NONLINEAR DEPENDENCE OF THE CURRENT ON THE ELECTRIC FIELD STRENGTH IN A THIN SEMICONDUCTING FILM IN A QUANTIZING MAGNETIC FIELD

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Transverse galvanomagnetic quantum phenomena in a thin semiconducting film are studied by the density matrix technique. Quantization of the transverse electron motion in the film is taken into account. Electron scattering is taken into account by perturbation theory methods. It is found that the dissipative current along the film depends on the applied electric field in a nonanalytic manner in the vicinity of zero, $I \sim \exp(-1/2\epsilon^2)$, and hence, in contrast to bulky semiconductors, Ohm's law does not hold in a thin semiconducting film. A monotonic dependence of the current on the magnetic field strength is obtained, the current decreasing exponentially at large magnetic field strengths. It is shown that the dissipative current increases when the film thickness is decreased. The Hall current is found to be the same as that in a bulky semiconductor.

THIS paper is devoted to an analysis of transverse galvanomagnetic phenomena in thin semiconducting films in strong magnetic fields. The magnetic fields $\mathcal{H}$ are assumed to be so strong that the relations

$$\omega \gg 1/\tau, \quad \hbar\omega > k_B T \tag{1}$$

are satisfied ($\omega = e\mathcal{H}/m^*c$ is the cyclotron frequency and τ is the relaxation time of the electron in the film). When conditions (1) are satisfied, the quantization of the electron motion in the magnetic field is significant and consequently Boltzmann's kinetic equation does not hold.

A similar problem for a bulky sample was first considered by Titeica.[1]. A rigorous quantum theory on the basis of the solution of the equation for the density matrix was proposed by Adams and Holstein.[2] They have shown that if the current flows along the magnetic field, Ohm's law is satisfied at least in weak fields. In semiconductors (non-degenerate gas), when one magnetic level is filled, the electric resistance R increases in general with increasing $\mathcal{H}$. In particular, $R \sim \mathcal{H}^2$ when the electron is scattered by acoustic lattice vibrations and by point defects.

On going over from a bulky semiconductor to a thin film, the transverse electron motion becomes quantized. Such a quantization leads to a specific dependence of the current on the electric and magnetic fields, and also on the thickness (see below). The electron energy in the absence of external fields is determined by the quasimomentum projection in the plane of the film and by a discrete quantum number N ($E = E(k_x, k_y, N)$).

In a thin semiconducting film, it may turn out that the electrons populate only the state with N = 1. This takes place[3] at low densities n and at low temperatures:

$$n < A/L_z^3, \quad k_B T < B\hbar^2/m^* L_z^2. \tag{2}$$

Here L_z is the thickness of the film, A and B are dimensionless quantities that depend on the dispersion law: in the case of a quadratic dispersion law $A \sim 10$ and $B \sim 50$. When the state with N = 1 is populated, the three-dimensional Brillouin zone reduces to a two-dimensional one. As a result, the motion of the electron in momentum space becomes planar, although the film itself remains a three-dimensional configuration, since $L_z \gg a$ (a = lattice constant).

We shall assume that the magnetic film is directed along the z axis and is perpendicular to the film, and that the electric field $\mathcal{E}$ is directed along the x axis. The magnetic field is described by a vector potential $\mathbf{A}(0, \mathcal{H}x, 0)$. We assume that conditions (1) are satisfied, i.e., the quantization of motion in the magnetic field is significant. We assume further that condition (2) holds and that the distances between the film levels are so large (see (14) below) that only one sub-band (N = 1) is populated, and transitions to other sub-bands under the influence of collisions can be neglected. Our problem is to find the current in the plane on the film. The current component I_y is the Hall current, and I_x, as will be shown below, is due to dissipation. The analysis pertains to carriers of arbitrary sign. In the case of holes, e and ω should be regarded as negative in all the expressions. Under the square root signs, however, ω is always taken to be the absolute value.

