THEORY OF NONLINEAR PHENOMENA IN FERROMAGNETS

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By use of the nonstationary density matrix $\rho(t)$, nonlinear phenomena are treated and the conditions for parametric excitation of quasiparticles are formulated; the stationary state of the excited waves is also determined. The relations obtained are applied to the study of parametric phenomena in ferromagnets. It is shown that for parametric excitation of spin waves by spin waves, it is necessary that the magnetic field intensity H_0 satisfy the condition $0 \leq H_0 \leq (4\pi/3 - \beta)M_0$, where M_0 is the saturation magnetization and where the anisotropy constant $\beta < 4\pi/3$. The excitation threshold attains its minimum value when the wave vector **f** of the pumped waves is perpendicular to the axis of easiest magnetization. Similar calculations are carried out for a system of spin waves plus photons.

INTRODUCTION

If a system of quasiparticles is subjected to the influence of an external source of energy that has sufficiently high power, then there can arise in such a system nonlinear parametric phenomena connected with an increase of the number of quasiparticles. Investigation of these phenomena makes it possible to study in detail the processes by which elementary excitations interact with one another.

Parametric resonance in ferromagnets was first discovered experimentally by Damon[1] and by Bloembergen and Wang.[2] Similar phenomena in antiferromagnets were investigated by Heeger[3] and by Borovik-Romanov and Prozorova.[4]

Nonstationary phenomena in magnetic materials were treated theoretically by Suhl,[5] Gurevich,[6] White and Sparks,[7] and Ozhogin.[8] In [5, 6] the calculation of these phenomena was carried out phenomenologically, by use of the Landau-Lifshitz[9] equation of motion of the magnetic moment, Maxwell's equations, and the equations of elasticity theory. The system of nonlinear equations thus obtained had a cumbersome form, and their solution was difficult. In [7, 8] the approach used for study of instability processes was a quantum-mechanical one; its advantage lay in the fact that instead of the equations of motion of the magnetic moments, it used the kinetic equation for the occupation numbers of the quasiparticles. Determination of the solution of such an equation encounters no difficulties. Among the shortcomings of such a treatment, however, are the incorrectness of the kinetic equation obtained and the phenomenological calculations of the damping of spin waves and phonons. Furthermore, although in [5-8] conditions for parametric excitation of quasiparticles were also found, it was not possible to determine the stationary amplitude of the excitation.

In the present paper, the investigation of parametric phenomena has been carried out by means of the nonstationary density matrix $\rho(t)$. This method makes it possible to treat nonlinear phenomena and to formulate the conditions for parametric excitation of the quasiparticles, and also to determine the stationary state of the excited waves. It is supposed that the amplitude of the external alternating field is constant in time.

The kinetic equations obtained make it possible to describe parametric phenomena in ferromagnets at frequency $\omega_0 = \epsilon_{\mathbf{k}} + \epsilon_{\mathbf{f-k}}$ and combinational effects with frequencies $\omega_\pm = \pm(\epsilon_{\mathbf{k-f}} - \epsilon_{\mathbf{k}})$, where ω_0 is the frequency of the pumped waves, and where $\epsilon_{\mathbf{k}}$ and $\epsilon_{\mathbf{f-k}}$ are the respective values of the energy of spin waves with wave vectors **k** and **f** − **k**. The threshold values of the amplitudes, the growth increment, and the stationary state of the amplified waves are found.

It is shown that when spin waves are excited by an alternating magnetic field of sufficiently high power, the expression for the imaginary part of the susceptibility, χ'', agrees with Damon's[10] formula,

$$\chi'' = \frac{(P/P_{\text{th}})^{1/2} - 1}{\alpha} \frac{P_{\text{th}}}{P},$$

if the parameter $\alpha = 2\omega\hbar\pi/\mu M_0$. Here ω is the frequency of the alternating magnetic field, μ is the Bohr magneton, M_0 is the saturation magnetization of unit volume of the ferromagnet, P is the power of the pumped waves, and P_{th} is the threshold power.

