SOVIET PHYSICS JETP VOLUME 24, NUMBER 2 FEBRUARY, 1967

DRIFT-CYCLOTRON OSCILLATIONS OF A COLLISION PLASMA PROPAGATING ACROSS A MAGNETIC FIELD

L. S. BOGDANKEVICH and A. A. RUKHADZE

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor March 16, 1966

J. Exptl. Theoret. Phys. (U.S.S.R.) 51, 628-638 (August, 1966)

Drift-cyclotron oscillations propagating across an external magnetic field in a spatially inhomogeneous low-pressure plasma with collisions are investigated. Particle collisions are taken into account by means of the Landau collision integral[10]. Short-wave oscillations with a wavelength smaller than the Larmor ion radius but larger than the electron Larmor radius are considered. The oscillation spectra are analyzed in the geometric optics approximation. Dispersion relations are obtained for the ordinary, extraordinary, and plasma waves and are used to determine the local spectra of drift-cyclotron oscillations of an inhomogeneous plasma and to estimate their increments. It is shown that with growth of the particle-collision frequency, the drift-cyclotron oscillations of a collisionless plasma are transformed into drift-dissipative oscillations that are characteristic of a collision plasma only. The stability of such oscillations depends significantly on the nonuniformity of the plasma-particle temperature.

1. INTRODUCTION

DRIFT-cyclotron oscillations of a collisionless inhomogeneous plasma of low pressure ($\beta = 8\pi P_0/B_0^2 \ll 1$) were investigated by Mikhaĭlovskiĭ and Timofeev[1,2] (see also[3]), who have shown that instabilities arise at the intersections of the drift and cyclotron branches of the oscillations. The maximum growth increments of the oscillations turned out to be equal to $\gamma \approx (m/M)^{1/4}\Omega_i$ for the longitudinal (plasma) wave and $\gamma \approx (m/M)^{1/2}\Omega_i$ for the ordinary wave (Ω_i—Larmor frequency of the ions).

The intersection regions in which drift-cyclotron instabilities develop in a collisionless plasma are very narrow and, as will be shown below, even rare particle collisions can greatly change the character of these instabilities. Drift-cyclotron oscillations of an inhomogeneous plasma, with allowance for Coulomb collisions, were already considered by one of the authors and Silin[1]. We confined ourselves, however, to only longitudinal plasma oscillations in the limit when the drift frequency is larger than the Larmor frequency of the ions. Nor were the conditions determined under which the particle collisions change the spectra of the drift-cyclotron oscillations of the collisionless plasma.

We investigate in the present paper arbitrary (non-potential) drift-cyclotron oscillations of an inhomogeneous plasma with collisions; the oscillations propagate transverse to the external magnetic field. We use the geometric-optics approximation[8], with the aid of which we determine the local spectra of the plasma oscillations[9]. When the characteristic dimension of the inhomogeneity of the plasma L_0 is much larger, and the oscillation wavelength much smaller, than the Larmor radius of the ions, such an approximation describes the spectra of the investigated oscillations not only qualitatively but also quantitatively.

In Sec. 2, by solving the kinetic equation with the Landau collision integral[10], we obtain expressions for the components of the dielectric tensor of the inhomogeneous plasma in different frequency regions and at different wavelengths of the oscillations in question. In Sec. 3 we investigate local spectra of drift-cyclotron oscillations propagating transversely to the external magnetic field, which is assumed to be homogeneous and directed along the z axis, while the direction of the plasma inhomogeneity is assumed to coincide with

[1)]Pogutse[5] attempted earlier to take into account the influence of particle collisions on drift-cyclotron plasma oscillations by using the Bantagar–Gross-Krook collision integral[6]. Unfortunately, such a collision integral is not suitable for a description of short-wave plasma oscillations[7], especially drift-cyclotron oscillations.

the x axis. The general dispersion equation for such oscillations breaks up into two equations—for the ordinary and extraordinary waves. An analysis of the dispersion equation for the ordinary wave has shown that the spectrum of the drift-cyclotron oscillations of a collisionless plasma changes already when $\nu_e \gtrsim (m/M)^{1/2}\Omega_i$, where ν_e is the frequency of the electron collisions; these oscillations go over into new drift-dissipative oscillations that depend essentially on the particle collisions in the plasma. The equation for the extraordinary wave breaks up in turn, under certain conditions, into equations for the transverse and longitudinal waves. With this, if $\nu_i \ll (m/M)^{5/4}\Omega_i$, then the spectrum of the longitudinal oscillations coincide with the spectrum of the drift-cyclotron oscillations of a collisionless plasma. If this inequality is violated, particle collisions become important, and these oscillations also go over into the longitudinal drift-dissipative oscillations of an inhomogeneous plasma. We obtain the conditions for buildup and estimate the growth increments of all the oscillations considered. In the appendix we present a solution of the kinetic equation for the ions in the case of short-wave oscillations with wavelength smaller than their Larmor radius.

