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WAVE SYNCHRONIZATION IN A GAS LASER WITH A RING RESONATOR

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Synchronization of two opposed waves in a ring laser is considered for the case where frequency detuning between the waves is possible. Polarization is computed with account of the zeroth and first harmonic of the elements of the density matrix. A formula has been obtained for the width of the synchronization range in single-mode operation, containing second-order terms of the mirror reflection coefficient. Results are given of an experimental investigation of the width of the synchronization range as a function of the magnitude and phase of the reflection coefficient. The reflection coefficient was varied by means of an auxiliary mirror. The observed dependence of the synchronization range on the reflection coefficient is in qualitative agreement with the analytical results.

THE investigation of wave processes in ring lasers has been the subject of a number of recent publications (see Fig. 1 below).[1-5] As we know, ring lasers excite two opposed waves whose frequencies may in general be different from one another. For example, rotation of the ring laser about an axis normal to the laser plane makes the conditions of propagation of the two opposed traveling waves unequal, because of the change in the equivalent perimeter of the circuit. This means that the natural frequencies of the resonator are split by a magnitude proportional to the angular velocity of rotation:

$$\Delta\omega_{1,2} = 8\pi n S / \lambda L. \tag{1}$$

Here s is the area of the circuit, L is the perimeter of the circuit, and n is the number of revolutions per second. This relationship has led many authors to conclude that the same frequency difference should also separate the frequencies of the two opposed waves excited in the laser. Nevertheless, the mutual synchronization of the opposed waves may disturb such a correspondence[4] at low angular velocities of rotation. A study of this problem is the subject of this paper.

We begin with the analysis of polarization of the active medium of the laser, due to the presence of opposed waves.

1. COMPUTATION OF THE POLARIZATION

We define the state of gas molecules by means of equations for the density matrix[2, 6] $\rho_{nm}(\mathbf{R}, \mathbf{p}, t)$:

$$\frac{\partial\rho_{nm}}{\partial t}+\mathbf{v}\frac{\partial\rho_{nm}}{\partial\mathbf{R}}=i\omega_{mn}\rho_{nm}+\frac{ie}{\hbar}\sum_k(\mathbf{r}_{nk}\rho_{km}-\rho_{nk}\mathbf{r}_{km})\mathbf{E}(\mathbf{R},t)-\frac{1}{\tau}(\rho_{nm}-\rho_n{}^0\delta_{nm}). \quad (1.1)$$

Here $\mathbf{v} = \mathbf{p}/m$ is the molecule velocity, $\mathbf{r}_{nm}$ is the matrix element of the displacement vector, $\omega_{mn} = (E_m - E_n)/\hbar$, $\mathbf{E}(\mathbf{R}, t)$ is the electric field intensity in the laser, and δ_{nm} is the Kronecker delta.

The last term in the right-hand side of this equation defines relaxation in the absence of the field $\mathbf{E}(\mathbf{R}, t)$ to a given distribution ρ_n^0, i.e., to the population distribution induced in the discharge tube. The corresponding relaxation times are designated by τ_{nm}. Only the two-level case will be discussed below, and therefore the quantum numbers n and m can assume only two values, 1 and 2.

The polarization vector is computed under the following assumptions: The density matrix is represented in the form

$$\rho_{nm}(\mathbf{R},\mathbf{p},t)=\rho_n(\mathbf{p})\delta_{nm}+\rho_{nm}^{(1)}(\mathbf{R},\mathbf{p},t),\quad \rho_{nm}^{(1)}=0 \quad \text{for} \quad n=m. \quad (1.2)$$

It is thus assumed that the diagonal elements of the density matrix are independent of time and coordinates. This means that only the zeroth and first harmonics of the elements of the density matrix are considered in the analysis. Such an approximation naturally fails to account for a number of phenomena, such as the reflection of waves from the periodic spatial structure in the population distribution of the molecules, due to the addition of the opposed waves in a fixed laser or in a laser rotating in synchronism. The second spatial harmonic is required to account for this phenomenon in the expression for the diagonal density matrix elements. This effect is not considered here, since it is assumed that mirror reflection plays the major role in the establishment of the synchronized state. The time-dependent effect of the second harmonic is also not accounted for. The validity of such an approximation in the case of a gas laser calls for special research and is beyond the scope of this work. (Analysis based on the perturbation method indicates that allowance for the second harmonic yields terms whose order of magnitude is not considered in our approximation.)

