INVESTIGATION OF THE LENGTH AND SHAPE OF A GIANT PULSE AS A FUNCTION OF THE POPULATION INVERSION COEFFICIENT

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Submitted to JETP editor December 13, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) **50**, 1458–1463 (June, 1966)

The possibility of an experiment that would allow for a quantitative verification of the conclusions of the giant pulse formation theory is investigated. It is shown that such a comparison is possible for the case of a laser with a passive Q-switch. The length and shape of a giant pulse from a neodymium glass laser have been measured for various values of population inversion and the results obtained have been compared with the theoretical conclusions. It is shown that the experimental results are in good agreement with theory when the population inversion coefficient is sufficiently large.

THE analysis of the giant pulse formation mechanism in Q-switched lasers and of the giant-pulse parameters as a function of laser parameters has been the subject of a number of theoretical papers.[1-3] The most interesting of these papers seems the work of Wagner and Lengyel,[2] because some of their results can be quantitatively compared with experimental results. These authors consider a two-level model of a laser operating in a single mode and neglect those effects which are slower than the length of a giant pulse (spontaneous emission, time-dependent change of pump power). Furthermore, they assume that the Q is restored instantaneously.

Based on these assumptions, Wagner and Lengyel obtained the following velocity equations for the photon density, Φ, in the resonator, and for the population inversion density, $N = N_2 - N_1$:

$$\frac{d\Phi}{dt} = \left(\frac{\alpha l}{t_r} - \frac{1}{T}\right)\Phi, \quad \frac{dN}{dt} = -\frac{2\alpha l}{t_r}\Phi, \tag{1}$$

where l is the length of the active laser rod, $\alpha = \alpha_0 N/N_0$ is the gain (α_0 is the coefficient of absorption by the active medium at the generation frequency with no pumping, $N_0 = N_1 + N_2$), t_r is the photon transit time of the optical path L of the resonator, and T is the photon lifetime in the resonator. In turn, $T = t_r/\gamma = L/c\gamma$, where c is the velocity of light, $\gamma = \gamma_1 + \gamma_2$ is the coefficient of total losses (fraction of photons lost in the time t_r), and γ_1 and γ_2 are coefficients of losses due to escape of radiation from the resonator and parasitic losses within the resonator.

A solution of the system (1) yields expressions for the total giant pulse energy, E, and peak pulse power, W_p, as functions of N_i, N_p, N_f, and V, where N_i is the population inversion density at the instant the Q is restored, N_p is population inversion density at the pulse peak, N_f is the final population inversion density, and V is the rod volume. Furthermore, Wagner and Lengyel used an electronic computer to obtain the lengths T_1 and T_2 of the leading and trailing edges of the pulse for given ratios of N_i/N_p. The results of the computation were tabulated, giving the numerical values of $t_1 = T_1/T$ and $t_2 = T_2/T$ for various ratios of N_i/N_p in the range from 1.1 to 12.0. In particular, the results obtained show that as the ratio of N_i/N_p increases, the total length of the pulse, $T_1 + T_2$, decreases, and the pulse itself becomes asymmetrical, assuming a steeper leading edge.

The comparison of theory with experiment is possible in principle by a direct measurement of the values of E, W_p, $t_1 + t_2$, and t_2/t_1, and by their comparison with the values computed according to Wagner and Lengyel[2] and based on the measurement of N_i, N_p, and V. Nevertheless, as far as we know, no rigorous quantitative comparison of these values has ever been made. In the first place, this is due to the known difficulties in measuring N_i and N_p, as well as the actually effective volume V of the active rod. Moreover, the majority of the existing Q-switches (rotating prism, Kerr cell, electro-optical and mechanical shutters) have a common characteristic in that they "switch in" all those regions of the active rod cross section in which the required population inversion $N_i > N_p$, has been reached. At the same time, as we know, the population inversion density N_i is inhomogeneously distributed over the volume of the rod, owing to a number of causes. This inhomogeneity is caused by an optical inhomogeneity of the rod, on the one hand, and by the varying pump energy density over the rod volume, on the other; it is also due to internal mode generation. As a result, the giant pulse is produced in different regions of the rod volume (and, consequently, of the rod cross-section) at different values of N_i, leading to an indeterminacy in the values of N_i, N_p, and V. The actually observed pulse, therefore, is averaged, as it were, over the entire cross-section of the rod, making it difficult to compare theory with experiment.