2. DIELECTRIC TENSOR OF A PLASMA IN THE REGION OF THE ION-CYCLOTRON RESONANCE FREQUENCIES

If the Larmor frequencies of the particles are large compared with the collision frequencies, the equilibrium distribution function can be written in the form

$$f_0 = \frac{N(c)}{[2\pi m T(c)]^{3/2}} \exp\left[-\frac{mv^2}{2T(c)}\right]. \tag{2.1}$$

The dependence of the density and temperature on x in this expression is determined by the characteristic equation

$$\frac{v_\perp \sin\varphi}{\Omega} - x = c = \text{const}. \tag{2.2}$$

Following the method of geometric optics[8], we can obtain from (2.1) and (2.2) the dielectric tensor of an inhomogeneous plasma with allowance for particle collisions. We write the tensor $\epsilon_{ij}(\omega, \mathbf{k}, x)$ in the form of a sum

$$\varepsilon_{ij}(\omega, \mathbf{k}, x) = \delta_{ij} + \varepsilon_{ij}^{(i)}(\omega, \mathbf{k}, x) + \varepsilon_{ij}^{(e)}(\omega, \mathbf{k}, x), \tag{2.3}$$

where $\epsilon_{ij}^{(i)}(\omega, \mathbf{k}, x)$ and $\epsilon_{ij}^{(e)}(\omega, \mathbf{k}, x)$ are the contributions due respectively to the ions and electrons of the plasma. We are interested in the frequency region $|\omega - n\Omega_i| \ll \Omega_e$ and the wavelength region $k_z = 0$, $k_\perp v_{Te} \ll \Omega_e$. Calculation of the electron contribution $\epsilon_{ij}^{(e)}(\omega, \mathbf{k}, x)$ under these conditions entails no difficulty and is carried out by usual methods. We therefore present only the results of such calculations (indicating how they are obtained).

In the frequency region $\omega \gg \nu_e$, where ν_e is the effective electron collision frequency, we can neglect the electron collisions in first approximation. The electron contribution to the dielectric tensor of the plasma is in this case a Hermitian tensor (since $k_z = 0$). Allowance for the electron collisions leads to the appearance of a small anti-Hermitian part in the tensor $\epsilon_{ij}^{(e)}(\omega, \mathbf{k}, x)$. The electron contribution to the dielectric constant of the plasma in this frequency region is calculated by perturbation theory by expanding the solution of the kinetic equation for the electrons in powers of ν_e/ω (see, for example,[11]). As a result we have

$$\begin{aligned}
\varepsilon_{11}^{(e)} &= \frac{\omega_{Le}^2}{\Omega_e^2}[1 - \mathcal{L}_\omega^e(1)] + i\frac{\nu_e\omega_{Le}^2}{\omega\Omega_e^2}[1 - \mathcal{L}_\omega^e(-1/2)],\\
\varepsilon_{22}^{(e)} &= \varepsilon_{11}^{(e)} - 2\frac{\omega_{Le}^2 k^2 v_{Te}^2}{\omega^2\Omega_e^2}\Big\{[1 - \mathcal{L}_\omega^e(2)]\\
&\quad - i\frac{\nu_e}{\omega}\frac{\sqrt{2}+2}{5}[1 - \mathcal{L}_\omega^e(0.57)]\Big\},\\
\varepsilon_{13}^{(e)} &= \varepsilon_{31}^{(e)} = \varepsilon_{23}^{(e)} = \varepsilon_{32}^{(e)} = 0,\\
\varepsilon_{12}^{(e)} &= -\varepsilon_{21}^{(e)} = i\frac{\omega_{Le}^2}{\omega\Omega_e}[1 - \mathcal{L}_\omega^e(1)]\\
&\quad - \frac{2\nu_e\omega_{Le}^2}{\Omega_e^3}[1 - \mathcal{L}_\omega^e(-1/2)],\\
\varepsilon_{33}^{(e)} &= -\frac{\omega_{Le}^2}{\omega^2}[1 - \mathcal{L}_\omega^e(1)] + i\frac{\nu_e\omega_{Le}^2}{\omega^3}[1 - \mathcal{L}_\omega^e(-1/2)].
\end{aligned} \tag{2.4}$$

Here

$$\mathcal{L}_\omega^e(\alpha) = \frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial \ln N T_e^\alpha}{\partial x},$$

ω_{Le} is the Langmuir frequency of the electrons, and

$$\nu_e = \frac{4}{3}\sqrt{\frac{2\pi}{m}}\frac{e^4 N L}{T_e^{3/2}}.$$

In the frequency region $\omega \sim \Omega_i \ll \nu_e$ we can calculate the electronic contribution to the dielectric constant of the plasma by using hydrodynamic equations[12,13]. We assume that $\omega \gg \nu_e k^2 \rho_{Te}^2/\Omega_e^2$. The transverse components $\epsilon_{12}^{(e)}$ and $\epsilon_{11}^{(e)}$ coincide in this case with those in (2.4), $\epsilon_{22}^{(e)} = \epsilon_{11}^{(e)}$, and for the longitudinal component we have

$$\varepsilon_{33}^{(e)} = 1.96 i \frac{\omega_{Le}^2}{\omega \nu_e} [1 - \mathcal{L}_\omega^e (1.71)]. \quad (2.5)$$

The remaining components, as above, are equal to zero.

