INFLUENCE OF A RADIAL ELECTRIC FIELD ON THE INSTABILITY OF AN INHOMOGENEOUS PLASMA

M. A. VLASOV and V. A. KRIVTSOV

Submitted to JETP editor March 22, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 730-739 (September, 1966)

The influence of a radial electric field E_r on the instability of an inhomogeneous plasma produced by an arc discharge in an equipotential cavity is studied. It is shown that a change of E_r results in a change of the critical magnetic field, and that a reversal of the electric field is accompanied by an abrupt transition of the plasma from one unstable state to another. This transition occurs during a time that is equal in order of magnitude to the inverse increment γ of the observed instability. Reversals in the direction of an electric field at frequencies of the order of γ made it possible to stabilize the plasma instability, study the dependence of the increments on the discharge parameters, and estimate the absolute values of these increments, which were found to be of the order of the drift frequencies.

1. INTRODUCTION

It has been shown[1] that the plasma of a low-pressure arc discharge is unstable. In a magnetic field stronger than a certain critical value intense plasma oscillations develop, leading to enhanced drift of the plasma particles across the magnetic field. The characteristic frequencies of the oscillations and the instability boundary, satisfying the condition $(\omega\tau)_i \sim 1$, suggested the hypothesis that this is a drift instability. To determine the character of the instability more specifically further experimental study was necessary, especially of the influence exerted by a radial electric field on the observed instability. We have the following initial considerations. The development of instability in an inhomogeneous plasma can be pictured as resulting from the polarization of initial density perturbations that arises out of differences in the drift velocities of ions and electrons. Instability should be observed if the polarization effect exceeds the effect associated with diffusive dissipation of the perturbations. In a homogeneous magnetic field the ion and electron drift velocities are given by

$$v_{\mathrm{dr}i} = \frac{(\omega\tau)_i^2}{1+(\omega\tau)_i^2}\left(1+\frac{m_i c^2 E_r}{eH^2 r}\right)\frac{cE_r}{H}, \qquad v_{\mathrm{dr}e} = \frac{cE_r}{H},$$

where $m_i c^3 E_r^2/eH^3 r$ is the centrifugal drift velocity.

It follows that if the nature of the instability is associated with the electric field[2] we should first expect to observe a strong influence of E_r on both the instability boundary and the oscillation frequencies. It also follows from the foregoing expressions that the radial electric field E_r can, depending on its sign, reduce (when $E_r > 0$) or enhance (when $E_r < 0$) the difference between the drift velocities, and, correspondingly, lead to stabilization or destabilization of the instability. Accordingly, instability in a plasma can be controlled through an electric field.

Experiments were performed on hydrogen at pressures from 6×10^{-4} to 6×10^{-3} Torr in magnetic fields up to 3000 Oe. The anode voltage and discharge current were 200 V and 100—200 mA, with the axial density of the plasma not exceeding $\sim 10^{11}$ particles/cm^3.

2. RADIAL ELECTRIC FIELD

The potential of a plasma can be controlled by applying a voltage to the conducting walls of a discharge chamber. This method has been considered with similar geometry by Bohm,[3] who showed that when a positive voltage is applied the plasma potential always follows the wall potential, but that when a negative voltage is applied the plasma potential remains practically unchanged. A similar method was used in experiments on the stabilization of a flute instability in the Ogra and Phoenix machines.[4] Figure 1 shows the electrical scheme of our experiment. The positive potential V_W was applied to the end electrodes and cylindrical wall of the discharge chamber relative to the anode, which remained at a constant potential. We shall show that in this case the radial electric field of a discharge can be controlled by varying the wall potential V_W.

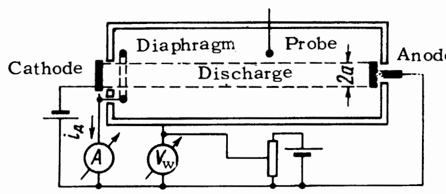

FIG. 1. Scheme of experiment with a constant potential on the discharge chamber wall. Chamber diameter 7.8 cm; cathode diameter 2a = 1 cm; length of discharge l = 40 cm.