Let us consider single-mode operation and thus define the field intensity in the laser as a sum of two opposed waves:

$$\mathbf{E}(\mathbf{R},t)=\sum_{1,2}\mathbf{E}_{1,2}(\mathbf{R},t),$$

$$\mathbf{E}_{1,2}(\mathbf{R},t)={}^1\!/_2(\mathbf{E}_{1,2}\exp\{i\omega t\mp i\mathbf{k}_{1,2}\mathbf{R}\}+\text{c.c.}). \quad (1.3)$$

Here and below, the upper sign refers to index 1 and the lower to index 2. Similarly for the polarization vector,

$$\mathbf{P}(\mathbf{R},t)=\sum_{1,2}\mathbf{P}_{1,2}(\mathbf{R},t),$$

$$\mathbf{P}_{1,2}(\mathbf{R},t)={}^1\!/_2(\mathbf{P}_{1,2}\exp\{i\omega t\mp i\mathbf{k}_{1,2}\mathbf{R}\}+\text{c.c.}). \quad (1.4)$$

As usual, we assume that $r_{nm} = 0$ when $n = m$.

Under the above conditions, the expression for the polarization vector can be written in the form,

$$\mathbf{P}_{1,2}=\frac{e^2n|\mathbf{r}|^2}{\hbar}\mathbf{E}_{1,2}\int_{-\infty}^{\infty}\frac{D(\mathbf{v})\,(\omega\mp\mathbf{k}_{1,2}\mathbf{v}-\omega_0+i/\tau_2)\,d\mathbf{v}}{(\omega\mp\mathbf{k}_{1,2}\mathbf{v}-\omega_0)^2+\tau_2^{-2}}. \quad (1.5)$$

Here, $D(\mathbf{v})$ is the difference in the level populations, taking account of saturation:

$$D(\mathbf{v})=D^0(\mathbf{v})\left\{1+\sum_{1,2}\frac{(\tau_1/\tau_2)\,e^2h^{-2}|\mathbf{r}|^2E_{1,2}^2}{(\omega\mp\mathbf{k}_{1,2}\mathbf{v}-\omega_0)^2+\tau_2^{-2}}\right\}^{-1}. \quad (1.6)$$

The velocity distribution of molecules is assumed maxwellian.

We change over to dimensionless variables and introduce the following symbols: E_0 is the amplitude of the forward and backward waves in the absence of mutual coupling;

$$a=\frac{\tau_1e^2|\mathbf{r}|^2E_0^2}{\tau_2\hbar^2\omega^2};\qquad b=\frac{e^2|\mathbf{r}|^2nD^0}{\hbar\omega},$$

$D^{(0)}$ is the molecule population difference in the absence of a field, summed over all velocities; the parameter a designates the role of saturation; parameter b designates the polarizability in a weak field; $2\beta = 1/\omega\tau_2$ is the characteristic of the relative natural line width of a molecule; $\alpha^2 = kT/mc^2$ defines the role of the Doppler effect; $(\omega_0 - \omega)/\omega = \mu$ is the relative detuning in the emission of the molecule; and $(\omega_1 - \omega_2)/\omega = \Delta\omega_{1,2}/\omega = \Omega$ is the relative frequency difference of the resonators. Here, $\omega_1 = k_1c$, $\omega_2 = k_2c$, $\Delta_{1,2} = (\omega_{1,2} - \omega)/\omega$ is the relative detuning $\mu \pm v/c = z$, $E_{1,2}/E_0 = e_{1,2}$.