GENERAL RELATIONS FOR THE CURRENT DENSITY

Condition (1) allows us to assume that the scattering potential $\hat{V}$ is small compared with the Hamiltonian $\hat{H}_0$ that includes the magnetic field. Therefore in the zeroth approximation the state of the electron can be regarded as stationary and an eigenstate for $\hat{H}_0$. However, the random character of the potential $\hat{V}$ causes the perturbed state not to be a pure state, and the electron system should be described by a density matrix.

We include in $\hat{H}_0$ the potential $\hat{U}_f$ of the electron in the film, and a term describing the interaction in the electric field

$$\hat{H}_0 = \frac{\hat{p}_x^2 + \hat{p}_z^2}{2m^*} + \frac{(p_y + m^*\omega x)^2}{2m^*} + \hat{U}_f - Fx, \tag{3}$$

where $F = -e\mathscr{E}$, $\hat{U}_f = \hat{U}(z)$ can be regarded for concreteness as an infinitely deep well of width L_z, but actually the concrete form of U(z) is of no fundamental significance for what follows. The periodic potential in the film depends on x and y and is taken into account, as is customary in the effective-mass method, by introducing m*.

The normalized solution and the spectrum of the Schrödinger equation with Hamiltonian (3) can be readily obtained in the form

$$\Psi_{M,k} = \frac{1}{\sqrt{L_z}}\psi(z)\frac{1}{\sqrt{L_y}}e^{iky}\frac{1}{\sqrt{l}}\varphi_M\left(\frac{x - x_k^0}{l}\right), \tag{4}$$
$$E_{M,k} = E_0 + (M + 1/2)\hbar\omega - Fx_k^0.$$

Here M is the magnetic quantum number; the index y of k_y has been left out; the function $\psi(z)$ differs from zero in the interval $(0, L_z)$ and is normalized to L_z in such a way that the mean value $|\psi(z)|$ is equal to unity; it is assumed that $L_x, L_y \gg L_z$; the magnetic length is $l = (\hbar/m^*)^{1/2}$; φ_M is the M-th Hermite function; the center of the oscillations is $x_k^0 = -l^2k + F/m^*\omega^2$, and E_0 is the energy of the electron at the first film level (N = 1).

From (4) we see that the stationary states are degenerate and that the factors that depend on x and z are real. Therefore the current along the x axis can appear only as a result of scattering. At the same time, the Hall current in the stationary state assumes a constant value.

The current density I can be written in terms of the average velocity: $I = -en\mathbf{v}$, where $\mathbf{v} = \mathrm{Tr}(\hat{\mathbf{v}}\hat{\rho}) = \mathbf{v}_{\mu\nu}\rho_{\mu\nu}$, and $\rho_{\mu\nu}$ are the elements of the density matrix, with $\mathrm{Tr}\,\rho = 1$.

The total Hamiltonian of one electron is written in the form

$$\hat{H} = \hat{H}_0 + \hat{V}. \tag{5}$$

We assume that the operator $\hat{V}$ depends only on the coordinates. Then

$$\hat{v}_x = \frac{i}{\hbar}[\hat{H}x] = \frac{\hat{p}_x}{m^*}, \quad \hat{v}_y = \frac{i}{\hbar}[\hat{H}y] = \frac{\hat{p}_y + m^*\omega x}{m^*},$$
$$\hat{v}^- = i\hat{v}_x + \hat{v}_y = \frac{\hbar}{m^*}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right) + \omega x.$$

Using the recurrence relations for the Hermite functions[4]

$$\sqrt{2(M+1)}\,\varphi_{M+1}\left(\frac{x - x^0}{l}\right) = \left(\frac{x - x^0}{l} - l\frac{\partial}{\partial x}\right)\varphi_M\left(\frac{x - x^0}{l}\right),$$
$$\sqrt{2M}\,\varphi_{M-1}\left(\frac{x - x^0}{l}\right) = \left(\frac{x - x^0}{l} + l\frac{\partial}{\partial x}\right)\varphi_M\left(\frac{x - x^0}{l}\right), \tag{6}$$

we obtain in the $\hat{H}_0$ representation the matrix elements of the operator $\hat{v}^-$:

$$\bar{v}_{M,k;M',k'} = \delta_{k,k'}\left(\frac{F}{m^*\omega}\delta_{M,M'} + \omega l\sqrt{2(M+1)}\,\delta_{M,M'-1}\right). \tag{7}$$