1. GENERAL STATEMENT OF THE PROBLEM

Let an isolated system, described by the Hamiltonian $\mathcal{H}$, be acted upon by an external force V(t), which can be represented in the form

$$V(t) = A_f b_f(t) + A_f^+ b_f^+(t), \tag{1.1}$$

here the quantities A_f and A_f^+ contain only operators of the quasiparticles of the system, whereas $b_f(t)$ and $b_f^+(t)$ are operators of the external perturbation. Then we get for the complete Hamiltonian

$$H = \mathcal{H} + A_f b_f(t) + A_f^+ b_f^+(t), \tag{1.2}$$

The reaction of the system to the external perturbation is expressed by the fact that the density matrix of the system becomes nonstationary. Supposing that the external perturbation is sufficiently small, we describe the nonstationary density matrix in the form[11]

$$\rho(t) = \rho_0 + \rho_1(t) + \rho_2(t), \tag{1.3}$$

where ρ_0 is the equilibrium density matrix, and where $\rho_1(t)$ and $\rho_2(t)$ correspond to the linear and quadratic perturbation approximations.

The operator $\rho(t)$ satisfies the following equation of motion:[1)]

$$\partial\rho / \partial t = i[\rho, H]. \tag{1.4}$$

By use of (1.4), the equations of motion for the operators $\rho_1(t)$ and $\rho_2(t)$ can be obtained:

$$\frac{\partial\rho_1}{\partial t} = i[\rho_1, \mathcal{H}] + i[\rho_0, A_f b_f(t) + A_f^+ b_f^+(t)], \tag{1.5}$$

$$\frac{\partial\rho_2}{\partial t} = i[\rho_2, \mathcal{H}] + i[\rho_1, A_f b_f(t) + A_f^+ b_f^+(t)]. \tag{1.5'}$$

The solution of equations (1.5) and (1.5′) has the form

$$\rho_1(t) = i\int_0^\infty [\rho_0, \hat{A}_f(-\tau)]\, b_f(t-\tau)d\tau + \text{Herm. conj.}, \tag{1.6}$$

$$\rho_2(t) = -\int_0^\infty d\tau_1 \int_0^\infty d\tau_2 \{[[\rho_0, \hat{A}_f(-\tau_1-\tau_2)],$$

$$\hat{A}_f(-\tau_2) b_f(t-\tau_2) + \hat{A}_f^+(-\tau_2) b_f^+(t-\tau_2)]\, b_f(t-\tau_1-\tau_2)\}$$

$$+ \text{Herm. conj.} \tag{1.7}$$

where $\hat{A}_f(\tau) = e^{i\mathcal{H}\tau} A_f e^{-i\mathcal{H}\tau}$ is the operator in the Heisenberg representation.

The change of the operator for the number of quasiparticles in state **k** can be written in the form

$$dn_k/dt = -i[n_k, \mathcal{H}] - i[n_k, A_f b_f(t) + A_f^+ b_f^+(t)]. \tag{1.8}$$

On averaging (1.8) by means of the nonequilibrium density matrix $\rho(t)$, we get the equation of motion for the mean of the quasiparticle operator $n_{\mathbf{k}}$:

$$\frac{dn_k}{dt} = -\gamma_k(n_k - \overset{0}{n}_k)$$

$$+ N_f \int_0^\infty d\tau\, e^{i\omega\tau} \langle[\hat{A}_f(-\tau), [n_k, A_f^+]]\rangle + \text{c.c.}, \tag{1.9}$$

where $\gamma_{\mathbf{k}}$ is the damping constant in the state **k**, $N_f = b_f^+ b_f$ is the number of quasiparticles entering from the external source, $\overset{0}{n}_{\mathbf{k}}$ is the Bose distribution function, and the brackets $\langle\ \rangle$ serve as a designation of the mean over the stationary density matrix ρ_0. In the derivation of (1.9) it was assumed that the source is in such a mode of operation that the time variation of b_f is $b_f(t) = b_f e^{i\omega t}$.