We now consider the ionic contribution $\epsilon_{ij}^{(i)}(\omega, \mathbf{k}, x)$. In the kinetic equation for the ions, in the region of short-wave oscillations of interest to us ($k_\perp v_{Ti} \gg \Omega_i$), we can use the simplified Landau collision integral obtained in [7,14]. As a result we obtain for the components of the tensor $\epsilon_{ij}^{(i)}(\omega, \mathbf{k}, x)$ in the frequency region $\omega \sim n\Omega_i$, the following expressions (see the appendix):

a) In the case $|\omega - n\Omega_i| \gg \nu_i z_i$

$$\varepsilon_{11}^{(i)} = \frac{\omega_{Li}^2}{k^2 v_{Ti}^2} \times \left\{ 1 - \mathcal{L}_\omega^i(0) - \frac{\omega}{(\omega - n\Omega_i)\sqrt{2\pi z_i}} [1 - \mathcal{L}_\omega^i(-1/2)] \right\} + i \frac{\nu_i \omega_{Li}^2}{\omega(\omega - n\Omega_i)^2 z_i^{1/2}} \frac{3(\pi+1)}{8\sqrt{\pi}} \left\{ 1 - \frac{k_y v_{Ti}^2}{\omega \Omega_i} \times \left[\frac{\partial \ln N}{\partial x} - \frac{3\pi + 2}{4(\pi+1)} \frac{\partial \ln T_i}{\partial x} \right] \right\},$$

$$\varepsilon_{13}^{(i)} = \varepsilon_{31}^{(i)} = \varepsilon_{23}^{(i)} = \varepsilon_{32}^{(i)} = 0,$$

$$\varepsilon_{22}^{(i)} = - \frac{\omega_{Li}^2}{\omega(\omega - n\Omega_i)\sqrt{2\pi z_i}} [1 - \mathcal{L}_\omega^i(1/2)] + i \frac{\nu_i \omega_{Li}^2 z_i^{1/2}}{\omega(\omega - n\Omega_i)^2} \frac{3(9\pi - 4)}{64\sqrt{\pi}} \left\{ 1 - \frac{k_y v_{Ti}^2}{\omega \Omega_i} \left[\frac{\partial \ln N}{\partial x} + \frac{40 - 11\pi}{2(9\pi - 4)} \frac{\partial \ln T_i}{\partial x} \right] \right\},$$

$$\varepsilon_{12}^{(i)} = - \varepsilon_{21}^{(i)} = i \frac{\omega_{Li}^2 n}{\omega(\omega - n\Omega_i) 2 z_i \sqrt{2\pi z_i}} [1 - \mathcal{L}_\omega^i(-1/2)],$$

$$\varepsilon_{33}^{(i)} = - \frac{\omega_{Li}^2}{\omega(\omega - n\Omega_i)\sqrt{2\pi z_i}} (\mathcal{L}_\omega^e(1) - \mathcal{L}_\omega^i(1/2)) + i \frac{\nu_i \omega_{Li}^2 z_i^{1/2}}{\omega(\omega - n\Omega_i)^2} \times \frac{3(3\pi + 20)}{64\sqrt{\pi}} \left\{ \mathcal{L}_\omega^e(1) - \mathcal{L}_\omega^i(0) \left[1 + \frac{7\pi - 8}{2(3\pi + 20)} \frac{\partial \ln T_i}{\partial \ln N} \right] \right\},$$

b) for $|\omega - n\Omega_i| \ll \nu_i z_i$

$$\varepsilon_{11}^{(i)} = i \frac{\omega_{Li}^2 n^2}{\omega \nu_i z_i^{5/2}} \frac{2}{3\pi} 0.914 \left\{ 1 - \mathcal{L}_\omega^i(0) \left[1 - 0.225 \frac{\partial \ln T_i}{\partial \ln N} \right] \right\},$$

$$\varepsilon_{22}^{(i)} = i \frac{\omega_{Li}^2}{\omega \nu_i z_i^{3/2}} \frac{1.6}{9\pi} 0.49 \left\{ 1 - \mathcal{L}_\omega^i(0) \left[1 + 0.74 \frac{\partial \ln T_i}{\partial \ln N} \right] \right\},$$

$$\varepsilon_{33}^{(i)} = i \frac{\omega_{Li}^2}{\omega \nu_i z_i^{3/2}} \frac{1.6}{2\pi} \left\{ \mathcal{L}_\omega^e(1) - \mathcal{L}_\omega^i(0) \left[1 + 1.38 \frac{\partial \ln T_i}{\partial \ln N} \right] \right\}. \quad (2.7)$$

The remaining components of the tensor $\epsilon_{ij}^{(i)}(\omega, \mathbf{k}, x)$ are in the latter case equal to zero. We have used the notation

$$\mathcal{L}_\omega^i(\alpha) = \frac{k_y v_{Ti}^2}{\omega \Omega_i} \frac{\partial \ln N T_i^\alpha}{\partial x},$$

$$\nu_i = \frac{4}{3} \sqrt{\frac{\pi}{M}} \frac{e_i^4 N L}{T_i^{3/2}}, \quad z_i = \frac{k^2 v_{Ti}^2}{\Omega_i^2}.$$

We note also that in the derivation of (2.7) it was assumed that $\Omega_i \gg \nu_i z_i$ (this has enabled us to neglect the contribution of all the nonresonant harmonics; see the appendix).

