooling of a magnetized plasma jet that expands into a vacuum is considered. The analysis shows that it is possible, in this way, to produce a medium with a nonequilibrium population inversion.

We wish to consider a plasma jet containing atoms or ions with several discrete levels, the populations of which are much smaller than those given by the Saha formula for a given density N and temperature T of the free electrons; this object has many interesting aspects aside from the verification of relaxation theory. A plasma jet of this kind is a high-temperature radiation source and can be used in experiments in which it is desired to fill a magnetic trap with hot plasma[1]; using a configuration of this kind it is also possible to simulate certain nonequilibrium states characteristic of astrophysical situations.[2] The basis of the method considered here, in which it is possible to obtain a relatively dense gas stream with unpopulated lower levels, is the utilization of the rapid recombination that occurs with a sharp reduction of the temperature of the free electrons in a plasma which initially exhibits a high degree of ionization. The populations in the lower levels cannot follow the transitions of electrons to the upper discrete levels; the nonequilibrium situation that is obtained in this way, if one achieves sufficiently rapid cooling of the free electrons, can lead to a population inversion,[3] thus making it possible to use this gas in a laser configuration.

The realization of rapid recombination of a highly ionized hydrogen plasma expanding into a vacuum depends on the possibility of cooling the plasma in a short time Δt from a temperature T_1 to a temperature T_2. Starting with the estimate in[2] we take $N = 10^{14} - 3 \times 10^{15}$ cm^{-3}, $T_2 = 10^3$ °K, $\Delta t \approx 10^{-7}$—10^{-8} sec and $T_1 \approx 5 \times 10^3$ °K, because at lower T_1 the degree of ionization does not remain high enough for a sufficient time immediately before the plasma cooling regime.

1. EXPANSION OF AN UNMAGNETIZED PLASMA

If the volume change occurs without energy exchange with the external medium and with essentially no change in ionization, the gas density N and temperature T are related by $TN^{1-\gamma}$ = const. For a monatomic gas and for a fully ionized plasma of atomic hydrogen we have $\gamma = 5/3$. We shall first discuss the experimental conditions that pertain to an unmagnetized plasma; in this case one can use directly the results of an analysis of the expansion of a neutral gas into a vacuum.[4] In the one-dimensional case the plasma boundaries move with a fixed velocity $v_m = 2v_0/(\gamma - 1)$ where $v_0 = (\gamma k T_0/m)^{1/2}$ is the speed of sound in the plasma, which is initially at rest; k is the Boltzmann constant, m is the mass of the hydrogen atom, and T_0 is the initial temperature of the gas. Initially a primary expansion wave propagates inside the layer with velocity v_0. After the expansion waves collide a new phase of the expansion appears; this is the stage which is of interest here. A similar pattern of behavior is obtained in a two-dimensional gas, in which case a long, uniform plasma cylinder expands into a vacuum, and in the three-dimensional case, in which a plasma sphere expands.

The relation between the temperatures T_1 and T_2 at times t_1 and $t_2 = t_1 + \Delta t$ in the expansion can be estimated from

$$\frac{T_2}{T_1} \approx \left[1 - \frac{4v_0\Delta t}{l_2(\gamma - 1)}\right]^{q(\gamma-1)}, \tag{1}$$

where l_1 and l_2 give the plasma thickness at time t_1 and t_2, and q is the number of dimensions in the problem. It is assumed here that the initial temperature T_0 (directly before the expansion of the plasma) is appreciably greater than $T_1 \approx 5000$°K. The minimum initial temperature $T_0^{(q)}$ required to provide the plasma cooling from T_1 to T_2 in the required time Δt for a given number of dimensions is given by the expression

$$T_0^{(q)} = \frac{m}{\gamma k}\left[\frac{l_2 Q_q(\gamma - 1)}{4\Delta t}\right]^2, \quad Q_q = 1 - (T_2/T_1)^{1/q(\gamma-1)}. \tag{2}$$

With $T_1 = 5T_2$, $\Delta t = 3 \times 10^{-8}$ sec, $\gamma = 5/3$, $m = 1.7 \times 10^{-27}$ g this expression yields

$$T_0^{(1)} = 2 \cdot 10^5 [l_2(\text{cm})]^2 \ {}^\circ\text{K}, \quad T_0^{(2)} = 10^5 [l_2(\text{cm})]^2 \ {}^\circ\text{K}.$$
$$T_0^{(3)} = 5 \cdot 10^4 [l_2(\text{cm})]^2 \ {}^\circ\text{K}.$$

Taking account of the complexity of a three-dimensional expansion into a vacuum under laboratory conditions we see that the practical utilization of adiabatic cooling of an unmagnetized high-temperature plasma for the purpose of producing a large amount of hydrogen with a nonequilibrium population inversion in the atomic levels is an extremely difficult problem.