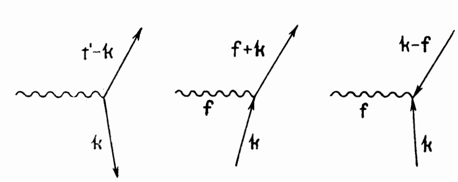
We shall consider only processes of the first order: that is, decomposition of a single quasiparticle, corresponding to the pumped wave, into two quasiparticles, corresponding to amplified waves; inelastic scattering of a quasiparticle of the amplified wave on a quasiparticle of the pumped wave; and virtual processes, connected with simultaneous absorption of three quasiparticles, one of which corresponds to the pump wave. Such processes are represented schematically in the figure. The operator A_f has the form

$$A_f = \sum_k \{\Psi_1(k, f-k; f)\, a_k^+ a_{f-k}^+ + \Psi_2(k; k-f; f)\, a_k^+ a_{k-f}$$

$$+ \Psi_3(k, -f-k; f)\, a_k a_{-f-k}\}, \tag{1.10}$$

where Ψ_1, Ψ_2, and Ψ_3 are amplitudes corresponding to the probabilities of the indicated processes.

For the value of $dn_{\mathbf{k}}/dt$ we get



[1)]Planck's constant is $\hbar = 1$.

$$\frac{dn_{\mathbf{k}}}{dt} = -2\gamma_{\mathbf{k}}(n_{\mathbf{k}} - \overset{0}{n}_{\mathbf{k}}) + 8N_{\mathbf{f}} \frac{|\Psi_1(\mathbf{k}, \mathbf{f}-\mathbf{k}; \mathbf{f})|^2 (n_{\mathbf{k}} - n_{\mathbf{f}-\mathbf{k}} + 1)}{(\omega - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{f}-\mathbf{k}})^2 + (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}})^2} \times (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}) - 2N_{\mathbf{f}} \frac{|\Psi_2(\mathbf{k}+\mathbf{f}; \mathbf{k}; \mathbf{f})|^2 (n_{\mathbf{k}} - n_{\mathbf{k}+\mathbf{f}})}{(\omega - \varepsilon_{\mathbf{k}+\mathbf{f}} + \varepsilon_{\mathbf{k}})^2 + (\gamma_{\mathbf{k}+\mathbf{f}} + \gamma_{\mathbf{k}})^2} \times (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}+\mathbf{k}}) + 2N_{\mathbf{f}} \frac{|\Psi_2(\mathbf{k}, \mathbf{k}-\mathbf{f}; \mathbf{f})|^2 (n_{\mathbf{k}-\mathbf{f}} - n_{\mathbf{k}})}{(\omega - \varepsilon_{\mathbf{k}} + \varepsilon_{\mathbf{k}-\mathbf{f}})^2 + (\gamma_{\mathbf{k}} + \gamma_{\mathbf{k}-\mathbf{f}})^2} \times (\gamma_{\mathbf{k}} + \gamma_{\mathbf{k}-\mathbf{f}}). \tag{1.11}$$

It is evident from equation (1.11) that the second term describes parametric phenomena at frequency $\omega_0 = \epsilon_{\mathbf{k}} + \epsilon_{\mathbf{f}-\mathbf{k}}$, whereas the third and fourth describe combinational effects with frequencies $\omega_\pm = \pm(\epsilon_{\mathbf{k}\pm\mathbf{f}} - \epsilon_{\mathbf{k}})$.

2. CONDITION FOR PARAMETRIC RESONANCE

We consider parametric phenomena in the neighborhood of resonance, $\omega = \omega_0 + \xi$, that is when the value of ξ is small. Then by use of (1.11) one can get the system of two equations

$$\begin{aligned} \frac{dn_{\mathbf{k}}}{dt} &= -2\gamma_{\mathbf{k}}(n_{\mathbf{k}} - \overset{0}{n}_{\mathbf{k}}) + B(n_{\mathbf{f}-\mathbf{k}} + n_{\mathbf{k}} + 1), \\ \frac{dn_{\mathbf{f}-\mathbf{k}}}{dt} &= -2\gamma_{\mathbf{f}-\mathbf{k}}(n_{\mathbf{f}-\mathbf{k}} - \overset{0}{n}_{\mathbf{f}-\mathbf{k}}) + B(n_{\mathbf{k}} + n_{\mathbf{f}-\mathbf{k}} + 1), \end{aligned} \tag{2.1}$$

where

$$B = 8|\Psi_1(\mathbf{k}, \mathbf{f}-\mathbf{k}; \mathbf{f})|^2 N_{\mathbf{f}} \frac{\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}}{\xi^2 + (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}})^2}$$