In terms of the above designations, expression (1.5) with allowance for (1.6), assumes the form

$$\mathbf{P}_{1,2}=-\frac{b\mathbf{E}_{1,2}}{\alpha\sqrt{2\pi}}\int_{-\infty}^{\infty}[z+(z-\mu)\Delta_{1,2}-2i\beta]\times\exp\left\{-\frac{(z-\mu)^2}{2\alpha^2}\right\}\times\Big\{[z+(z-\mu)\Delta_{1,2}]^2+4\beta^2+a\Big[e_{1,2}^2$$

$$+ e_{2,1}^2 \frac{[z+(z-\mu)\Delta_{1,2}]^2+4\beta^2}{[2\mu - z-(z-\mu)\Delta_{1,2} \pm \Omega(z-\mu)]^2+4\beta^2}\Big]\Big\}^{-1} dz. \quad (1.7)$$

Let us simplify this expression, using the conditions

$$\beta \ll \alpha, \quad \mu \ll \alpha, \quad a \ll 4\beta^2, \quad \Omega \ll \beta. \quad (1.8)$$

The first and second conditions are always well satisfied in gas lasers. The third and fourth conditions impose corresponding limitations upon the magnitude of the field in the laser and upon the mismatch Ω, which can, for example, be due to rotation.

After expansion of the integrand in (1.7) in terms of the small parameters $a/4\beta^2$ and Ω/β and change of the integration variable,

$$(z+(z-\mu)\Delta_{1,2})/2\beta \to z,$$

expression (1.7) assumes the form,

$$\mathbf{P}_{1,2} = i\frac{b\mathbf{E}_{1,2}}{\alpha\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\exp\{-(z-x)^2/\eta^2\}}{1-iz}\Big\{(1-\Delta_{1,2})$$

$$\times\Big[1-\frac{ae_{1,2}^2}{4\beta^2(z^2+1)} - \frac{ae_{2,1}^2}{4\beta^2(z^2-4xz+4x^2+1)}\Big]$$

$$\mp \frac{2ae_{2,1}^2(z^2-3zx+2x^2)}{4\beta^2(z^2-4xz+4x^2+1)}\Omega\Big\}dz. \quad (1.9)$$

Here

$$\eta = \alpha/\beta\sqrt{2}; \qquad x = \mu/2\beta. \quad (1.10)$$

All integrals in (1.9) are expressed by a probability integral with complex argument of the form $(1 \mp ix)/\eta$. An expansion of these expressions in terms of the modulus of the complex argument, which is small in view of (1.8) and (1.10), leads to the following expression for the complex polarization, $\kappa_{1,2} = \mathbf{P}_{1,2}/\mathbf{E}_{1,2}$:

$$\varkappa_{1,2} = -\frac{\gamma}{4\pi}\Big\{\Big[\sqrt{\frac{2}{\pi}}\frac{\mu}{\alpha} - \frac{\mu a e_{1,2}^2}{4\beta\alpha^2}$$

$$-\frac{\mu a e_{2,1}^2}{4\beta(4\beta^2+\mu^2)}\Big](1-\Delta_{1,2}) \mp \frac{\mu a e_{2,1}^2(\beta^2-\mu^2)\Omega}{8\beta^3(\mu^2+4\beta^2)}\Big\}$$

$$+ i\frac{\gamma}{4\pi}\Big\{\Big[1-\frac{ae_{1,2}^2}{8\beta^2} - \frac{ae_{2,1}^2}{2(4\beta^2+\mu^2)}\Big](1-\Delta_{1,2})$$

$$\mp \frac{ae_{2,1}^2(2\beta^2-\mu^2)}{8\beta^2(\mu^2+4\beta^2)}\Omega\Big\}. \quad (1.11)$$

It should be noted that the contribution from wave 2 (backward wave) to the real part of the polarizability κ_1 is α^2/β^2 times larger in order of magnitude than the contribution from wave 1 (forward wave). In formula (1.11), $\gamma = 2\pi\sqrt{2\pi}b/\alpha$.

2. DETERMINATION OF THE SYNCHRONIZATION RANGE

The real part of polarizability κ contributes a field-dependent correction to the resonator frequency. The imaginary part, proportional to the number of excited molecules, contains the energy source and determines the excitation threshold in the laser and the magnitude of stationary amplitude.