The quantity v_m refers to the minimum velocity of the excited plasma jet or plasmoid away from the "wall" which separates the plasma from the vacuum before the start of expansion: in particular, if the plasma expansion is realized by expelling the plasma through a weaker region of a dense magnetic "wall," the condition for rapid expansion of the plasma into the vacuum assumes the form

$$l_1 u / \Lambda > v_m, \tag{3}$$

where Λ is the effective dimension of the external boundary of the magnetic field and u is the velocity of the plasmoid.

2. EXPANSION OF A MAGNETIZED PLASMOID

We now wish to discuss the cooling conditions characteristic of the expansion of a magnetized plasma into a vacuum. The magnetization criterion here is the fact that the gas pressure $p = NkT$ must be small compared with the magnetic pressure $p_m = H^2/8\pi$ which is exerted perpendicularly to the lines of the force of the field. Assume that the layer or pinch of magnetized plasma is confined by a field which is everywhere parallel to a fixed direction along the plasma-vacuum boundary; we assume that expansion before the time $t = 0$ is prevented by an external magnetic field which exceeds the field within the plasma by a factor which provides precise equilibration of the gas pressure p. At time $t = 0$ the external field is turned off rapidly and a nonmagnetized vacuum is formed at the plasma boundary, in which case the magnetized plasma starts to expand into the void. We assume that initially the plasma is ideally conducting. To make estimates we shall use the results of the analogous "unmagnetized" problem in (1) except that the quantity v_0 is replaced by the initial Alfvén velocity $v_{A_0} = H_0/2(\pi N_0 m)^{1/2}$. The characteristic linear dimension of the layer or pinch which is used for the rapid recombination is given by

$$l_2^{(q)} = \frac{2H_0 \Delta t}{(\pi m N_0)^{1/2} (\gamma - 1) Q_q} \tag{4}$$

where H_0 and N_0 are the values of the field and plasma density immediately before expansion. Substituting in (4) $T_1 = 5T_2$, $\Delta t = 3 \times 10^{-8}$ sec, $H_0 = 2 \times 10^4$ G, $N_0 = 10^{17}$ cm^{-3}, $\gamma = 5/3$ and $m = 1.7 \times 10^{-24}$ g, we have

$$l_2^{(1)} = 2 \text{ cm}, \quad l_2^{(2)} = 3 \text{ cm}. \tag{5}$$

A comparison of (2) and (5) shows that the strong magnetic field, which will be frozen in the plasma, allows a sharp increase in the initial plasma pressure, and thus intensifies the cooling process in the expansion into the vacuum. Another important feature is the fact that the expansion of a magnetized plasma is subject to control; this feature is important in the recombination stage. In accordance with (4) we see that the volume of cooled plasma can, in principle, be increased by increasing H_0 by a large factor, a procedure which is completely feasible technically. However, we are still subject to limitations such as those indicated by (3) which are formally independent of H_0 but which lead to great difficulties associated with the necessity of rapid displacement, in a vacuum, of a strong external magnetic field, if one is to release the compression on the plasma rapidly enough.[1)]

The magnetic field of the expanding plasma is reduced by virtue of the expansion and as a result of diffusion; this leads to a reduction in the magnetic pressure in the plasma and to a corresponding reduction in the expansion velocity as well as the appearance of a self-induction back current in the peripheral skin layer of the plasma. The region of field smearing corresponds roughly to the skin depth:

$$\delta \sim \frac{c}{2} \sqrt{\frac{\Delta t}{\pi \sigma} \frac{p}{p_m}}$$

where c is the velocity of light in vacuum and σ is the conductivity; for a fully ionized hydrogen plasma $\sigma(\text{sec}^{-1}) = 1.5 \times 10^8 [T({}^\circ\text{K})]^{3/2}/\ln\Lambda$. We then substitute everywhere the parameters corresponding to the time t_2: $H_2 = 200$ G, $T_2 = 10^3$ °K, $N_2 = 10^{14}$ cm^{-3}, $\ln \Lambda(t_2) = 6$, finding $\delta = 0.2$ cm. A comparison of the skin depth with the characteris-