In order to find the threshold values of the amplitude of quasiparticle excitation and the limits of parametric resonance, it is sufficient to solve Eqs. (2.1) for the occupation numbers of the quasiparticles, $n_{\mathbf{k}}$ and $n_{\mathbf{f}-\mathbf{k}}$. The characteristic equation of the system (2.1) has the form

$$(\lambda + 2\gamma_{\mathbf{k}} - B)(\lambda + 2\gamma_{\mathbf{f}-\mathbf{k}} - B) - B^2 = 0. \tag{2.2}$$

On solving (2.2) for λ, we get

$$\lambda_{1,2} = B - (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}) \pm [(\gamma_{\mathbf{k}} - \gamma_{\mathbf{f}-\mathbf{k}})^2 + B^2]^{1/2}. \tag{2.3}$$

It is not difficult to see that λ_2 is always negative ($\lambda_2 < 0$); therefore the condition for parametric excitation of quasiparticles can be described in the form $\lambda_1 > 0$, or

$$B > 2\gamma_{\mathbf{k}}\gamma_{\mathbf{f}-\mathbf{k}} / (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}). \tag{2.4}$$

The region of parametric resonance is determined by the inequality

$$-(\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}})(N_{\mathbf{f}}/N_{\text{th}} - 1)^{1/2} < \xi < (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}})(N_{\mathbf{f}}/N_{\text{th}} - 1)^{1/2}, \tag{2.5}$$

where

$$N_{\text{th}} = \gamma_{\mathbf{k}}\gamma_{\mathbf{f}-\mathbf{k}}/4|\Psi_1(\mathbf{k}, \mathbf{f}-\mathbf{k}; \mathbf{f})|^2 \tag{2.5'}$$

is the threshold number of quasiparticles corresponding to the pumped wave.

If the magnitude of the wave vector of the pumped wave is appreciably smaller than the wave vector of the amplified wave ($\mathbf{f} \ll \mathbf{k}$), then the growth increment of the wave is $\Gamma_{\mathbf{k}} = \lambda_1/2$, or

$$\Gamma_{\mathbf{k}} = \frac{4\gamma_{\mathbf{k}}^3}{\xi^2 + 4\gamma_{\mathbf{k}}^2}\left[\frac{N_{\mathbf{f}}}{N_{\text{th}}} - \frac{\xi^2 + 4\gamma_{\mathbf{k}}^2}{4\gamma_{\mathbf{k}}^2}\right]. \tag{2.6}$$

3. STATIONARY MODE

If the frequency of the external source (the pump) satisfies condition (2.5), then parametric resonance occurs. In this case an exponential increase of amplitude of the excited waves will be observed until such time as nonlinear interactions in the system of quasiparticles begin to play a fundamental role. Thereafter, the increase of amplitude will cease, and the system will go over to the stationary mode.

The kinetic equations that describe the transition of the system to a stationary state can be obtained from equations (1.11). The terms that are nonlinear in the occupation numbers $n_{\mathbf{k}}$ and $n_{\mathbf{f}-\mathbf{k}}$ are determined from the collision integrals of the quasiparticles, $\dot{n}_{\mathbf{k}}$ and $\dot{n}_{\mathbf{f}-\mathbf{k}}$, account being taken of the width of the levels $\mathbf{k}$ and $\mathbf{f}-\mathbf{k}$ of the system; that is,

$$\delta(\omega - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{f}-\mathbf{k}}) \to \frac{1}{\pi}\frac{\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}}{(\omega - \varepsilon_{\mathbf{k}} - \varepsilon_{\mathbf{f}-\mathbf{k}})^2 + (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}})^2}.$$