As a result we get the following expression for the current density:

$$I^- = I_y + iI_x = enc\frac{\mathscr{E}}{\mathscr{H}} - en\omega l\sum_{M,k}\sqrt{2(M+1)}\,\rho_{M+1,k;M,k}. \tag{8}$$

CALCULATION OF THE DENSITY MATRIX

The quantum equation of motion for the density matrix is

$$i\hbar\partial\hat{\rho}(t)/\partial t = [\hat{H}\hat{\rho}(t)], \tag{9}$$

where $\hat{H}$ is the total Hamiltonian (5). This Hamiltonian does not take into account the thermostat processes responsible for the establishment of the specified temperature and preventing the electrons from becoming heated. It is usually assumed that this neglect is permissible for weak electric fields, since the Joule heat $j\mathscr{E}$ is proportional to $\mathscr{E}^2$. In our case, however (see below), the current j is not proportional to $\mathscr{E}$, and remains close to zero up to sufficiently large fields ($\mathscr{E}_0$). Therefore the approximation considered is valid up to fields $\mathscr{E}_0$. It is precisely this section of $j(\mathscr{E})$ which is of greatest interest. The results for stronger fields make no claim to any accuracy and are qualitative in character.

We shall solve (9) by a method proposed by Kohn and Luttinger[5] and used by Adams and Holstein.[2] The stationary solution as $t \to \infty$ does not depend on the choice of the initial conditions, since the information contained in the initial matrix $\rho(0)$ is completely lost as a result of the scattering process. We therefore choose for $\rho(0)$ the equilibrium distribution function f_M which obtains in a system without an electric field:[1)] $\rho_{M,k;M'k'}(0) = f_M \delta_{m,M'}$. In particular, f_M can stand for the Boltzmann distribution

$$f_M = \exp\left[(\eta - (M + {}^1/_2)\hbar\omega - E_0) / k_B T\right].$$

From (9) we obtain an equation for the Laplace transform

$$\mathscr{F}(s) = s\int_0^\infty \rho(t) e^{-st} dt,$$
$$\mathscr{F}(s) = \rho(0) - \frac{i}{\hbar s}[\hat{H}\hat{\mathscr{F}}(s)]. \qquad (10)$$

It is required to find $\mathscr{F}(0)$, since in the theory of Laplace transforms there is a limiting relation (see [6])

$$\lim_{t\to\infty} \rho(t) = \lim_{s\to+0} \mathscr{F}(s). \qquad (11)$$

We put

$$\mathscr{F}_{M,k;M',k'}(s) - f_{M'}\delta_{M,M'} = G_{M,k;M',k'}(s).$$

Substituting $\mathscr{F}(s)$ in (10) and using (4), we write an exact expression for G(s):

$$[i\hbar s + (M' - M)\hbar\omega + Fl^2(k' - k)]G_{M,k;M',k'}(s)$$
$$= V_{M,k;M',k'}(f_{M'} - f_M) + [VG(s)]_{M,k;M',k'}. \qquad (12)$$

We seek a solution of (12) in powers of the potential $\hat{V}$. The linear term in G makes no contribution to the current, owing to the random scattering potential. We therefore confine ourselves to the quadratic term. Going in it to the limit as $s \to 0$ and bearing (11) in mind, we finally obtain the following expression for the ρ matrix elements that make a contribution to the current (8):

$$\rho_{M+1,k;M,k} = \frac{i\pi}{\hbar\omega}\sum_{\Lambda,k'} V_{M+1,k;\Lambda,k'} V_{\Lambda,k';M,k}$$
$$\times\{\delta[(M + 1 - \Lambda)\hbar\omega + Fl^2(k - k')](f_{M+1} - f_\Lambda)$$
$$- \delta[(M - \Lambda)\hbar\omega + Fl^2(k - k')](f_M - f_\Lambda)\}. \qquad (13)$$

In going to the limit we have omitted terms containing limits in the sense of the principal value o: quantities of the type $1/(z - i|s|)$, since they do not reverse sign upon time reversal or when $s \to -s$, and therefore do not affect the magnitude of the current.