3. SPECTRA OF DRIFT-CYCLOTRON PLASMA OSCILLATIONS

We now proceed to investigate the spectra of the drift-cyclotron oscillations propagating transverse to the magnetic field in an inhomogeneous plasma with collisions. As already noted, we investigate the local spectra of the oscillations [9], which we determine directly from the eikonal equation [8]

$$| k^2 \delta_{ij} - k_i k_j - \frac{\omega^2}{c^2} \varepsilon_{ij}(\omega, \mathbf{k}, x) | = 0. \quad (3.1)$$

For waves transverse to the magnetic field, Eq. (3.1) breaks up into two:

$$k^2 - \frac{\omega^2}{c^2} \varepsilon_{33} = 0, \quad (3.2)$$

$$k^2 \varepsilon_{11} - \frac{\omega^2}{c^2} (\varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2) = 0, \quad (3.3)$$

the first of which describes the ordinary wave (which is purely transverse) in the plasma, and the second the extraordinary wave (which generally speaking is neither purely transverse nor purely longitudinal).

Let us consider first the simpler equation (3.2) for the ordinary wave. In the frequency region $\omega - n\Omega_i \approx \Delta \gg \nu_i z_i$ (far from cyclotron resonance) and under the condition $\omega \gg \nu_e$, the local spectrum of the plasma oscillation defined by this equation has the following form ($\omega - n\Omega_i = \Delta + i\gamma$):

a) for $\beta \ll z_i m/M$

$$\Delta = - \frac{n\Omega_i \omega_{Li}^2}{\sqrt{2\pi z_i}\, k^2 c^2} [1 - \mathcal{L}_n^i(1/2)],$$

$$\gamma = - \nu_i z_i \frac{3(9\pi - 4)}{32\sqrt{2}} \frac{1 - \mathcal{L}_n^i(0.11)}{1 - \mathcal{L}_n^i(1/2)}; \quad (3.4)$$

b) for $\beta \gg z_i m/M$

$$\Delta = - \frac{mn\Omega_i}{M\sqrt{2\pi z_i}} [1 - \mathcal{L}_n^i(1/2)] \left[1 + \frac{T_e}{T_i} \frac{\partial \ln N T_e}{\partial \ln N T_i} \mathcal{L}_n^i(1) \right]^{-1},$$

$$\gamma = - \nu_i z_i \frac{3(20 + 3\pi)}{32\sqrt{2}} \frac{1 - \mathcal{L}_n^i(0.24)}{1 - \mathcal{L}_n^i(1/2)}. \quad (3.5)$$

Here we put

$$\mathscr{L}_n{}^i(\alpha)=\frac{k_y v_{Ti}{}^2}{n\Omega_i{}^2}\frac{\partial}{\partial x}\ln NT_i{}^{\alpha}.$$

If we completely neglect the particle collisions in the plasma, the oscillations in question are stable. The oscillations can become unstable only if ion collisions are taken into account, provided the ion temperature is inhomogeneous. Even in this case they occur in a narrow wavelength interval, where

$$[1-\mathscr{L}_n{}^i(1/2)][1-\mathscr{L}_n{}^i(\alpha)]<0, \tag{3.6}$$

where $\alpha = 0.11$ when $\beta \ll z_i m/M$ and $\alpha = 0.24$ when $\beta \gg z_i m/M$. The maximum growth increments of the oscillations for these two limiting cases are respectively equal to

$$\gamma\sim\nu_i\left(\frac{\beta\Omega_i}{\nu_i}\right)^{2/5},\quad \gamma\sim\nu_i\left(\frac{m}{M}\frac{\Omega_i}{\nu_i}\right)^{2/3}.$$

Besides the solutions indicated above, Eq. (3.2) has far from cyclotron resonance also a solution in the resonant region (double resonance), when the drift frequency intersects the cyclotron frequency, namely

$$\frac{k_y v_{Te}{}^2}{\Omega_e}\frac{\partial \ln NT_e}{\partial x}\approx n\Omega_i.$$

Such resonant oscillations can be unstable only in a plasma with $\beta \gg z_i m/M$, wherein

$$\Delta=\pm i\sqrt{\frac{m}{M}}\frac{n\Omega_i}{(2\pi z_i)^{1/4}}\left(1+\frac{T_i}{T_e}\frac{\partial \ln NT_i{}^{1/2}}{\partial \ln NT_e}\right)^{1/2}. \tag{3.7}$$

It is precisely this instability for the ordinary wave that was observed by Mikhaĭlovskiĭ in the case of a collisionless plasma[2]. The maximum growth increment of such an instability is of the order of $\gamma \sim \sqrt{m/M}\,\Omega_i$. With increasing particle collision frequency, the resonant region in which these oscillations are possible narrows down and disappears completely when $\gamma_e > \sqrt{m/M}\,\Omega_i$, i.e., the resonant instability becomes stabilized. On the other hand, nonresonant dissipative instabilities, described by formulas (3.4) and (3.5), obviously remain in this case.