In the plasma column based on the anode (r < a) the electron current density j is given by

$$\frac{1}{r}\frac{\partial r j_{1r}}{\partial r}+\frac{\partial j_{1z}}{\partial z}=Z,$$

where Z is the rate of ionization. Since j_r and Z vary only weakly along the length of the chamber we have

$$\frac{1}{r}\frac{\partial r j_{1r}}{\partial r}=-\frac{1}{l}j_1|_{z=0}+Z,$$

where $j_1|_{z=0}$ is the density of the electron current to the anode. Similarly, for r > a we have

$$\frac{1}{r}\frac{\partial r j_{2r}}{\partial r}=-\frac{1}{l}j_2|_{z=0}.$$

In our case the plasma is always positive relative to the surrounding walls; therefore

$$j_1|_{z=0}=\frac{n_1v_1}{4}e^{-\varphi_1/T_1},\qquad r<a,$$

$$j_2|_{z=0}=\frac{n_2v_2}{4}e^{-(\varphi_2-V_c)/T_2},\qquad r>a$$

and

$$\frac{1}{r}\frac{\partial r j_{2r}}{\partial r}=-j_{1z}{}^0\frac{1}{l}+\frac{n_2v_2}{4}\frac{1}{l}e^{-\varphi_{10}/T_1}(1-e^{-\psi_1/T_1})+Z,$$

$$r<a,$$

$$\frac{1}{r}\frac{\partial r j_{2r}}{\partial r}=-j_{2z}{}^0\frac{1}{l}+\frac{n_2v_2}{4}\frac{1}{l}e^{-\varphi_{20}/T_2}(1-e^{-(\psi_2-V_c)/T_2}),$$

$$r>a, \tag{1}$$

where φ is the potential of the perturbed plasma, $j_z^0 = \frac{1}{4}nve^{-\varphi_0/T}$ is the density of the electron current to the wall for an unperturbed plasma potential φ_0, and $\psi = \varphi - \varphi_0$ is the perturbation of the plasma potential. The subscript "1" pertains to plasma parameters in the arc column, while "2" pertains to the corresponding parameters at the periphery of the discharge.

We shall assume henceforth that in both of the indicated regions the plasma parameters n and T depend only weakly on the potential and do not vary radially, with a sharp transition between the respective values occurring at the separation of the two regions. The transition region will be left out of consideration. Then, using the expression

$$j_r=\frac{D_\perp}{T}n\frac{\partial\varphi}{\partial r}-D_\perp\frac{\partial n}{\partial r} \tag{2}$$

in a magnetic field and assuming

$$\frac{1}{r}\frac{\partial r j_r{}^0}{\partial r}=-\frac{1}{l}j_z{}^0+Z, \tag{3}$$

in the unperturbed state, simple transformations of (1) yield

$$\frac{\partial^2\psi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_1}{\partial r}=T_1m_1{}^2(1-e^{-\psi_1/T_1}),\qquad r<a,$$

$$\frac{\partial^2\psi_2}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_2}{\partial r}=T_2m_2{}^2(1-e^{-(\psi_2-V_c)/T_2}),\qquad r>a, \tag{4}$$

where

$$m^2=\frac{v}{4lD_\perp}e^{-\varphi_0/T}. \tag{5}$$

The function $\psi(r)$ of interest can be derived from (4). The present problem formally resembles that considered by Fetisov for a flat probe in a magnetic field[5] and possesses analytic solutions only in certain special cases.