In electric and magnetic fields the δ function in (13) expresses the law of electron energy conservation. Unlike the case of a bulky semiconductor, the δ-function argument does not contain the energy of motion of the electron along the z axis. Strictly speaking, the δ function should also have as an argument the change in the energy of the film, but if this change is larger than the magnetic and electric energy, then the δ function is equal to zero and the transitions between the different sub-bands are forbidden. Bearing in mind the criterion (1) and that $l(k - k') \sim (l^2 m^* k_B T/\hbar^2)^{1/2}$, we can assume that it is permissible to neglect the transitions between the different sub-bands, at least if the following condition is satisfied

$$\Delta E_N \gg \hbar\omega, \quad \Delta E_N \gg e\mathscr{E}l. \qquad (14)$$

Owing to the lack of degeneracy in our case, ρ cannot be expanded in powers of the electric field. The coefficients of the powers of $\mathscr{E}$ would contain derivatives of the δ function, which make impossible any transitions between states and, as already noted, lead to the absence of current along the x axis. Consequently we can expect Ohm's law not to hold in the case of a thin semiconducting film. The results obtained below confirm this premise.

CALCULATION OF THE CURRENT

To find the current, we shall use formulas (8) and (13), and will go over with the aid of the recurrence relations (6) to the indices M and Λ in the scattering matrix elements. If we put $M + 1 = M'$, the summation over M′ can again be carried out from zero, i.e., the term in the sum corresponding to $M' = 0$ vanishes. We symmetrize these in-

[1)]In the Kohn and Luttinger method, which is used below, it is assumed that there is no electric field at the initial instant, after which the field is turned on adiabatically. This determines the choice of the initial distribution function, whereby the random scattering can be taken into account by perturbation theory. The subsequent use of the Laplace transform implies the use of this method of turning on the electric field. A detailed corroboration of the method is given in[5].

dices under the double summation sign. As a result we obtain the expression

$$I^- = enc\frac{\mathscr{E}}{\mathscr{H}} - i\pi\frac{enl^2}{\hbar}\sum_{M,\Lambda,k,k'}(f_M - f_\Lambda)\,\delta[(M-\Lambda)\hbar\omega + Fl^2(k-k')]\cdot\Big\{(k-k')V_{M,k;\Lambda,k'}V_{\Lambda,k';M,k} + \frac{1}{2}\Big[\Big(\frac{\partial V}{\partial x}\Big)_{M,k;\Lambda,k'}V_{\Lambda,k';M,k} - V_{M,k;\Lambda,k'}\Big(\frac{\partial V}{\partial x}\Big)_{\Lambda,k';M,k}\Big]\Big\}. \quad (15)$$

For convenience in the calculation of the matrix elements, we expand the potential V in a Fourier series

$$V = (L_xL_y)^{-1/2}\sum_{q_x,q_y}V_{q_x,q_y}(z)\exp(iq_xx + iq_yy).$$

For the matrix element we obtain

$$V_{M,k;\Lambda,k'} = \sum_{q_x,q_y}\delta_{q_y,k-k'}\,v_q(\exp iq_xx)_{M,\Lambda}. \quad (16)$$

In (16), v_q specifies the transitions between states in the absence of the fields:

$$v_q = \frac{1}{L_xL_yL_z}\int V|\psi(z)|^2\exp(iq_xx + iq_yy)\,d^3x,$$

$$q_x = k_x - k_x', \quad q_y = k - k'.$$

In the products of the matrix elements $V_{M,k;\Lambda,k'}$ $V_{\Lambda,k';M,k}$ we must put, owing to the random character of the scattering potential,

$$V_{q_x,q_y}(z)V_{q_{x'},q_{y'}}(z) = |V_{q_x,q_y}(z)|^2\delta_{q_{x'},-q_x,}\delta_{q_{y'},-q_y,}.$$

As a result we obtain

$$V_{M,k;\Lambda,k'}V_{\Lambda,k';M,k} = \sum_{q_x,q_y}\delta_{q_y,k-k'}|v_q|^2\,|(\exp iq_xx)_{M,\Lambda}|^2. \quad (17)$$

The other matrix elements in (15) are obtained similarly.