If the excitation conditions are satisfied, two opposed standing waves are established in the laser. The coupling between the opposed waves can be provided by reflection from auxiliary mirrors, for example. If the interaction is sufficiently small, it can be expressed linearly. Taking such a linear coupling into account, the field intensity equations for two opposed waves in a laser assume the form,

$$\ddot{\mathbf{E}}_1 + \frac{\omega}{Q}\dot{\mathbf{E}}_1 + \omega_1^2\mathbf{E}_1 = -4\pi\ddot{\mathbf{P}}_1,$$
$$\ddot{\mathbf{E}}_2 + \frac{\omega}{Q}\dot{\mathbf{E}}_2 + \omega_2^2\mathbf{E}_2 = -4\pi\ddot{\mathbf{P}}_2, \quad (2.1)$$

where

$$\mathbf{P}_1 = \varkappa_1(\mathbf{E}_1 + \bar{m}_1\mathbf{E}_2), \qquad \mathbf{P}_2 = \varkappa_2(\mathbf{E}_2 + \bar{m}_2\mathbf{E}_1), \quad (2.2)$$

$\bar{m}_{1,2}$ are the complex coefficients of reflection and Q is the quality factor of the resonator.

As usual, assuming that

$$\mathbf{E}_1 = e_1\mathbf{E}_0\exp\{i(\omega t+\varphi)\}, \qquad \mathbf{E}_2 = e_2\mathbf{E}_0\exp\{i(\omega t+\psi)\},$$
$$\bar{m}_1 = m_1\exp\{i\vartheta_1\}, \qquad \bar{m}_2 = m_2\exp\{i\vartheta_2\}$$

and considering that e_1, e_2, φ, and ψ are slowly varying time functions, we obtain simplified equations for these functions:

$$\dot{e}_1 = \frac{\omega}{2}\Big[\Big(4\pi\,\mathrm{Im}\,\varkappa_1 - \frac{1}{Q}\Big)e_1 - 4\pi|\varkappa|m_1e_2\sin(\Phi-\chi_1)\Big], \quad (2.3)$$

$$\dot{e}_2 = \frac{\omega}{2}\Big[\Big(4\pi\,\mathrm{Im}\,\varkappa_2 - \frac{1}{Q}\Big)e_2 + 4\pi|\varkappa|m_2e_1\sin(\Phi+\chi_2)\Big], \quad (2.4)$$

$$\dot{\varphi} = \omega\Big[\Delta_1 - 2\pi\,\mathrm{Re}\,\varkappa_1 - 4\pi|\varkappa|\frac{m_1e_2}{2e_1}\cos(\Phi-\chi_1)\Big], \quad (2.5)$$

$$\dot{\psi} = \omega\Big[\Delta_2 - 2\pi\,\mathrm{Re}\,\varkappa_2 - 4\pi|\varkappa|m_2\frac{e_1}{2e_2}\cos(\Phi+\chi_2)\Big], \quad (2.6)$$

where*

$$\Phi = \varphi - \psi, \qquad \chi_{1,2} = \mathrm{arctg}\frac{\mathrm{Im}\,\varkappa_{1,2}}{\mathrm{Re}\,\varkappa_{1,2}} + \vartheta_{1,2}.$$

If two opposed waves E_1 and E_2 propagating in the laser are added together, beats should result

*$\mathrm{arctg} \equiv \tan^{-1}$.

in the general case; the beat frequency should be determined by the equation,

$$\Phi = \omega\left\{\Omega - 2\pi(\mathrm{Re}\,\varkappa_1 - \mathrm{Re}\,\varkappa_2) - 2\pi|\varkappa|\left[\frac{e_2}{e_1}m_1\cos\alpha_1 - \frac{e_1}{e_2}m_2\cos\alpha_2\right]\right\};$$

$$\alpha_1 = \Phi - \chi_1, \qquad \alpha_2 = \Phi + \chi_2. \tag{2.7}$$

In the synchronized state, $\dot{\Phi} = 0$ and (2.7) assumes the form,

$$\Omega = 2\pi(\mathrm{Re}\,\varkappa_1 - \mathrm{Re}\,\varkappa_2) + 2\pi|\varkappa|\left[\frac{e_2}{e_1}m_1\cos\alpha_1 - \frac{e_1}{e_2}m_2\cos\alpha_2\right]. \tag{2.8}$$

It is necessary to substitute in Eq. (2.8), which determines the phase difference between the opposed waves as a function of the detuning between the resonators, the expressions for e_1 and e_2 taken from (2.3) and (2.4) with $\dot{e}_1 = 0$, $\dot{e}_2 = 0$, and $\omega = (\omega_1 + \omega_2)/2$ (stationary mode).