The spectrum of the ordinary wave has an entirely different form in the frequency region $\omega \ll \nu_e$ (such oscillations are possible only in a collision plasma). Here the decisive terms in (3.2) are already those due to electronic dissipation. Drift-cyclotron waves are possible in this region of frequencies only if $\beta > \sqrt{m/M}\,(\nu_i/\Omega_i)z_i$, and their spectrum is given by

$$\omega=\omega^*-i\frac{k^2c^2\nu_e}{\omega_{Le}{}^2}-i\frac{m}{M}\frac{\nu_e\omega^*}{(\omega^*-n\Omega_i)\sqrt{2\pi z_i}}\times\left(1+\frac{T_i}{T_e}\frac{\partial \ln NT_i{}^{1/2}}{\partial \ln NT_e{}^{1.71}}\right), \tag{3.8}$$

where

$$\omega^*=\frac{k_y v_{Te}{}^2}{\Omega_e}\frac{\partial}{\partial x}\ln NT_e{}^{1.71}.$$

The imaginary part of the frequency in this expression is small compared with the real part. We see that if $\beta < z_i^{5/2}\nu_i/\Omega_i$ the oscillations in question are damped; under the inverse condition, the oscillations are unstable provided only $\omega^* < n\Omega_i$. (We note that $|\omega^* - n\Omega_i| > \nu_i z_i$.) Instability is possible in a rather broad region of wavelengths, and the maximum increment for the development of the instability can become of the order of $\gamma \sim \sqrt{m/M}\,\Omega_i$.

We proceed now to investigate ordinary drift-cyclotron waves in the frequency region $|\omega - n\Omega_i| < \nu_i z_i$, i.e., in the direct vicinity of the cyclotron frequency $n\Omega_i$. In the limit when $\omega \gg \nu_e$, we confine ourselves for simplicity to the case when $\nu_i/\Omega_i > z_i^{-3/2}m/M$ (or $\beta \ll z_i^{5/2}\nu_i/\Omega_i$). The imaginary terms in (3.2) are then small compared with the real ones. As a result we obtain for the spectrum of the ordinary wave

$$\omega=\frac{1}{1+k^2c^2/\omega_{Le}{}^2}\frac{k_y v_{Te}{}^2}{\Omega_e}\frac{\partial \ln NT_e}{\partial x},$$

$$\gamma=\frac{\nu_e}{1+k^2c^2/\omega_{Le}{}^2}\left[1-\left(1+\frac{k^2c^2}{\omega_{Le}{}^2}\right)\frac{\partial \ln NT_e{}^{-1/2}}{\partial \ln NT_e}\right]+\frac{m}{M}\frac{n^2\Omega_i{}^2}{\nu_i z_i{}^{3/2}}\frac{0.16}{1+k^2c^2/\omega_{Le}{}^2}\times\left[1+\frac{T_i}{T_e}\left(1+\frac{k^2c^2}{\omega_{Le}{}^2}\right)\frac{\partial \ln NT_i{}^{1.38}}{\partial \ln NT_e}\right]. \tag{3.9}$$

These formulas continue the spectrum (3.5) from the frequency region $\Delta \gg \nu_i z_i$ into the region $\Delta \ll \nu_i z_i$. We see that the ion collisions always have a tendency to build up the oscillations, while the electron collisions stabilize them when $\partial \ln T_e/\partial \ln N < 2$ and exert an unstabilizing action when $\partial \ln T_e/\partial \ln N > 2$, just like the ion collisions. We note that the contribution of the ion collisions to γ prevails over the contribution of the electron collisions when $\Omega_i > \nu_e(M/m)^{1/4}z_i^{3/4}$. The maximum growth increment of the oscillation is in this case of the order of $\gamma \sim (\Omega_i^2/\nu_e)\sqrt{m/M} < \Omega_i$. The instability under consideration is quite dangerous, since it encompasses a rather broad region of wavelengths and can develop practically at any time.

In the frequency region $\omega < \nu_e$, and under the condition $\Delta \ll \nu_i z_i$, the ordinary wave is always damped and its spectrum takes the form (continuation of the spectrum (3.8))

$$\omega=\frac{k_y v_{Te}{}^2}{\Omega_e}\frac{\partial}{\partial x}\ln NT_e{}^{1.71}-i\frac{k^2c^2\nu_e}{\omega_{Le}{}^2}. \tag{3.10}$$

We now investigate the equation (3.3) for the extraordinary wave. In the frequency region far from cyclotron resonance, when $|\omega - n\Omega_i| \gg \nu_i z_i$, the term ϵ_{12}^2 in (3.3) can be neglected. Then this equation breaks up in turn into two equations:

$$\varepsilon_{11} = 0, \qquad (3.11)$$

$$k^2 - \omega^2 c^{-2} \varepsilon_{22} = 0, \qquad (3.12)$$

which describe respectively purely longitudinal and purely transverse extraordinary waves in the plasma. The spectra of these waves do not change on going from frequencies $\omega > \nu_e$ to frequencies $\omega < \nu_e$, since the contribution of the electrons to the components ϵ_{11} and ϵ_{22} can be neglected if $|\omega - n\Omega_i| > \nu_i z_i$.