We shall confine ourselves henceforth to the nondegenerate case, when the Boltzmann statistics are applicable. Since $k_BT < \hbar\omega$, only one magnetic level will be populated in practice ($f_M = 0$ when $M \neq 0$), and therefore contributions to the current will be made by transitions from the zeroth magnetic level to all the remaining levels ($M = 0$, $\Lambda \neq 0$). Using the generating functions for the system of orthonormal Hermite functions,[4] we obtain for the matrix element the following expression:

$$|(\exp iq_xx)_{0,\Lambda}|^2 = \frac{1}{2^\Lambda\Lambda!}[(lq_y)^2 + (lq_x)^2]^\Lambda \times \exp\{-\tfrac{1}{2}[(lq_y)^2 + (lq_x)^2]\}. \quad (18)$$

Now, summing in (15) over k′ and q_y, and then over k, and bearing (17) and (18) in mind, we obtain for the current density

$$I_y + iI_x = enc\frac{\mathscr{E}}{\mathscr{H}} - i\frac{en}{\hbar F}L_y\sum_{\Lambda,q_x}\frac{|v_{\tilde{q}}|^2}{2^\Lambda\Lambda!}(\tilde{q}_y + iq_x) \times [(lq_y)^2 + (lq_x)^2]^\Lambda\exp\{-\tfrac{1}{2}[(lq_y)^2 + (lq_x)^2]\}. \quad (19)$$

Owing to energy conservation, the momentum k changes by a definite amount $\tilde{q}_y = \Lambda\hbar\omega/Fl^2$ when the electron goes over to the magnetic level Λ. The scattering matrix element is taken in this case between states differing by q_x and $\tilde{q}_y$. The real part of the second term in (19) is odd relative to q_x, since v_q is always symmetrical in q. This term drops out after summing over q_x. Thus, the scattering does not influence the Hall current, and I_y is the same in our case as for a bulky sample regardless of the scattering mechanism:

$$I_y = enc\,\mathscr{E}/\mathscr{H}. \quad (20)$$

DISCUSSION OF RESULTS

Let us discuss our results as applied to different scattering mechanisms.

As seen from (19), the current along the electric field is due to collisions and depends on the concrete scattering mechanism (the factor $|v_{\tilde{q}}|^2$). The value of v_q does not depend explicitly on the electric or magnetic field, and is in general of the same order in the film as in a bulky sample, since we have eliminated the dimensional quantities. The dependence of $v_{\tilde{q}}$ on the magnetic and electric fields enters only via the value of $\tilde{q}$.

In thin films, a large contribution is made to the scattering by collisions between electrons and point defects. It is important that in this case v_q does not depend on the momentum transfer q. The same holds also for scattering of electrons by acoustic oscillations in a semiconductor at almost all temperatures (T > 1°K).[7] This leads to the relation

$$|v_q|^2 = \gamma/\tau L_xL_y = \gamma e/m^*\mu L_xL_y,$$

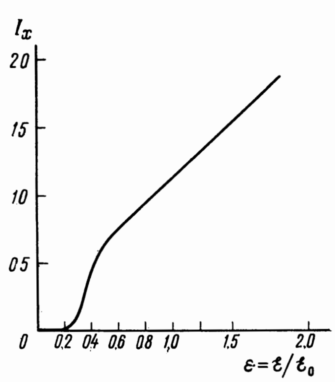
where τ and μ are respectively the relaxation time and the mobility of the electron in the film in the absence of an electric or magnetic field, and the proportionality coefficient is $\gamma = \hbar^3/6m^*$ (see [8]).