Subtracting (2.4) from (2.3), and considering that $|\kappa| \approx \gamma/4\pi$ (see (1.11)) and $\Delta_{1,2} = \Omega/2$, we obtain,

$$\frac{a\mu^2(e_2^2 - e_1^2)}{8\beta^2(4\beta^2+\mu^2)} = \left[1 - \frac{a(4\beta^2+3\mu^2)}{8\beta^2(\mu^2+4\beta^2)}\right]\Omega + m_1\frac{e_2}{e_1}\sin\alpha_1 + m_2\frac{e_1}{e_2}\sin\alpha_2. \tag{2.9}$$

Let us substitute expressions (1.11) and (2.9) into (2.8), taking account of conditions (1.8):

$$\Omega = \gamma\left\{\frac{\beta}{\mu}\left[1 - \frac{a(4\beta^4+3\beta^2\mu^2+\mu^4)}{8\beta^4(\mu^2+4\beta^2)}\right]\Omega + \frac{m_1e_2}{e_1}\left(\frac{\beta}{\mu}\sin\alpha_1 + \frac{1}{2}\cos\alpha_1\right) + \frac{m_2e_1}{e_2}\left(\frac{\beta}{\mu}\sin\alpha_2 - \frac{1}{2}\cos\alpha_2\right)\right\}. \tag{2.10}$$

We shall now consider that the coupling between the waves is weak. If the wave coupling is totally absent, the amplitudes of both waves have the same value E_0, given by

$$a = \frac{\tau_1 e^2|\mathbf{r}|^2E_0^2}{\tau_2\hbar^2\omega^2} = \frac{(\gamma Q - 1)8\beta^2(4\beta^2+\mu^2)}{\gamma Q(8\beta^2+\mu^2)}. \tag{2.11}$$

Consequently, $e_{10} = e_{20} = 1$ when there is no coupling. On the other hand, if coupling is present but is small, it can only slightly affect the wave amplitude;[1)] consequently, one can set $e_1 = 1 + A$ and $e_2 = 1 + B$, where $A \ll 1$ and $B \ll 1$.

As we shall see below, such an assumption is valid if μ is not too small, i.e., if the laser frequency is not too close to the mean frequency of molecular transition (center of the Doppler line). This is the case that will now be discussed. Small μ call for the investigation of the amplitude stability of two opposed waves, which is beyond the scope of this paper.

Accurate to first-order terms we have

$$e_2/e_1 = 1 + (B - A), \quad e_1/e_2 = 1 - (B - A).$$

The value of $B - A$ is found from (2.3) and (2.4), also accurate to first-order terms:

$$B - A = \frac{4\beta^2(4\beta^2+\mu^2)}{a\mu^2}(m_1\sin\alpha_1 + m_2\sin\alpha_2) \tag{2.12}$$

(the term containing Ω has been neglected, since $\Omega \ll m$). Substituting (2.12) into (2.10), we find

$$\Omega = \frac{\gamma}{2K}\left\{m_1\left(\frac{2\beta}{\mu}\sin\alpha_1 + \cos\alpha_1\right) + m_2\left(\frac{2\beta}{\mu}\sin\alpha_2 - \cos\alpha_2\right) + \frac{2\beta^2(4\beta^2+\mu^2)}{a\mu^2}\left[m_1^2\left(\frac{4\beta}{\mu}\sin^2\alpha_1 + \sin 2\alpha_1\right) - m_2^2\left(\frac{4\beta}{\mu}\sin^2\alpha_2 - \sin 2\alpha_2\right) + 2m_1m_2\sin(\alpha_1+\alpha_2)\right]\right\}, \tag{2.13}$$

where

$$K = 1 - \frac{\gamma\beta}{\mu}\left[1 - \frac{a(4\beta^4+3\beta^2\mu^2+\mu^4)}{8\beta^4(\mu^2+4\beta^2)}\right]$$

Analyzing expressions (2.12) and (2.13), we can see that they are valid ($B - A \ll 1$) only if

$$\frac{4\beta^2(4\beta^2+\mu^2)}{a\mu^2}m_{1,2} \ll 1.$$

This is the condition that imposes definite limitations upon the value of μ.