Taking into account the smallness of the imaginary part of $\epsilon_{11}^{(i)}$ compared with the real part, we obtain the following local spectrum of the longitudinal oscillations of the plasma in the frequency region under consideration:

$$\Delta = \frac{n\Omega_i}{\sqrt{2\pi} z_i} \frac{1 - \mathcal{L}_n^i(-1/2)}{1 + k^2 r_{Di}^2 - \mathcal{L}_n^i(0)}$$

$$\gamma = -\frac{\nu_i z_i}{n^2} \frac{3(\pi+1)}{4\sqrt{2}} \frac{1 - \mathcal{L}_n^i(-0.69)}{1 - \mathcal{L}_n^i(-1/2)}. \qquad (3.13)$$

In the limit when the drift frequency is greatly larger than the cyclotron frequency, i.e., $\omega_{dr} \sim k_y v_{Ti}^2/\Omega_i L_i \gg n\Omega_i$, the spectrum (3.13) goes over into the spectrum obtained in[4]. We see from (3.13) that the oscillations in question can be essentially unstable only under this condition (more accurately, it is sufficient to satisfy the simple inequality), with buildup of the oscillations taking place if $1.45 < \partial \ln T_i/\partial \ln N < 2$. The growth increment of the oscillations can in this case be of the order of $\gamma \sim \nu_i z_i \lesssim (\Omega_i^2 \nu_i)^{1/3}$.

Besides the instability due to longitudinal waves, which we have just considered, resonant instability is also possible (at the intersection of the drift branch with the cyclotron branch), and was first investigated by Mikhaĭlovskiĭ and Timofeev [1]. The growth increment of the oscillations for such an instability is of the order of $\gamma \sim (m/M)^{1/4}\Omega_i$. However, as noted above, the resonant region in which the instability develops is very narrow, and even rare collisions can eliminate such an instability. Indeed, a simple analysis of (3.11) shows that even when $\nu_i \gtrsim (m/M)^{3/2}\Omega_i$ the instability region narrows down, and when $\nu_i \gtrsim (m/M)^{5/4}\Omega_i$ there is no resonant region at all—the instability is eliminated. On the other hand, the non-resonant dissipative instability with the spectrum (3.13) still remains.

The local spectrum of the transverse oscillations described by (3.12) coincides in the frequency region under consideration, when $\Delta \gg \nu_i z_i$, with the spectrum of the ordinary wave (3.4), both when $\omega \gg \nu_e$ and when $\omega \ll \nu_e$.

In conclusion let us consider the extraordinary wave in the frequency region $\Delta \ll \nu_i z_i$, i.e., near cyclotron resonance. In this case Eq. (3.3) no longer breaks up into two equations for the longitudinal and transverse waves, since the term ϵ_{12}^2 can, generally speaking, not be neglected. An analysis of this equation in the general case is therefore quite cumbersome. However, for a real low-pressure plasma, when

$$\beta \nu_i z_i^{3/2}/\Omega_i \ll \beta z_i^{1/2} \lesssim 1,$$

and for the most interesting wavelength region

$$\frac{m}{M}\frac{\nu_i z_i^{5/2}}{\Omega_i} \ll \frac{m}{M} z_i^{3/2} \lesssim 1$$

Eq. (3.3) simplifies appreciably and leads to the following local spectrum of the drift-cyclotron oscillations, which is valid for both $\omega < \nu_e$ and $\omega > \nu_e$ (in the latter case it is necessary also to satisfy the inequality $\beta \ll \nu_i z_i^{5/2}/\Omega_i$):

$$\omega = \frac{k_y v_{Ti}^2}{\Omega_i}\frac{\partial}{\partial x} \ln N T_i^{-0.225},$$

$$\gamma \approx 0.5\frac{m}{M}\nu_i z_i^{5/2}\left[\frac{\Omega_e^2}{\omega_{Le}^2} + 1 + \frac{T_e}{T_i}\frac{\partial \ln N T_e}{\partial \ln N T_i^{-0.225}} + \frac{\omega_{Le}^2}{k^2c^2}\left(1 + \frac{T_e}{T_i}\frac{\partial \ln N T_e}{\partial \ln N T_i^{-0.225}}\right)^2\right]. \qquad (3.14)$$

The oscillations under consideration are in practice always unstable, and their maximum growth increment can become of the order of $\gamma \sim \nu_i (M/m)^{2/3} < \Omega_i$.