We introduce the parameter $\mathscr{E}_0 = (e\hbar)^{1/2}(H/c)^{3/2}/m^*$; $\mathscr{E}_0$ is the electric field in which the electron acquires an energy $\hbar\omega$ in a magnetic unit path ($e\mathscr{E}_0 l = \hbar\omega$, $l\tilde{q} = \Lambda\mathscr{E}_0/\mathscr{E}$). For $\mathscr{H} \sim 10^5$ Oe and $m^* = 0.1m_0$, we have $\mathscr{E}_0 \sim 12\,500$ V/cm. For fields $\mathscr{E} < \mathscr{E}_0$ we can confine ourselves in (19), with a high degree of accuracy, to the first term of the sum in Λ. Integrating with respect to q_x, we obtain for the dissipative current I_x, in the case of scattering by point defects, the formula

$$I_x = en\frac{\gamma e}{2\sqrt{2\pi}\hbar^2\omega l m^*}\frac{1}{\mu}\{\varepsilon^{-2} + \varepsilon^{-4}\{\exp(-1/2\,\varepsilon^2), \qquad (21)$$

where the dimensionless parameter is $\epsilon = \mathscr{E}/\mathscr{E}_0$. We see from (21) that I_x is not an analytic function of ϵ, thus confirming the impossibility of expanding in powers of ϵ, as predicted above on the basis of qualitative considerations.

For weak electric fields, the current is close to zero up to 0.2, after which it begins to grow. Physically this dependence is determined by two factors: on the one hand, the displacement of the oscillator in one transition decreases with increasing field, and on the other hand the frequency of the transitions, determined in this case by expression (18) (v_q does not depend on q_y) increases continuously.



Dependence of the dissipative current on the electric field in the scattering of electrons by point defects and acoustic lattice vibrations. The scale is in arbitrary units.

For strong fields it is necessary to take into account in (19) several terms in the sum over Λ. The form of the obtained relation is shown in the figure. A similar dependence of the current on the field is obtained for the case of electron scattering by acoustic lattice vibrations, because the matrix element v_q is likewise independent of q.

In the case when the current I_x is due to scattering of electrons by ionized impurities, it is necessary to put $v_q \sim q^{-4}$ in (19).[2] The dependence of the matrix element on q causes the transition frequency to increase additionally with increasing field. After simple calculations we obtain

$$I_x \sim \exp(-1/2\varepsilon^2) \qquad (22)$$

In (22) we took account of transitions only to the nearest magnetic level. This is justified when $\epsilon < 1$. We see from (22) that the current increases monotonically with increasing field.

The dependence of the current on the magnetic field is determined by expressions (21) and (22) and by the figure, since $\mathscr{H} \sim \epsilon^{-2/3}$. This dependence will be different for different values of the electric field. In all cases, however, the current decreases monotonically with increasing field, going over for large fields into an exponential dependence.

$$I_x \sim \exp(-\alpha\mathscr{H}^3), \qquad \alpha = e\hbar/2m^*c^3\mathscr{E}^2. \qquad (23)$$

It follows from (21) that in scattering by point defects, the current is inversely proportional to the thickness of the film L_z, since $\mu \sim \tau \sim L_z$. This is a feature common to all the scattering mechanisms, because the square of the modulus of the matrix element $|v_q|^2$ is inversely proportional to the volume. It is of interest to compare this result with the dependence of the current on the thickness in the absence of a magnetic field. It was indicated in [8] that the current decreases with decreasing film thickness. In our case the opposite was obtained. The reason is that in the absence of a magnetic field the scattering decreases the current, but in a strong magnetic field, as noted above, the electron scattering produces current.

In conclusion let us estimate the currents for different scattering mechanisms. For pure films, where only scattering by acoustic oscillations is important, the mobility is [8]

$$\mu = e\hbar^3\rho c_l^2 L_z / 3m^* E_1^2 k_B T,$$

and for $\epsilon \sim 1$, $n = 10^{18}$ cm^{-3}, $T \sim 4°$K, $\mathscr{H} \sim 10^5$ Oe, $m^* = 0.1m_0$, $L_z \sim 10^{-6}$ cm, a coupling constant $E_1 \sim 3$ eV, and a longitudinal-wave velocity $c_l \sim 10^5$ cm/sec, the current per unit length due to scattering by the acoustic waves will be ~ 1 mA. On the other hand, the current due to scattering by point defects (the case of relatively dirty films $\mu \sim 10^3$ cm^3/V-sec) will be ~ 0.1 A. Under these conditions, the Hall current is ~ 1 A.

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60