In the stationary mode, the system of kinetic equations has the form

$$\begin{aligned} &-2\gamma_{\mathbf{k}} n_{\mathbf{k}}^{\text{st}} + B(n_{\mathbf{f}-\mathbf{k}}^{\text{st}} + n_{\mathbf{k}}^{\text{st}})\left[1 + \frac{B}{\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}}\frac{n_{\mathbf{k}}^{\text{st}} + n_{\mathbf{f}-\mathbf{k}}^{\text{st}}}{N_{\mathbf{f}}}\right]^{-1} - \frac{2B}{N_{\mathbf{f}}} n_{\mathbf{k}}^{\text{st}} n_{\mathbf{f}-\mathbf{k}}^{\text{st}} = 0, \\ &-2\gamma_{\mathbf{f}-\mathbf{k}} n_{\mathbf{f}-\mathbf{k}}^{\text{st}} + B(n_{\mathbf{k}}^{\text{st}} + n_{\mathbf{f}-\mathbf{k}}^{\text{st}}) \times \left[1 + \frac{B}{\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}-\mathbf{k}}}\frac{n_{\mathbf{k}}^{\text{st}} + n_{\mathbf{f}-\mathbf{k}}^{\text{st}}}{N_{\mathbf{f}}}\right]^{-1} - 2\frac{B}{N_{\mathbf{f}}} n_{\mathbf{k}}^{\text{st}} n_{\mathbf{f}-\mathbf{k}}^{\text{st}} = 0, \end{aligned} \tag{3.1}$$

where $n_{\mathbf{k}}^{\text{st}}$ and $n_{\mathbf{f}-\mathbf{k}}^{\text{st}}$ are the stationary values of the occupation numbers of the excited quasiparticles $(n_{\mathbf{k}}^{st} \gg n_{\mathbf{k}}^{0})$.

Of special interest is the case in which the magnitude of the wave vector of the pumped wave, **f**, is considerably smaller than that of the wave vector of the amplified wave $(f \ll k)$. Then the value of $n_{\mathbf{k}}^{st}$ is determined from formulas (3.1) and has the form

$$n_{\mathbf{k}}^{st} = N_{th}[(N_{\mathbf{f}}/N_{th})^{1/2} - 1]. \qquad (3.2)$$

We shall consider phenomena connected with the scattering of excited quasiparticles on quasiparticles corresponding to pumped waves. Let the frequency $\omega = \omega_+ + \xi$. Then by use of equations (1.11), we can derive the following system of equations, which describe the change of the number of quasiparticles with time in the states **k** and **f** + **k**:

$$\frac{dn_{\mathbf{k}}}{dt} = -2\gamma_{\mathbf{k}}(n_{\mathbf{k}} - n_{\mathbf{k}}^0) + \widetilde{B}(n_{\mathbf{f}+\mathbf{k}} - n_{\mathbf{k}}),$$
$$\frac{dn_{\mathbf{f}+\mathbf{k}}}{dt} = -2\gamma_{\mathbf{k}+\mathbf{f}}(n_{\mathbf{f}+\mathbf{k}} - n_{\mathbf{f}+\mathbf{k}}^0) - \widetilde{B}(n_{\mathbf{f}+\mathbf{k}} - n_{\mathbf{k}}), \qquad (3.3)$$

where

$$\widetilde{B} = 2N_{\mathbf{f}}\frac{|\Psi_2(\mathbf{k}+\mathbf{f};\mathbf{k};\mathbf{f})|^2}{\xi^2 + (\gamma_{\mathbf{k}+\mathbf{f}} + \gamma_{\mathbf{k}})^2}(\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}+\mathbf{k}}). \qquad (3.4)$$

The roots of the characteristic equation of this system are

$$\lambda_{1,2} = -\widetilde{B} - (\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}+\mathbf{k}}) \pm [(\gamma_{\mathbf{k}} - \gamma_{\mathbf{f}+\mathbf{k}})^2 + \widetilde{B}^2]^{1/2}. \qquad (3.5)$$

It follows that the number of excited quasiparticles cannot increase exponentially with time, since $\lambda_{1,2} < 0$. Therefore at frequency ω_+ (and similarly at frequency ω_-) resonance phenomena are absent.