The synchronization range is determined from the stability condition of the synchronized state:

$$d\Omega/d\Phi \geqslant 0 \tag{2.14}$$

(it is assumed that amplitude stability exists). Thus, the boundary of the synchronization range is determined by the equation

$$d\Omega/d\Phi = 0. \tag{2.15}$$

The synchronization range Ω_0 can be obtained by eliminating Φ from (2.15) and (2.13), in which

$$\mu/2\beta = \tau_2(\omega_0 - {}^1\!/_2(\omega_1+\omega_2)).$$

The system (2.13) and (2.15) can in general not be solved analytically. Let us therefore consider three special cases.

1. $m_2 = 0$; $m_1 = m$; $\alpha_1 = \alpha$. Equations (2.13) and (2.15) here assume the form

$$\Omega_0 = \frac{\gamma}{2K}m\left(\frac{2\beta}{\mu}\sin\alpha + \cos\alpha\right), \tag{2.16}$$

[1)]See the thesis of V. I. Parygin, Physics Department of the Moscow State University, 1955.

$$\frac{2\beta}{\mu}\cos\alpha - \sin\alpha = 0. \tag{2.17}$$

The second-order term in m has been neglected here, since the linear term cannot vanish for any values of the parameters.

Eliminating the angle α from (2.16) and (2.17), we get

$$\Omega_0 = \frac{\gamma}{2K}\frac{\sqrt{4\beta^2+\mu^2}}{\mu}m. \tag{2.18}$$

2. $m_1 = m_2 = m$. Designating

$$\Phi - \frac{\chi_1-\chi_2}{2} = \Psi, \quad \frac{\chi_1+\chi_2}{2} = \chi,$$

$$\frac{2\gamma\beta}{\mu K}\cos\chi + \frac{\gamma}{K}\sin\chi = C,$$

$$\frac{4\gamma\beta^2(4\beta^2+\mu^2)}{K a\mu^2}\cos^2\chi\left(1-\frac{2\beta}{\mu}\operatorname{tg}\chi\right) = D, \tag{2.19}$$

we rewrite (2.13) and (2.15) in the form

$$\Omega_0 = Cm\sin\Psi + Dm^2\sin 2\Psi,$$

$$0 = C\cos\Psi + 2Dm\cos 2\Psi. \tag{2.20}$$

The term of the order of m^2 was retained in (2.20), because at a certain relation between phases χ_1 and χ_2 the coefficient C of the linear term may approach zero, leaving the squared term to play the major role. C is close to zero if

$$\operatorname{tg}\chi \approx -2\beta/\mu. \tag{2.21*}$$

Eliminating the angle ψ from (2.20), we get

$$\Omega_0 = Cm\left(\frac{3}{4}+\frac{1}{4}\sqrt{1+32\frac{D^2m^2}{C^2}}\right)\left\{\frac{1}{2}-\frac{C^2}{32D^2m^2} + \frac{C^2}{32D^2m^2}\sqrt{1+\frac{32D^2m^2}{C^2}}\right\}^{1/2}. \tag{2.22}$$

The following limiting cases can be considered in the analysis of (2.22):

a) $C \neq 0$ and $Dm \ll C$. Then, $\Omega_0 = Cm$;

b) the coupling is sufficiently strong, so that $C \ll Dm$, with $\Omega_0 = Dm^2$.

In the intermediate case, when C is comparable to Dm, the dependence of the synchronization range on the reflection coefficient m is more complex.