From the foregoing analysis of the spectra of the drift-cyclotron oscillations propagating transverse to the external magnetic field we can draw the following conclusions:

a) Even relatively rare collisions of the particle lead to stabilization of the resonant instabilities which take place in a collisionless plasma. Thus, the oscillations investigated in[1] become stabilized when $\nu_i \gtrsim (m/M)^{5/4}\Omega_i$ and those considered in[2] become stabilized when $\nu_e \gtrsim \sqrt{m/M}\,\Omega_i$.

b) On the other hand, allowance for collisions leads to the appearance of new drift-dissipative instabilities in the region of cyclotron frequencies; these instabilities remain even when the resonant instabilities become stabilized. The drift-dissipative instabilities take place both at homogeneous and inhomogeneous plasma-particle temperatures. In the case of inhomogeneous temperature, however, the region of instability with respect to different plasma parameters is much broader.

From our point of view, drift-dissipative instabilities are quite dangerous under conditions when they can develop, since they encompass a rather broad region of wavelengths and have sufficiently large growth increments.

APPENDIX

To derive (2.6) and (2.7), we start from the linearized kinetic equation for δf_i:

$$-i(\omega-\mathbf{kv})\delta f_i-\Omega\frac{\partial\delta f_i}{\partial\varphi}=-\frac{e_i}{M}\left\{\mathbf{E}+\frac{[\mathbf{v}[\mathbf{kE}]]}{\omega}\right\}\frac{\partial f_{0i}}{\partial\mathbf{v}}+I_{ii}(f_{0i},\delta f_i)+I_{ie}(f_{0i},\delta f_e),\qquad (\text{A.1})^*$$

where[7,14]

$$I_{ii}(f_0,\delta f)=\frac{2\pi e_i^4NL}{M^2}\frac{1}{v}\left\{\left(A-B\frac{v_\perp^2}{v^2}\right)\cdot\frac{\partial^2\delta f_i}{\partial v_\perp^2}+\frac{A}{v_\perp^2}\frac{\partial^2\delta f_i}{\partial\varphi^2}\right\},$$

$$I_{ie}=-\frac{4\pi e_i^2e^2L}{m}\frac{\partial f_{0i}}{\partial\mathbf{p}}\int d\mathbf{p}\frac{\mathbf{v}}{v^3}\delta f_e,$$

$$A=\frac{1}{\sqrt{\pi}}\left\{\sqrt{\pi}\Phi(t)\left(1-\frac{1}{2t^2}\right)+\frac{e^{-t^2}}{t}\right\},$$

$$B=\frac{1}{\sqrt{\pi}}\left\{\pi\Phi(t)\left(1-\frac{3}{2t^2}\right)+3\frac{e^{-t^2}}{t}\right\},$$

$$\Phi(t)=\frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}dx,\quad t=\frac{v}{\sqrt{2}\,v_{Ti}}.\qquad (\text{A.2})$$

Here f_{0i} is the equilibrium distribution function of the ions, defined by (2.1). According to (2.2)

$$\frac{\partial f_{0i}}{\partial v_i}=\left[-\frac{v_i}{v_{Ti}^2}+\frac{\delta_{yi}}{\Omega_i}a(v^2)\right]f_{0i},$$

$$a(v^2)=\frac{\partial\ln N}{\partial x}+\frac{\partial\ln T_i}{\partial x}\left(-\frac{3}{2}+\frac{v^2}{2v_{Ti}^2}\right).\qquad (\text{A.3})$$

Substituting (A.3) in (A.1), we seek a solution in the form

$$\delta f_i=-\frac{ie_i}{T_i}\frac{av_{Ti}^2}{\omega}\frac{E_y}{\Omega_i}f_{0i}+Ff_{0i}.\qquad (\text{A.4})$$

We obtain here for F the equation

$$F=\sum_n\frac{i}{\omega-n\Omega_i}\exp\left\{-in\varphi+i\frac{k_\perp v_\perp}{\Omega_i}\sin\varphi\right\}\times\frac{1}{2\pi}\int_{-\pi}^{+\pi}d\varphi'\exp\left\{in\varphi'-i\frac{k_\perp v_\perp}{\Omega_i}\sin\varphi'\right\}$$
$$\times\left\{\frac{e_i}{T_i}\left(1-\frac{k_yv_{Ti}^2}{\omega\Omega_i}a\right)\mathbf{Ev}_{\varphi'}+\frac{2\pi e_i^4NL}{M^2v}\times\left[\left(A-B\frac{v_\perp^2}{v^2}\right)\frac{\partial^2F}{\partial v_\perp^2}+\frac{A}{v_\perp^2}\frac{\partial^2F}{\partial\varphi'^2}\right]+\frac{4\pi e_i^2e^2L}{mT_i}\mathbf{v}\int d\mathbf{p}\frac{\mathbf{v}}{v^3}\delta f_e\right\}.\qquad (\text{A.5})$$

*$[\mathbf{v}[\mathbf{kE}]]\equiv\mathbf{v}\times[\mathbf{k}\times\mathbf{E}]$.