[1)]It is interesting to consider the expansion of a straight pinch after the termination of a power pulse with a sharp trailing edge. The structure of the magnetic field in a pinch is different than that which has been assumed here for the purpose of making estimates. However, the possibilities of producing a nonequilibrium population inversion on the basis of the pinch effect are not exhausted by the scheme being discussed here.

tic dimensions of the plasmoid in (4) shows that the skin effect plays an important but not decisive role.

3. RAPID FLOW OF A MAGNETIZED PLASMA INTO A VACUUM

There is no doubt that the rapid release of a plasmoid from external compression in free expansion is extremely difficult. A possible means of overcoming this fundamental difficulty would appear to be a device in which a high-velocity magnetized jet of fully ionized hydrogen is introduced into a vacuum with a magnetic field that falls off with distance. If the plasma jet travels along a uniform magnetic field (or if the plasma is formed in the field) it is magnetized. The cooling occurs when the plasma expands, moving along the lines of force of the decaying field. We assume at the outset that the plasma magnetized by the field H_0 moves into the vacuum with a velocity $\mathbf{u} \parallel H_0$ through a long slit of width l_0 in a diaphragm. Ideally the diaphragm, which is perpendicular to u, shields the plasma from the external magnetic field. We denote by N_0 and T_0 the density and temperature of the plasma before it escapes through the diaphragm, the plasma being assumed to be uniform in cross section. The axis Oz is parallel to u, the axis Oy is along the slit, and the axis Ox is perpendicular to the slit.

Moving into the unmagnetized vacuum region and expanding, the plasma gradually loses internal field and is accelerated. We assume that at some distance from the diaphragm, at which the magnetic field has been reduced appreciably, the z-component of the mean velocity of the plasma particles becomes essentially independent of coordinates; this velocity is denoted by u_c. In the coordinate system that moves with velocity u_c along the axis Oz, the displacement of the particles is analogous to the one-dimensional expansion of a plasma in a vacuum; in the fixed coordinate system xyz one gets essentially a time sweep of the expansion process along the axis Oz. Making use of these considerations we can write the following expression for the density of the main plasma mass:

$$N(x, z) = \frac{3N_0 l_0}{2l(z)}\left\{1 - \left[\frac{2x}{l(z)}\right]^2\right\}$$

$$\text{for } |x| < \frac{l(z)}{2}, \quad z > 0,$$

where $l(z)$ is the width of the plasma jet:

$$l(z) = 3\frac{v_A z}{u_c} + \Lambda(z), \quad \Lambda(0) = l_0, \quad |\Lambda(z)| \lesssim l_0.$$

As before, the distribution of temperature is obtained by means of the relation $TN^{1-\gamma} = \text{const}$, $\gamma = 5/3$. The boundary of the jet is determined by the expression $x(z) = l(z)/2$; asymptotically ($z \gg l_0$) the jet assumes the shape of a wedge bounded by the plane angle $2\vartheta_m$, $\tan\vartheta_m = 3v_{A_0}/u_c$; as the plasma flows through the slit the density falls off as $1/z$. If the plasma issues from a circular aperture of diameter $2R_0$ the motion is axisymmetric. In cylindrical coordinates (r, z) the density distribution assumes the form

$$N(r, z) = 2N_0\left[\frac{R_0}{R(z)}\right]^2\left\{1 - \left[\frac{r}{R(z)}\right]^2\right\}, \quad z > 0,$$

where $R(z) = 3v_{A_0}z/u_c + \rho(z)$, $\rho(0) = R_0$, $|\rho(z)| \lesssim R_0$. When $r \gg R_0$ the jet assumes the form of a cone with opening angle $2\vartheta_m$, $\tan\vartheta_m = 3v_{A_0}/u_c$: the plasma density goes as $1/z^2$.