3. Only first-order terms in m are retained in Eq. (2.13). This may be done when either $m_1 \gg m_2$ (or $m_2 \gg m_1$) or the phase difference χ of the reflected rays is such that the linear term is always larger than the squared term. Setting $m_2 = m_1(1+M)$, and designating

$$C_1 = \frac{\gamma}{K}\left(\frac{2\beta}{\mu}\cos\chi + \sin\chi\right)\left(1+\frac{M}{2}\right), \tag{2.23}$$

$$C_2 = \frac{\gamma}{2K}M\left(\frac{2\beta}{\mu}\sin\chi - \cos\chi\right), \tag{2.24}$$

we rewrite (2.13) in the form

$$\Omega_0 = C_1 m_1\sin\Psi + C_2 m_1\cos\Psi. \tag{2.25}$$

Substituting $\sin\psi$ and $\cos\psi$ from (2.15) into (2.25),

$$C_1\cos\Psi - C_2\sin\Psi = 0,$$

we obtain

$$\Omega_0 = (C_1^2+C_2^2)^{1/2}m_1 = \frac{\gamma m_1}{K}\left\{(1+M)\left(\frac{2\beta}{\mu}\cos\chi+\sin\chi\right)^2 + \frac{M^2}{4}\left(\frac{4\beta^2}{\mu^2}+1\right)\right\}^{1/2}. \tag{2.26}$$

We note in this case that the synchronization range undergoes a nonlinear change when one of the coupling coefficients (say m_2) changes and m_1 remains constant.

When $m_2 \approx m_1$ ($M \ll 1$), the synchronization is a linear function of M, whose slope depends upon the phase of reflected rays:

$$\Omega_0 \approx \frac{\gamma m_1}{K}\left(\frac{2\beta}{\mu}\cos\chi+\sin\chi\right)\left(1+\frac{M}{2}\right). \tag{2.27}$$

As M grows, the slope of the curve increases and when M is large ($M \gg 1$) it no longer depends upon the phase of the reflected ray; at the same time, the function $\Omega_0(M)$ again becomes linear:

$$\Omega_0 \approx \frac{\gamma m_1}{2K}\left(\frac{4\beta^2}{\mu^2}+1\right)^{1/2}M. \tag{2.28}$$

3. EXPERIMENTAL PART

In the majority of experiments performed to study the locking phenomenon, the ring laser (Fig. 1) was suspended so as to permit torsional

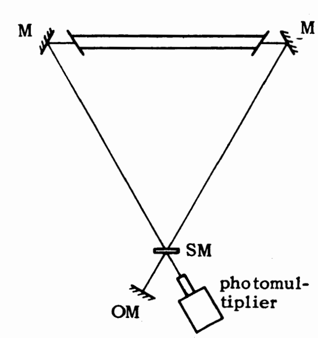


FIG. 1

*tg ≡ tan.

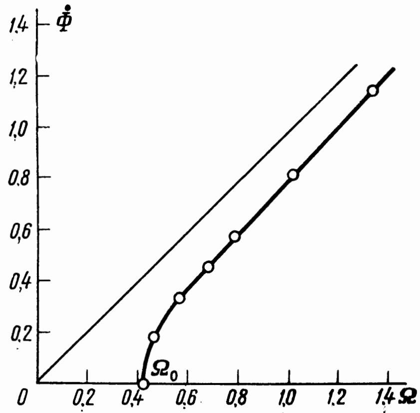


FIG. 2

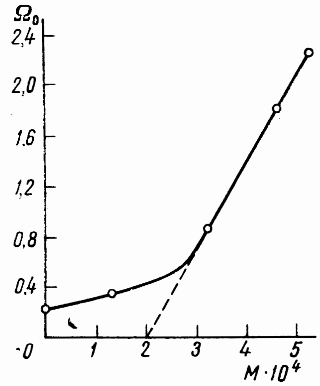


FIG. 4

oscillations in the plane of the loop. The torsional oscillations of the platform had a fairly low damping and a practically sinusoidal form. Therefore, the detuning frequency $\Delta\omega_{1,2}$ of the opposed beams was sinusoidal.

Interference of two opposed beams emerging from the ring laser produced a difference-frequency signal at the cathode of a photomultiplier. The difference-frequency signal was recorded with a loop oscilloscope and processed. A typical diagram of the observed beat frequency as a function of the detuning is given in Fig. 2. The diagram shows that the difference frequency is zero at low detunings. This then is the range of total mutual synchronization of the opposed-beam frequencies. Large frequency detuning of the opposed beams results in beats whose frequency asymptotically approaches the detuning frequency as the latter increases.

