# *THE POSSIBILITIY OF AN OSCILLATORY NATURE OF GRAVITATIONAL COLLAPSE*

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Some arguments are advanced which indicate that gravitational collapse is not an irreversible attraction of matter towards the center but is in reality an oscillatory equilibrium. Newtonian motion at zero pressure which is the classical analog of collapse is of an oscillatory nature; the latter persists in the co-moving coordinate systems of the general theory of relativity. For a spherically symmetric, conservative motion the transition from compression to expansion admits of two possibilities: either passage through the center with a nonzero velocity, or a turning point at a finite distance from the center. It is shown that the first case holds in the relativistic case for all possible relativistically invariant equations of state such that $p = \beta\epsilon$, $0 \le \beta \le 1$. Oscillatory collapse regarded as a quasar model is discussed. Such a model should be possible if oscillatory collapse can be observed from the outside, i.e., from the R-region. However, this problem remains open.

THE equations of the general theory of relativity are invariant with respect to the replacement $t \to -t$;[1] the irreversible nature of the gravitational closure of a star[2, 3] is for this reason a paradox. The beginning of the gravitational closure of a star is a slow-quasi-static passage of a massive star through the Oppenheimer-Volkoff limit (OV limit[3, 4]), during which the velocity of the matter is very small: $v = 0$; consequently the initial conditions are also invariant with respect to the replacement $t \to -t$. In investigating collapse it is usual to neglect[2, 3] dissipative processes in the matter and the irreversibility of the equations is therefore unimpaired. This article is devoted to an explanation of this paradox.

The problem of what happens to the collapsing matter inside the Schwarzschild sphere remains unsolved.[3] In this article we shall advance arguments favoring the assumption that collapse represents an oscillatory equilibrium.[5] It will be shown that when dissipation is neglected there occurs pulsation of stars from comparatively large dimensions, corresponding to the OV limit, to dimensions of the order of the gravitational radius.

Gravitational collapse is due to the fact that there are in nature no forces which are capable of withstanding the strong gravitational field which compresses the star; the motion during collapse is determined only by the gravitational field and by the inertia of the matter. For this reason it is natural to consider initially the nonrelativistic problem on the motion of matter under the action of gravitation and inertia only, i.e., at zero pressure. Let us consider the spherically symmetric problem. The equation for the radius of the star will be (see [5]; Sec. 5.3):

$$d^2R/dt^2 = Ce^S/R^3 - kM/R^2, \tag{1}$$

where S is the entropy, C is a constant (at zero pressure $C = 0$), k is the gravitational constant, and M is the mass of the star (accurate to within a factor of the order of unity).

As we shall see below, in the course of collapse the matter passes through the center ($R = 0$). It is therefore essential to change the form of the spherical coordinate system in such a way that on passage through the center the polar angles should not change jump-wise. We therefore set

$$-\infty < R < +\infty, \quad 0 < \theta < \pi, \quad 0 < \varphi < \pi, \tag{2}$$

so that on passing through the center R changes sign and the polar angles remain unchanged. Equation (1) remains valid. Multiplying it by $\dot{R}$ and integrating, we obtain

$$\dot{R}^2 = 2\left(E + \frac{kM}{|R|}\right), \qquad E = -\frac{kM}{R_0}, \tag{3}$$

where E is an integration constant representing the total energy per unit mass. For $E < 0$ oscillatory motion takes place with $-R_0 < R < R_0$ ($R_0$ is the maximum radius). Near the center

$$R \sim (t - t_0)^{2/3}, \qquad \dot{R} \sim (t - t_0)^{-1/3} \to \infty \text{ as } R \to 0. \tag{4}$$

The period of the oscillations is:[5, 6]

$$T = 2\int_0^{R_0}\left[2kM\left(\frac{1}{R} - \frac{1}{R_0}\right)\right]^{-1/2} dR = \frac{\pi}{2}\sqrt{\frac{R_0^3}{kM}}. \tag{5}$$

The fact that the motion will be oscillatory is clear from physical considerations, since dissipative processes have not been taken into account.

We shall now show that the assumption about the oscillatory nature of the collapse does not contradict the general theory of relativity. The general theory of relativity is distinguished by the fact that in the first place the equations of motion must be relativistic and take into account the tensor character of gravitation and the velocity dependence of the mass, and secondly that it is essential to take into account the distortion of the observed picture due to the bending of the light rays and the Doppler effect in a strong gravitational field.

Since the processes of collapse which interest us take place inside the Schwarzschild sphere (in the T-region according to the terminology of [3,7]), we must choose a coordinate system which describes processes in this region.[2,3] The Schwarzschild solution is unsuitable because it corresponds to an observation at infinity[8] for which the interior of the Schwarzschild sphere is unobservable.