It follows from measurements of plasma parameters in a stable regime that in an arc column with $T_1 \approx 10$ eV and $\varphi_{10} \approx 2$–3 V we can also expect to find little dependence of the plasma potential in this region on V_W; therefore $\psi_1/T_1 \ll 1$ and the exponential in (4) can be expanded in a series. We can proceed similarly for r > a, since in this case for sufficiently large V_W the plasma potential practically equals the wall potential, according to Bohm.[3] In this approximation the problem is reduced to the solution of the simpler equation

$$\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}-m^2\psi=0.$$

As a result we have

$$\psi_1=c_1I_0(m_1r),\qquad \psi_2=-c_2K_0(m_2r),$$

or, connecting the solutions at r = a, we obtain

$$\psi_1=V_c\left[\frac{K_1(m_2a)}{I_1(m_1a)K_0(m_2a)+I_0(m_1a)K_1(m_2a)}\right]I_0(m_1r),$$

$$\psi_2=V_c\left[1-\frac{I_1(m_1a)}{I_1(m_1a)K_0(m_2a)+I_0(m_1a)K_1(m_2a)}\right]K_0(m_2r) \tag{6}$$

where I_0 and I_1 are Bessel functions with an imaginary argument; K_0 and K_1 are Macdonald functions.

We shall now estimate the order of magnitude of m_1 and m_2. From (5), in conjunction with (2) and (3), we obtain (with H measured in kilo-oersteds)

$$m_1^2 \approx \frac{j_{1z}^0}{nD_\perp l} \approx \frac{Z}{D_\perp n_1} \approx 0.3 \cdot 10^{16} \frac{I_a}{n_1} \frac{\sigma_i}{\sigma_{col}} \frac{H^2}{\sqrt{T_1}}, \qquad (7)$$

where σ_i and σ_{col} are the cross sections for ionization and for electron collision with neutral atoms, and T is the electron temperature; similarly

$$m_2^2 \approx 1/q, \qquad (8)$$

where q is the density decay constant.

For $p = 6 \times 10^{-3}$ Torr, H = 340 Oe, and I_a = 100 mA we have the experimental values $n_1 = 10^{11}$ particles/cm^3 and $q \approx 2$ cm. We then obtain $m_1 = 3.4$ and $m_2 = 0.5$. The corresponding curve of $\psi(r)/V_W$ based on (6) for a = 0.6 cm is shown in Fig. 2. The potential perturbation is seen to differ for different points of the discharge. This must lead to a nonuniform change of the initial potential pattern and thus to a change of the radial electric field E_r. The sharpest changes of E_r will occur close to the boundary of the arc column at r = a; this is precisely the region where instability arises.

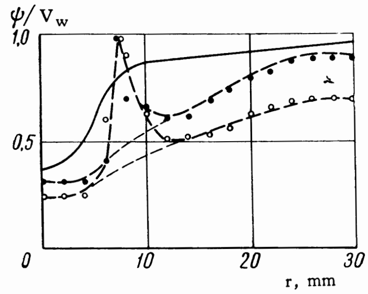

FIG. 2. Relative perturbation of the potential plasma, $\psi(r)/V_w$, as a function of the radius. Solid curve – theoretical dependence based on (6). The experimental points correspond (with $p = 6 \times 10^{-3}$ Torr and H = 340 Oe) to: ● – V_w = 1V, ○ – V_w = 3V.

In order to determine how well the foregoing results correspond to reality we performed control measurements of the radial potential distribution for different values of V_W. The potential was determined from the bending of the electron branch of the probe characteristic; subcritical measurements are represented in Fig. 3. The radial electric field near the arc column is seen to depend strongly on V_W. If in the unperturbed state E_r within this region is positive and of the order 10 V/cm, then with increase of the wall potential the absolute electric field will diminish and will be reversed at $V_W \approx 3$ V. As a rule strong oscillations will arise in the discharge, preventing the consistent observation of variations in E_r at large wall potentials. Figure 2 shows the corresponding experimental values of $\psi(r)/V_W$; qualitative agreement is observed between the experimental and theoretical curves except in the region near $r \approx a$. The difference for $r \sim a$ is associated with the properties of the radial potential distribution in the undisturbed state, which were neglected when the simplified model was used.

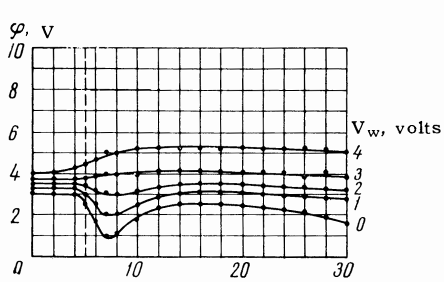

FIG. 3. Radial distribution of potential plasma vs wall potential V_w (indicated at the right-hand end of each curve) for $p = 6 \times 10^{-3}$ Torr, H = 340 Oe.