Upon using the stationarity conditions $\dot{n}_{\mathbf{k}} = \dot{n}_{\mathbf{f}+\mathbf{k}} = 0$, we get

$$n_{\mathbf{k}} = n_{\mathbf{k}}^0 - \frac{\widetilde{B}\gamma_{\mathbf{f}+\mathbf{k}}}{2\gamma_{\mathbf{k}}\gamma_{\mathbf{f}+\mathbf{k}} + \widetilde{B}(\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}+\mathbf{k}})}(n_{\mathbf{k}}^0 - n_{\mathbf{f}+\mathbf{k}}^0),$$
$$n_{\mathbf{f}+\mathbf{k}} = n_{\mathbf{f}+\mathbf{k}}^0 + \frac{\widetilde{B}\gamma_{\mathbf{k}}}{2\gamma_{\mathbf{k}}\gamma_{\mathbf{f}+\mathbf{k}} + \widetilde{B}(\gamma_{\mathbf{k}} + \gamma_{\mathbf{f}+\mathbf{k}})}(n_{\mathbf{k}}^0 - n_{\mathbf{f}+\mathbf{k}}^0). \qquad (3.6)$$

From equations (3.6) it is clear that in an alternating field at the difference frequency ω_+ there occurs a redistribution of the population of the levels of the system.

4. THRESHOLD PHENOMENA IN FERROMAGNETS

We consider parametric excitation of spin waves with wave vector **k** by spin waves with wave vector **f** $(f \ll k)$. In this case the quasiparticle operators $b_{\mathbf{f}}$ will coincide with the spin-wave generation and absorption operators $a_{\mathbf{f}}^{+}$, $a_{\mathbf{f}}$. The amplitude $\Psi_1(\mathbf{k}, -\mathbf{k}; 0)$ has the form[12]

$$\Psi_1(\mathbf{k}, -\mathbf{k}; 0) = -\pi\mu(2\mu M_0/V)^{1/2}\sin 2\vartheta_{\mathbf{k}} e^{-i\varphi_{\mathbf{k}}}, \qquad (4.1)$$

where V is the volume of the crystal.

Let the frequency of the pump spin waves, $\epsilon_{\mathbf{f}}$, coincide with the frequency of parametric resonance $\epsilon_{\mathbf{f}} = 2\epsilon_{\mathbf{k}}$; $\epsilon_{\mathbf{k}}$ is the energy of a spin wave vector **k**:

$$\varepsilon_{\mathbf{k}} = \varepsilon_0(1 + \eta\sin^2\vartheta_{\mathbf{k}})^{1/2},$$
$$\varepsilon_0 = \mu(H_0 + \beta M_0), \quad \eta = 4\pi M_0/(H_0 + \beta M_0), \qquad (4.2)$$

where H_0 is the constant magnetic field within the body, directed along the axis of easiest magnetization; β is the anisotropy constant; and $\theta_{\mathbf{k}}$ is the angle between the wave vector **k** and the magnetic field $\mathbf{H}_0$.

On taking account of the equality $\epsilon_{\mathbf{f}} = 2\epsilon_{\mathbf{k}}$ and using formula (4.2), we find that the angle between the wave vector **f** and the magnetic field H_0 must satisfy the condition

$$\sin\vartheta_{\mathbf{f}} > \left[\frac{3}{4\pi M}(H_0 + \beta M_0)\right]^{1/2}. \qquad (4.3)$$

We remark that the inequality (4.3) holds only for those ferromagnets in which $\beta < 4\pi/3$. Furthermore, for the magnitude of the magnetic field we get from (4.3)

$$0 \leqslant H_0 \leqslant (4\pi/3 - \beta)M_0. \qquad (4.4)$$

In the stationary state, the number of spin waves with wave vector **k** is determined by formula (3.2), in which the threshold for excitation of spin waves is

$$N_{th} = V\gamma_{\mathbf{k}}^2/8\pi^2\mu^3 M_0\sin^2 2\vartheta_{\mathbf{k}}. \qquad (4.5)$$

From the equality $\epsilon_{\mathbf{f}} = 2\epsilon_{\mathbf{k}}$ it follows that the value of the excitation threshold has a minimum value, at angles $\theta_{\mathbf{f}} = \pi/2$,

$$N_{th} = V\frac{8\gamma_{\mathbf{k}}^3}{3\mu^3 M_0}\left(4\pi + \beta + \frac{H_0}{M_0}\right)^{-1}\left(4\pi - 3\beta - 3\frac{H_0}{M_0}\right)^{-1}. \qquad (4.6)$$