In estimating the cooling it is now convenient to use (4) in which we can take a rather high value $H_0 \gtrsim 10^5$ G; the values then obtained, $l_2^{(1)} \gtrsim 10$ cm and $l_2^{(2)} \gtrsim 15$ cm, show that in principle this expansion scheme could solve the problem of producing a large amount of hydrogen plasma for rapid recombination. An ideal diaphragm which cuts off the magnetic field external to the plasma for fields $H \gtrsim 10^5$ G is probably not as easy to obtain as a rapidly displaced large field. However, the diaphragm has been introduced here only for clarifying the analysis of the expansion of a magnetized high-velocity jet. A magnetic-field configuration suitable for stationary flow of a magnetized plasma jet through an aperture (uniform for $z < 0$ and expanding for $z > 0$) can be formed by means of appropriate coils. The highly ionized plasma moves along a long segment of uniform field and becomes magnetized; it then enters the region $z > 0$, following the expanding structure of the lines of force. If we neglect the short time interval for the expansion of the surplus of electrons in the forward region, there will be no important transient effects which might lead, for example, to the detachment of lines of force frozen in the plasma. The considerations given above show that it would be possible to use plasma jets of rather small length ~ 1 m. In these jets it would be easy to realize magnetizations $H_0 \gtrsim 10^5$ G because for transit times $\sim 10^{-6}$ sec, a pulsed magnetic field can be used for compressing the jet directly in front of the point $z = 0$.

For hydrogen plasma temperatures which are of interest here, a comparison of the heat flow due to the thermal conductivity $q_\kappa = \kappa\nabla T$ with the heat carried directly by the moving jet $q_{u_c} = mNu_c c_v T$

yields $q_\kappa/q_{u_c} \ll 1/300$ for a cylindrical jet and a value still smaller for a plane jet. Consequently, it may be assumed that thermal conductivity has essentially no effect on the temperature distribution and does not reduce the cooling efficiency.

We note that a rigorous solution of the plane or axisymmetric problems of rapid expansion of a magnetized plasma jet would scarcely be of great value if one does not also take account of recombination in the jet, heating of the free electrons in recombination and in collisions with recombined atoms, and energy exchange between electrons and heavy particles. It should be emphasized that a full calculation of recombination is a multilevel problem,[3] hence, the rather detailed analysis given by Kuznetsov and Raĭzer[5] is also to some extent an approximation. This analysis shows that the expansion of a high-velocity plasma is not favored by direct recombination; thus, after preparation (by means of an appropriate magnetic field configuration) the velocity of the cold plasma should be reduced or expansion terminated. This control of the rate of expansion of a magnetized plasma can be achieved by an appropriate magnetic field; in order to make an optimum choice of the magnetic field configuration it is necessary to carry out detailed calculations of both cooling and recombination.

The authors are indebted to V. S. Komel'kov, M. I. Pergament, S. B. Pikel'ner, S. I. Syrovatskiĭ and S. S. Tserevitinov for discussion of this work.

[1] N. V. Sidorenko, V. A. Ankudinov and R. N. Il'in, ZhTF **35**, 585 (1965), Soviet Phys. Tech. Phys. 10, 461 (1965).

[2] L. I. Gudzenko and L. A. Shelepin, Proc. Commission on Spectroscopy, Acad. Sci. USSR, No. 1, 15-th Conference on Spectroscopy, Minsk 1963, Moscow **2**, 1964, p. 195; J. R. Hurle and A. Hertzberg, Phys. Fluids **8**, 1601 (1965).

[3] L. I. Gudzenko and L. A. Shelepin, DAN SSSR **160**, 1296 (1965), Soviet Phys. Doklady 10, 147 (1965).

[4] Ya. B. Zel'dovich and Yu. P. Raĭzer, Fizika udarnykh voln i vysokotemperaturnykh gidrodinamicheskikh yavleniĭ (Physics of Shock Waves and High Temperature Phenomena), Fizmatgiz, 1963.

[5] N. M. Kuznetsov and Yu. P. Raĭzer, PMTF (Appl. Math. Theor. Phys.) 4, 10 (1965).

Translated by H. Lashinsky
134