When $|\omega-n\Omega_i|>\nu_iz_i$, Eq. (A.5) can be solved by successive approximations. In first approximation we obtain the solution for a collisionless plasma; the next approximation gives the correction due to the ion collisions. Substituting the solution (A.4) obtained in this manner into the density of the ion current induced in the plasma, we obtain after simple calculations the sought expressions (2.6) (in the frequency region $\omega\sim n\Omega_i$).

In the opposite limit, when $|\omega-n\Omega_i|<\nu_iz_i$ and $\Omega_i\gg\nu_iz_i$, it is sufficient to retain one resonant term when solving (A.5). As a result we obtain

$$F=\frac{e_i}{T_i}\left(1-\frac{k_yv_{Ti}^2}{\omega\Omega_i}a\right)\exp\left\{i\frac{k_\perp v_\perp}{\Omega_i}\sin\varphi-in\varphi\right\}\times\left[J_n(z_i)\left(v_zE_z+n\frac{\Omega_i}{k_\perp}E_x\right)+iJ_n'v_\perp E_y+\frac{4\pi e_i^2e^2L}{mT_i}v_z\int d\mathbf{p}\frac{v_z}{v^3}\delta f_e\right]$$
$$\cdot\left[\frac{4\pi e_i^4NL}{M^2v}\frac{k_\perp^2}{\Omega_i^2}\left(A-\frac{3}{4}B\frac{v_\perp^2}{v^2}\right)\right]^{-1}.\qquad (\text{A.6})$$

Formulas (A.6) and (A.4) lead to the expressions in (2.7). In deriving these expressions we evaluated the following integrals numerically:

$$J_1=\int_0^\infty\frac{e^{-t^2}t^2dt}{[A(A-{}^3/_4B)]^{1/2}}\approx 0.914,$$

$$J_2=\frac{3}{2}-\frac{1}{J_1}\int_0^\infty\frac{e^{-t^2}t^4\,dt}{[A(A-{}^3/_4B)]^{1/2}}\approx 0.225,$$

$$J_3=\int_0^\infty\frac{t^4dt}{B}e^{-t^2}\left[\frac{A}{[A(A-{}^3/_4B)]^{1/2}}-1\right]\approx 0.49,$$

$$J_4=\frac{1}{J_3}\int_0\frac{t^6\,dt\,e^{-t^2}}{B}\left[\frac{A}{[A(A-{}^3/_4B)]^{1/2}}-1\right]-\frac{3}{2}\approx 0.74,$$

$$J_5=\int_0^\infty t^4dte^{-t^2}\left[\frac{1}{[A(A-{}^3/_4B)]^{1/2}}+\frac{4}{3}\left(1-\frac{A}{[A(A-{}^3/_4B)]^{1/2}}\right)\right]\approx 0.4,$$

$$J_6 = -\frac{3}{2} + \frac{1}{J_5}\int_0^\infty e^{-t^2} t^6 dt \left\{ \frac{4}{3}\left(1 - \frac{A}{[A(A - {}^3/_4 B)]^{1/2}}\right) + \frac{1}{[A(A - {}^3/_4 B)]^{1/2}} \right\} \approx 1.38. \qquad (A.7)$$

[1] A. B. Mikhaĭlovskiĭ and A. V. Timofeev, JETP **44**, 919 (1963), Soviet Phys. JETP **17**, 626 (1963).

[2] A. B. Mikhaĭlovskiĭ, JETP **44**, 1552 (1963), Soviet Phys. JETP **17**, 1043 (1963).

[3] A. B. Mikhaĭlovskiĭ, Nuclear Fusion **5**, 125 (1966).

[4] A. A. Rukhadze and V. P. Silin, DAN SSSR **169**, 71 (1966), Soviet Phys. Doklady, in press.

[5] O. P. Pogutse, JETP **47**, 941 (1964), Soviet Phys. JETP **20**, 630 (1965).

[6] P. Batnagar, E. Gross, and M. Krook, Phys. Rev. **94**, 511 (1954). E. Gross and M. Kroof, Phys. Rev. **102**, 593 (1956).

[7] L. P. Pitaevskiĭ, JETP **44**, 969 (1963), Soviet Phys. JETP **17**, 658 (1963).

[8] A. A. Rukhadze and V. P. Silin, UFN **82**, 499 (1964), Soviet Phys. Uspekhi **7**, 209 (1964).

[9] A. B. Mikhaĭlovskiĭ, Voprosy teorii plazmy (Problems in Plasma Theory) **3**, 141 (1963).

[10] L. D. Landau, JETP **7**, 206 (1936).

[11] V. P. Silin and A. A. Rukhadze, Elektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of Plasma and Plasmalike Media), Atomizdat, 1961.

[12] S. I. Braginskiĭ, in: Voprosy teorii plazmy (Problems in Plasma Theory), ed. by M. A. Leontovich, v. 1, Atomizdat, 1963, p. 183.

[13] A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, Atomnaya énergiya **15**, 451 (1963).

[14] Yu. B. Ivanov, and A. A. Rukhadze, Izv. Vuzov Radiofizika **7**, 332 (1964).

Translated by J. G. Adashko
70