We consider excitation of spin waves by an alternating external magnetic field **h**(t). The interaction Hamiltonian in this case has the form

$$V(t) = -\int \mathbf{M}\mathbf{h}(t)\,dV,$$
$$\mathbf{h}(t) = i\mu\left(\frac{2\pi\omega}{V}\right)^{1/2}[\mathbf{f}_0\mathbf{e}_{\mathbf{f}}]\,b_{\mathbf{f}}e^{i\mathbf{f}\mathbf{r} - i\omega t} + \text{Herm. conj.},$$

where **M** is the magnetic moment of unit volume, $\mathbf{f}_0 = \mathbf{f}/|\mathbf{f}|$, and $\mathbf{e}_{\mathbf{f}}$ is the polarization vector of the alternating magnetic field. On going over from the operators **M** to the spin-wave generation and absorption operators $a_{\mathbf{k}}^{+}$ and $a_{\mathbf{k}}$, we get

$$\Psi_1(\mathbf{k}, -\mathbf{k}; 0) = i\mu\left(\frac{\pi\omega}{8V}\right)^{1/2}$$

$$\times [\mathbf{f}_0\mathbf{e}_f]_z \frac{\eta \sin^2\vartheta_\mathbf{k}}{(1+\eta\sin^2\vartheta_\mathbf{k})^{1/2}} \exp(-2i\varphi_\mathbf{k}),$$

$$\Psi_2(\mathbf{k}; \mathbf{k}; 0) = -i\mu\left(\frac{2\pi\omega}{V}\right)^{1/2} [\mathbf{f}_0\mathbf{e}_f]_z \frac{1+{}^1/_2\eta\sin^2\vartheta_\mathbf{k}}{(1+\eta\sin^2\vartheta_\mathbf{k})^{1/2}}. \quad (4.7)$$

Let the frequency ω of the alternating magnetic field satisfy the condition

$$\omega = 2\varepsilon_\mathbf{k}, \quad (4.8)$$

then for the value of the angle between the direction of the vector $\mathbf{k}$ of the spin wave and the axis of easiest magnetization, we get

$$\sin\vartheta_\mathbf{k} = \frac{1}{4}\left[\frac{\omega^2 - 4\mu^2(H_0+\beta M_0)^2}{\pi\mu^2(H_0+\beta M_0)M_0}\right]^{1/2}. \quad (4.9)$$

Hence the frequency ω must be determined from the inequality

$$2\mu(H_0+\beta M_0) < \omega < 2\mu(H_0+\beta M_0) \times\left(1+\frac{4\pi M_0}{H_0+\beta M_0}\right)^{1/2}. \quad (4.10)$$

On using the expressions (2.5′) and (4.9), we find the value of the threshold for excitation of spin waves by an alternating magnetic field:

$$N_{\text{th}} = V\frac{8}{\pi}\frac{\varepsilon_0^2\omega\gamma_\mathbf{k}^2}{\mu^2(\omega^2-4\varepsilon_0^2)^2[\mathbf{f}_0\mathbf{e}_f]_z^2}. \quad (4.11)$$

The threshold value of the amplitude of the magnetic field is

$$h_{\text{th}} = 4\frac{(H_0+\beta M_0)\omega\gamma_\mathbf{k}}{[\omega^2-4\mu^2(H_0+\beta M_0)^2][\mathbf{f}_0\mathbf{e}_f]_z}. \quad (4.12)$$

From formula (4.12) it is clear that the threshold value for excitation of spin waves by an alternating magnetic field will be least if the pump vector $\mathbf{f}$ and the polarization vector $\mathbf{e}_f$ lie in the basal plane.

We remark in closing that by use of expression (4.11) for the threshold value for excitation of spin waves, we can find the imaginary part of the susceptibility, χ'':

$$\chi'' \approx \frac{\mu M_0}{2\pi\omega N_f} n^{\text{st}},$$

or on taking account of (3.2), we get

$$\chi'' \approx \frac{P_{\text{th}}}{\alpha P}\left[\left(\frac{P}{P_{\text{th}}}\right)^{1/2} - 1\right];$$

$$\alpha = 2\pi\omega / \mu M_0, \quad P = \omega N_f, \quad P_{\text{th}} = \omega N_{\text{th}|}. \quad (4.13)$$

Formula (4.13) agrees with the expression for the imaginary part of the susceptibility, χ'', found by Damon.[10]

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