3. INFLUENCE OF RADIAL ELECTRIC FIELD ON INSTABILITY

The reduction of the radial electric field as the discharge-chamber wall potential is enhanced must affect the drift instability of the plasma. First, we can expect an elevation of the critical magnetic field H_{cr} as V_W increases. This agrees with the experimental dependence of H_{cr} on V_W that is shown in Fig. 4. However the change of H_{cr} is not great as would be expected for a centrifugal instability; as E_r drops from 10 V/cm to practically zero H_{cr} is multiplied by at most 1.5. Further reduction of the potential leads to a sharp drop of H_{cr} and the plasma passes abruptly to a new unstable state characterized by very low critical magnetic fields. The potential V_W^* at which the abrupt change of instability characteristics is observed varies with the discharge parameters in the range 2–3 V and, as shown by the measurements, corresponds to the instant when the electric field is reversed (Fig. 3).

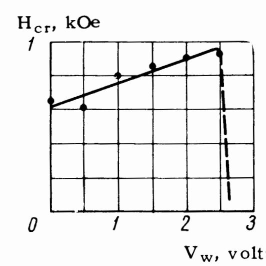

FIG. 4. Critical magnetic field H_{cr} vs V_w; $p = 10^{-3}$ Torr.

The transition can be observed most clearly near the boundary of the instability, which is here manifested by the existence of a torch rotating steadily in the direction of ion motion.[6] Under these conditions we measured the dependence on V_W that is exhibited by the rotational frequency f_{rot} of the torch, the ratio i_e/i_i of electron current

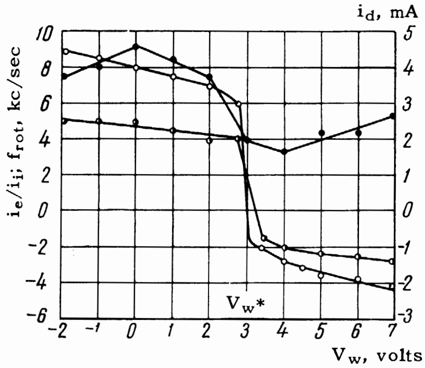

FIG. 5. Rotational frequency f_{rot} of torch, ratio i_e/i_i of electron current to saturation ion current, and current in diaphragm versus wall potential V_w; ○ – f_{rot}, ● – i_e/i_i, ○ – i_d; $p = 10^{-3}$ Torr, H = 730 Oe.

to saturation ion current in a probe positioned outside the arc column, and the current i_d in the annular diaphragm having a potential under that of the anode (Fig. 1). The results, shown in Fig. 5, indicate that the abrupt change of the instability properties at V_W^* is accompanied by reversal of the direction of torch rotation from that of ion motion to that of electron motion and by reversal of the diaphragm current. At the same time the form of the torch is altered.

The plasmograms recorded under the foregoing conditions showed that for $V_W < V_W^*$ the torch is a relatively narrow plasma structure (Fig. 6a) propagating transversely to the magnetic field toward the chamber wall. For $V_W > V_W^*$ it is considerably broader, while its height is considerably less than the dimensions of the system (Fig. 6d). When the wall potential was V_W^* in some cases the observed discharge configurations corresponded to an unperturbed state of the column boundary (Fig. 6c). This permitted that hypothesis that a transition between two unstable states passes through a stable plasma state, after which instability develops (Fig. 6). However, oscillograms of oscillations in the probe current showed that in this case the plasma passes spontaneously from one unstable state to another, and that stability can be said to exist only at the instants when these transitions occur (Fig. 6g). It should also be noted that the characteristic frequencies of the observed oscillations are practically independent of E_r when $E_r > 0$, but usually increase with E_r when $E_r < 0$.