Let us now take the co-moving coordinate system[1] and let us now assume that close to the center the motion is isotropic. This makes the equation for the isotropic model of the universe[1] valid in the co-moving coordinate system:

$$R_0^0 - \frac{1}{2}R = -\frac{3}{a^4}\left[\left(\frac{a}{c}\right)^2\left(\frac{da}{d\tau}\right)^2 + a^2\right] = -\frac{8\pi k}{c^4}\varepsilon, \qquad (6)$$

or

$$\frac{3}{a^2}\left[\left(\frac{da}{cd\tau}\right)^2 + 1\right] = \frac{8\pi k}{c^4}\varepsilon,$$

where $\tau$ is the proper time in the co-moving coordinate system, $R_k^i$ is the contracted curvature tensor, $R$ is the scalar curvature, $a$ is the radius of the universe, and the range of $a$ is defined in accordance with (2). The line element is

$$ds^2 = c^2 d\tau^2 - dl^2. \qquad (7)$$

Neglecting dissipative processes, the following expression is valid:[1]

$$3\ln|a| = -\int \frac{d\varepsilon}{p+\varepsilon}, \qquad (8)$$

where the pressure $p$ is a function of the energy $\epsilon$. We consider the case of a linear dependence:

$$p = \beta\varepsilon, \quad 0 \leqslant \beta \leqslant 1; \qquad (9)$$

$\beta_1 = 0$ for cold nonrelativistic matter, $\beta_2 = 1/3$ for ultrarelativistic matter and electromagnetic radiation, and $\beta_3 = 1$ for the relativistic limit of Zel'dovich.[9] Formula (8) yields:

$$|a|^{3(1+\beta)}\varepsilon = \text{const.} \qquad (10)$$

Substituting (10) in (6), we have

$$\frac{1}{c^2}\left(\frac{da}{d\tau}\right)^2 = \left[-1 + \frac{c}{3|a|^{3(1+\beta)-2}}\right] \sim 2[E - U(a)]. \qquad (11)$$

In the isotropic model all linear dimensions are proportional to a; one can therefore take a to be the Lagrange coordinate of some body; a is the Euler coordinate at the initial time.

Equation (11) coincides with the formula for the velocity $\dot{a}$ of a body of unit mass moving in a spherical well with a potential energy $U(a)$ with $U(0) = -\infty$. Therefore $\dot{a}$ increases without bound for $a \to 0$. (We note that $\dot{a} \to \infty$ does not contradict the fact that the velocity cannot be larger than the speed of light, since a is determined from the condition that the surface of the sphere is $4\pi a^2$ and not from radial distances.) The motion of bodies in a potential well has been studied in detail—periodic oscillations take place in this case.

For $a \to 0$ only the second term in the square brackets of (11) is appreciable. The solution near a singular point is of the form:

$$a \sim |\tau - \tau_0|^\alpha \operatorname{sign}(\tau - \tau_0), \quad a(\tau_0) = 0,$$

$$\alpha = 2/3(1+\beta) \geqslant 1/3 > 0,$$

$$\alpha_1 = 2/3, \quad \alpha_2 = 1/2, \quad \alpha_3 = 1/3. \qquad (12)$$

The function $a(\tau)$ defined by (12) is an odd function of the difference $\tau - \tau_0$. As can also be seen from (12), an arbitrary particle reaches the center after a finite time.[1] It is also readily seen that in the vicinity of the point $\tau = \tau_0$ the derivative $\dot{a}$ does not change sign—the particles move in one direction without stopping or turning.

Let us now consider the relativistic problem of the spherically-symmetric motion of matter with zero pressure. The expression for the line element is (see [1]):

$$ds^2 = c^2 d\tau^2 - r^2(R,\tau)\, d\sigma^2 - e^{\omega} dR^2,$$

$$d\sigma^2 = d\theta^2 + \sin^2\theta\, d\varphi^2, \qquad (13)$$

where R is a coordinate characterizing the distance from the center. If the initial distribution was "classical," then R can be defined as the distance from the center at the initial time; r is the "radius" at the instant $\tau$, defined in such a way that the circumference of the circle with its center at the origin is $2\pi r$. The equations of the general theory of relativity lead to the relation:[1]

$$\dot{r}^2 = f(R) + F(R)\,/\,r, \quad F > 0, \tag{14}$$

where f and F are arbitrary functions of R. If infinite values of R are not reached, then $f < 0$. From dimensionality considerations it follows that $F \sim kM$. Obviously Eqs. (14) and (3) are very similar and (14) therefore also yields an oscillatory solution. An equation of the type of Eq. (5) is obtained for the period. Allowance for the pressure leads to an increase in the period. Equation (5) differs from the equation for the pulsations of the cepheids by a numerical factor and it leads to sensible values of the period[5] of the order of several days and weeks, depending on the values of the parameters employed; this is close to the values observed for quasars.[10]