A series of experiments was carried out to study the width of the locking band as a function of the modulus of the coupling coefficient m_2. For this purpose, an auxiliary mirror AM was mounted behind one of the ring laser mirrors. The auxiliary mirror was adjusted for exact backward reflection of one of the laser output beams (Fig. 3). This arrangement established a directed coupling between the rays propagating in the right and left directions. Such a coupling was equivalent to an increase in one of the coupling coefficients (m_2, for example) due to internal resonator scattering. The additional coupling coefficient introduced into the system can be readily determined, given the transmission of the laser output mirror, the transmission of filter F, and the reflection coefficient of the auxiliary mirror. Filter F is introduced to vary the strength of the coupling. Owing to scatter in the measurement of Ω, every experimental point plotted in Fig. 4 represents an average of many measurements.

It is apparent that the slope of the curve in Fig. 4 depends on the additional reflection coefficient M, so that a transition from a lower (smaller M) to a higher (larger M) slope of the straight line is noted. The experimental conditions, on the whole, correspond to the third case considered in the theoretical section of this paper. The shape of the experimental curve is in qualitative agreement with the theoretical function $\Omega_0(M)$ derived from (2.26).

Of particular interest is the fact that at large reflection coefficients a change takes place in the nature of the beat frequency function, depending upon the detuning. If the coupling coefficient is low, the beat frequency sweeps through all values from ψ_{max} to zero on passing into and out of the locking band (see Fig. 2); a high coupling coefficient, on the other hand, gives rise to a sharp cutoff of the beats at a certain threshold frequency, the beats failing to assume any values below the

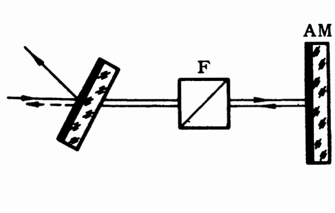


FIG. 3

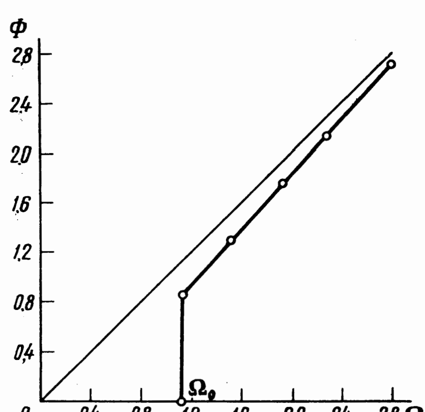
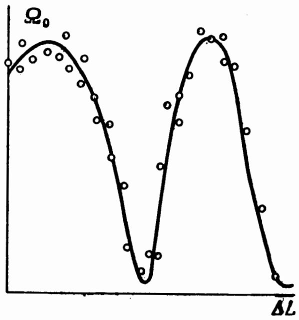


FIG. 5

FIG. 6



threshold (Fig. 5). A similar change in the locking characteristic, at different coupling coefficients, occurs also in radio engineering,[7] furnishing yet another justification of the suggested analogy between processes occurring in the ring laser and in radio circuits.

We have already noted the interesting dependence of the width of the locking band upon the phase of reflected beams, which follows from formula (2.20). Experimental variation of the phase of scattered rays is feasible in the case when two return mirrors are used to provide two-way coupling, by moving one mirror parallel to itself. The width of the locking band as a function of phase difference of reflected beams is given in Fig. 6. The diagram clearly shows the periodic character of the curve, which is qualitatively close to the theoretical law (wide maximum and narrow minimum) derived from (2.19) and (2.20). If the quadratic term is neglected, then $\Omega_0 \sim |\cos(\chi + \delta)|$, where $\tan\delta = -\mu/2\beta$. The presence of the quadratic term prevents Ω_0 from decreasing to zero, even when $m_1 = m_2$. The fact that Ω_0 never reaches zero in Fig. 6, can be attributed either to the above circumstance, or to inexact equality of the reflection coefficients m_1 and m_2.

Thus, in spite of the approximate character of the theoretical section of this work, the basic features of the phenomenon discussed show a qualitative agreement with the experimental results.

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