For $V_W > V_W^*$ the radial electric field in a discharge is reversed, becoming negative. This permits the existence of the instability observed in Penning-type discharges, which is associated with the difference between the drift velocities of ions and electrons in a transverse electric field as a consequence of friction between these particles and

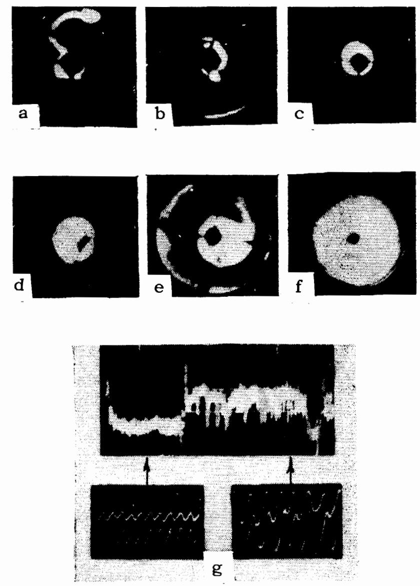

FIG. 6. Typical plasmograms. V_w = (a) 0, (b) 2.8 V, (c) 3 V, (d) 3.5 V, (e) 4 V, (f) 6 V; (g) oscillograms of oscillations at probe when V_w = 3 V.

the neutral gas.[7] A similar effect for $E_r > 0$ has a stabilizing effect. This type of instability may possibly occur in our case when $V_W > V_W^*$. The reversal of the direction of rotation can be associated with reversal of the electrodynamic force **iH**/c that leads to the rotation,[8] and is associated with the reversal of the radial current i_r to the chamber wall; this current is proportional to the diaphragm current i_d (Fig. 5).

4. STABILIZATION OF THE INSTABILITY AND ESTIMATE OF THE INCREMENT

An abrupt plasma transition from one unstable state to another should occupy a time equal in order of magnitude to the reciprocal increment of the observed instability. This means that if the direction of E_r is reversed periodically at frequencies exceeding the increment, the instability can be stabilized. For this purpose we performed the experiment represented schematically in Fig. 7. An audio-generator voltage having an amplitude $U_\sim$ of

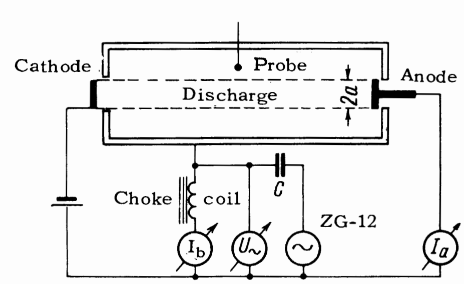

FIG. 7. Experimental scheme with variable potential of discharge chamber wall.

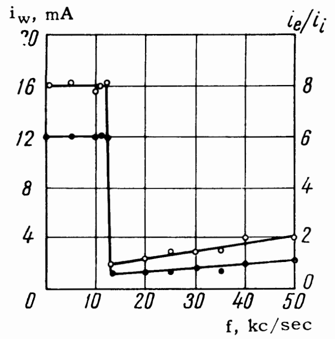

FIG. 8. Current i_w to discharge chamber wall and ratio i_e/i_i versus frequency of alternating voltage; $U_\sim = 7$ V, $p = 3 \times 10^{-3}$ Torr, H = 1700 Oe, ○ – i_w, ● – i_e/i_i.

as much as 10 V at frequencies f up to 200 kc/sec was applied to the chamber wall. The stabilization criteria were the changes in the wall current i_W and the ratio i_e/i_i. The measurements showed that in some cases stabilization of the instability occurs for $U_\sim > 6$ V, being manifested through an abrupt reduction of i_W and i_e/i_i to values close to those observed under subcritical conditions (Fig. 8). Stabilization is also confirmed by the plasmograms recorded in these cases.