Spherically-symmetric, conservative motion in a potential well admits of two alternatives: passage through the center with a velocity $\dot{r} \neq 0$ and turning of the particle at a finite distance from the center. It follows from the above that the first of these is realized; it is seen from (12) that account of the pressure or of any relativistically invariant interaction also does not lead to the second alternative: $\dot{r} \neq 0$ always and the center is reached. The fact that a turning point of the type $\dot{r} = 0$ is impossible has been shown in [7, 11]; in relativistic collapse none of the classical (in the sense of "non-quantum") interactions is capable of stopping the matter which by virtue of the spherical symmetry should pass through the center. All three examples which have been considered have led to an oscillatory character of the motion in the associated coordinate system. Thus, in the associated coordinate system the course of the relativistic and nonrelativistic collapse (for $p = 0$) is identical—the solution is of an oscillatory nature with a finite period.

The Newtonian problem of the collapse coincides formally with the problem of the motion of a test particle in a field with a potential $1/r$ in plane Euclidean space. In the relativistic case one must analogously consider the motion of a test particle in the Schwarzschild field. Let us consider the motion of a test particle in the coordinates of Finkelstein who uses the variable $\tau$ in place of the Schwarzschild time:[1, 12]

$$ct = c\tau \pm \ln\left(1 - \frac{|r|}{r_0}\right) \operatorname{sign} r, \quad -r_0 < r < r_0; \tag{15}$$

we use, as in (2), negative r. For definiteness we take the plus sign in (15). The argument of the logarithm has been written in such a way that t be real inside the Schwarzschild sphere, in the T-region ($|r| < r_0$). Then

$$cdt = cd\tau - \frac{dr}{1 - |r|/r_0}, \tag{16}$$

and it follows that the function $t(r, \tau)$ has a continuous first derivative everywhere in the T-region including at the point $r = 0$. The continuity at the point $r = 0$ is seen from the fact that for $r \ll r_0$ we have

$$ct = c\tau - |r| \operatorname{sign} r = c\tau - r. \tag{17}$$

From (15) we see that in passing through the center when r changes sign the sign in front of the logarithm also changes. This means that the motion towards the center is replaced by motion from the center.[1, 12] Formula (17) has no singularities whatsoever at $r = 0$.

Thus we can consider the oscillatory nature of the motion in the co-moving system of coordinates under the condition of spherical symmetry proved for the case of a conservative system. In how far are these assumptions fulfilled?

The assumption that the motion is isotropic and the assumption of spherical symmetry are very crude. The density at the center turns out to be infinite, since at $\tau = \tau_0$ all particles pass through the center. Inasmuch as the compressed models are unstable,[1, 3] the radial symmetry is destroyed, clusters are formed, and the density of the matter does not reach infinity at the origin.[13] During the half-period of expansion a portion of the clusters having a sufficient energy may leave the collapsing mass. Since small fluctuations in the expansion are dissipated, the clusters formed during the compression should be rather large. From what has been said it is also clear that precisely oscillatory collapse and not anticollapse,[3] in the course of which a spherically symmetric shell can be discarded but not clusters of matter, is required for the formation of stars. In the absence of symmetry the density is finite everywhere and the "gravitational well" has a finite depth. If the motion in the infinitely deep well was oscillatory, it remains oscillatory in the well of finite depth and the matter passes by the origin.

Similarly the system is not conservative. The expansion phase cannot end in the T-region;[3, 11] emergence into the R-region is unavoidable ($|r| > r_0$) and therefore energy losses to radiation and ejection of clusters are possible. Owing to energy dissipation processes the outer radius decreases steadily ($r \to r_0 + 0$) and in the final analysis the remaining part of the collapsing matter pulsates within the Schwarzschild sphere. After the maximum radius becomes of the order of the gravitational radius, the energy dissipation ceases asymptotically and the process becomes purely oscillatory without damping. The pulsating matter enters the R-region periodically and the picture for an outside observer depends essentially on the structure of the R-region. Two cases are possible here: a) the structure of the R-region is such that the external observer sees only one period—the anticollapse going over into the collapse. The time of the intense luminosity is of the order of T according to (5);[3] b) the structure of the R-region is such that an external observer sees many periods of oscillation. The question of which of the alternatives is admitted by the equations of the theory of general relativity remains open and is outside the scope of this article. We note merely that the second alternative is very attractive since it explains well the observed phenomena. The periodic change in the light yield of quasars[10] is determined by the period of collapse. The turbulent oscillatory nature of the collapse leads to intense radiation and to the possibility of ejection of a large amount of matter from the core of the star which contains heavy elements. There is no sharp, almost instantaneous, self-closure which excludes radiation of a considerable portion of the energy[3] and this explains the intense luminosity of quasars. We emphasize again that the question remains open and that the picture of the oscillatory collapse for an outside observer is being investigated further.

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