Stabilization was not observed in all operating regimes of an arc discharge, being possible only for a narrow region of the parameters (Fig. 9). There is a characteristic absence of stabilization in high magnetic fields at low pressures. It has been suggested that this property is associated with a reduction, in this region, of the amplitude of potential oscillations at the periphery of the discharge. Indeed, the rate of potential variation in regions where ionization is absent is determined by the diffusion rate transversely to the magnetic field. If the period of variation in the wall potential $U_\sim$ is smaller than the diffusion time $t = r^2/D_\perp$,

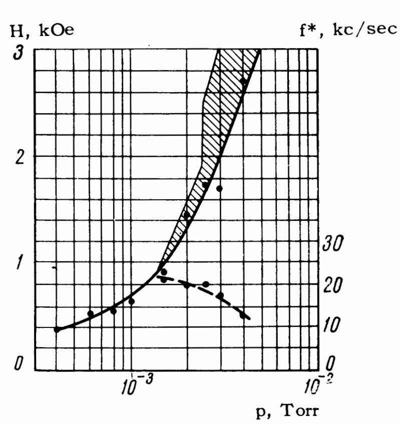

FIG. 9. Stabilization region and pressure dependence of the increment $\gamma = 2\pi f^*$. Solid curve – pressure dependence of H_{cr}; dashed curve – $f^*(p)$ near the instability boundary. The stabilization region is shaded.

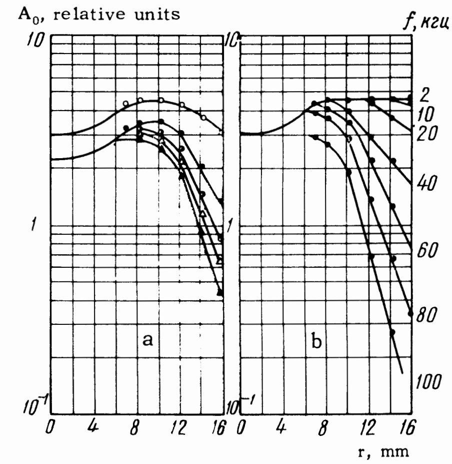

FIG. 10. Amplitude A_0 of oscillations vs radius. a – ○ – $p = 8 \times 10^{-3}$ Torr, H = 560 Oe; ● – $p = 3 \times 10^{-3}$ Torr, H = 560 Oe, ◑ – H = 780 Oe, △ – H = 1000 Oe, ▲ – H = 1230 Oe, b – $A_0(r)$ for different frequencies (indicated at the end of each curve.

where r is the distance from the observation point to the ionization zone, then the plasma potential at that point will not be able to follow the wall potential and will have a smaller oscillatory amplitude. This does not mean that a potential φ_0 corresponding to the unperturbed state is established, because the plasma would thus become charged negatively with respect to the chamber wall, and this would be impossible under our conditions. The constant potential φ' established in the plasma is modulated by oscillations of amplitude A_0 always satisfying the inequality $\varphi' + A_0 \geq U^0_\sim$, where $U^0_\sim$ is the amplitude of the wall potential. With increasing magnetic field strength and decreasing pressure the time t is increased; this diminishes A_0 and the corresponding value of φ'. As a result, for certain values of H and p (Fig. 9) the amplitude of potential oscillations is insufficient for modifying the potential contour associated with stabilization of the instability. According to the foregoing hypothesis the amplitude A_0 of plasma potential oscillations at the periphery of the discharge should decrease as the magnetic field, radius, and oscillation frequency

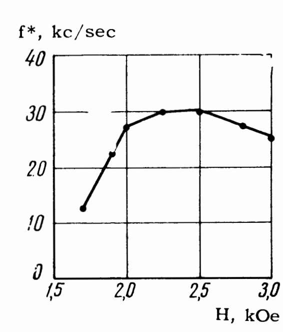

FIG. 11. Dependence of the increment $\gamma = 2\pi f^*$ on the magnetic field; $p = 3 \times 10^{-3}$ Torr.

are increased or the pressure is decreased. This hypothesis is confirmed qualitatively by results obtained for a stable regime at $U_\sim = 5$ V (Figs. 9, 10). An increase of the amplitude $U_\sim$ somewhat expands the stabilization zone; however, it was shown experimentally that the absolute value of U has a limit ≈ 10 V, above which an intense buildup of oscillations is observed in the plasma instead of stabilization.

The frequency f* at which stabilization occurs depends on pressure (Fig. 9) and the magnetic field (Fig. 11), but is practically independent of the other discharge parameters. It is reasonable to assume that f* is of the order of the increment γ of the observed instability; then the experimental curves presented here give the absolute values of the increment and show its dependence on the discharge parameters. It follows from the theory of drift instability that γ should be of the order of the drift frequency $\omega_{dr} = k_\varphi cT/Hq$, where q is the density decay constant. For $p = 2.5 \times 10^{-3}$ Torr and $H = 2000$ Oe we have $q = 1$ cm. Then, with $\lambda_\varphi = 2\pi a$ and $T_e = 2$ eV we have $\omega_{dr} \approx 2 \times 10^5$ sec^{-1}, which agrees in order of magnitude with $\gamma = 2\pi f^*$ $\approx 1.7 \times 10^5$ sec^{-1} in this case, thus providing good confirmation that the observed instability is of dissipative-drift character.

5. CONCLUSION

The following conclusions have been derived from our measurements.

1. An increasing potential V_W of the discharge chamber wall results in a reversal of the radial electric field E_r from positive values for $V_W < 3$ V to negative values for $V_W > 3$ V. A lower electric field is accompanied by a higher critical magnetic field for the drift instability, while the reversal of E_r leads to an abrupt transition between two unstable states of the plasma. In the course of this transition the plasma passes through a stable state in which it remains only during a time that equals the inverse increment of instability in order of magnitude. At $V_W \approx 3$ V the abrupt change of instability properties occurs spontaneously and is of relaxational character.

2. By applying an alternating voltage with the frequency $\sim\gamma$ to the chamber wall we were enabled to stabilize the instability within a relatively narrow region of the discharge parameters near the instability boundary. The confinement of stabilization within this region appears to be associated with a reduction of the oscillatory amplitude outside the arc column whenever the potential oscillation period is comparable with the diffusion time. The stabilization procedure enabled us to trace the dependence of the drift instability increment γ on the discharge parameters and to estimate the absolute value of the former, which was found to be of the order of the drift frequency.

3. The transitions between unstable states occur at very low values of the radial electric field. It has therefore been established that the derived values of the increment do not coincide with the drift frequencies in the electric field, and that the instability for $E_r > 0$ is of a dissipative-drift character [9] rather than of a centrifugal character. This conclusion also agrees with the absence of any strong dependence of H_{cr} or the oscillatory frequencies on the electric field strength. When $E_r < 0$ we evidently observe the instability that characterizes Penning-type discharges.[7]

The authors are indebted to E. I. Dobrokhotov and A. V. Zharinov for valuable discussions.

[1] M. A. Vlasov, JETP Letters 2, 274 (1965), transl. p. 174.

[2] M. N. Rosenbluth and A. Simon, Phys. Fluids 8, 1300 (1965).

[3] D. Bohm et al., The Characteristics of Electrical Discharges in Magnetic Fields, edited by A. Guthrie and R. K. Wakerling, McGraw-Hill, New York, 1949.

[4] G. F. Bogdanov, I. N. Golovin, Yu. A. Kucheryaev, and D. A. Panov, Yadernyĭ sintez (Nuclear Fusion), Supplement, 1, 215 (1962); L. G. Kuo, E. G. Murphy, M. Petravić, and D. R. Sweetman, Phys. Fluids 7, 988 (1964).

[5] I. K. Fetisov, JETP 36, 1110 (1959), Soviet Phys. JETP 9, 789 (1959).

[6] M. A. Vlasov, E. I. Dobrokhotov, and A. V. Zharinov, Yadernyĭ sintez (Nuclear Fusion) 6, (1966).

[7] A. Simon, Phys. Fluids 6, 382 (1963); F. C. Hoh, Phys. Fluids 6, 1184 (1963).

[8] M. A. Vlasov, JETP 51, 715 (1966), this issue, p. 475.

[9] A. V. Timofeev, DAN SSSR 152, 84 (1963). Soviet Phys. Doklady 8, 890 (1964).

Translated by I. Emin
84