Passive shutters with phototropic substances[4-6] have an advantage from the viewpoint of the feasibility of comparing theory with experiment. Q-switches of this type make it possible to set up the operation of a laser so as to operate the shutter (to render the filter transparent) as the result of self-excitation of the system. In such a case, a slight excess of pump power over threshold will cause generation only in those regions of the rod cross section where gain has exceeded total losses, once population inversion has reached the required density. Under such conditions, therefore, the giant-pulse formation will be limited to definite regions of the rod cross section having a maximal value of N_i.[1)]

The presence of such limited regions of the rod cross section, participating in the formation of the giant pulse, is confirmed experimentally by photographing the emission energy distribution in the near field. Thus, Fig. 1 shows the emission energy distribution over the end face of a neodymium rod, clearly displaying the limited regions participating in the generation. In practice, it is obviously very difficult to determine that volume of the rod which is actually active in the formation of the pulse, and, therefore, it is difficult to compare theory with experiment in terms of quantities that depend upon the volume V, i.e., E and W_p.[2)] On the other hand, when passive Q-switches are used, it is easy to determine the ratio N_i/N_p, and to compare the theoretical and experimental values of $t_1 + t_2$ and t_2/t_1.

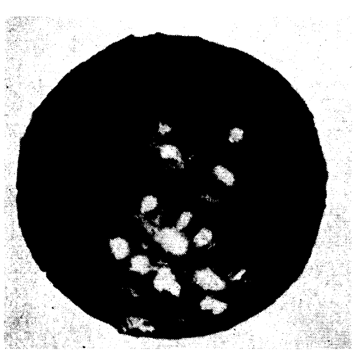

FIG. 1. Near-field distribution of a giant pulse emission energy ($\tau_f = 0.7$, rod diameter 11 mm).

The experimental determination of N_i/N_p depends upon the following conditions. In the case of a passive shutter, assuming that the main losses in the resonator are due only to absorption losses in the opaque shutter and to losses from radiation leakage from the shutter through the mirrors, the generation threshold criterion can be written as,

$$R_1R_2\tau_f \exp(2\sigma_a N_i l) = 1, \qquad (2)$$

where R_1 and R_2 are reflection coefficients of the resonator mirrors, τ_f is the transmittance of the filter in the opaque state at the generation frequency, and σ_a is the absorption cross section for active particles in the laser transition.

At the peak power of the giant pulse (it is assumed that the shutter is fully bleached at adequate speed), the total losses will equal the gain in the active medium having a population inversion density N_p; therefore, the following relationship should be satisfied for that instant of time:

$$R_1R_2 \exp(2\sigma_a N_p l) = 1. \qquad (3)$$

Equations (2) and (3) readily yield an expression for the population inversion coefficient N_i/N_p in the form of a function of simple parameters:

$$N_i / N_p = \ln R_1R_2\tau_f^2 / \ln R_1R_2. \qquad (4)$$

Consequently, by varying the initial transmittance τ_f of the passive filter one can change the ratio of N_i/N_p with fixed R_1 and R_2, and, in agreement with the theory,[2] also change the length $t_1 + t_2$ of the giant pulse and the ratio t_2/t_1.

The experimental arrangement was the same here as in our previous work.[6] Cylindrical neodymium glass rods of various dimensions were used in the experiment. The passive filter consisted of a solution of one of the analogs of poly-

[1)]It is this situation that can apparently explain the fact that with formerly used shutter types (rotating prism, Kerr cell, etc.), the shortest achieved length of a giant pulse was 20–30 nsec. At the same time, passive shutters resulted in pulse lengths of the order of 10 nsec with comparable pump power.

[2)]In this case, however, one can pose an inverse problem: estimate the effective volume V_{eff} of the rod participating in the generation, by measuring the total energy E of the pulse and the initial population inversion N_i.

methine dye in nitrobenzene. Varying concentration of this substance changed the transmittance of the solution within the limits from 0.35 to 0.98. The value of τ_f was measured with a spectrophotometer whose absolute error did not exceed 0.005. The time resolution of the receiver system was not worse than 10^{-9} sec. The laser parameters were selected so as to obtain a more rigorous agreement with (2) and (3).

First, relationships (2) and (3) will be satisfied if the losses due to radiation leakage from the resonator, $\gamma_1 = \frac{1}{2}[(1 - R_1) + (1 - R_2)]$, are much larger than the internal parasitic losses, γ_2, in the resonator (parasitic absorption, scattering, diffraction losses, etc.). Since $\gamma_2 \lesssim 10^{-2}$, and $R_1 \approx 1.0$ in our experiment, the reflection coefficient R_2 of the output mirror should be $\lesssim 0.8$. On the other hand, to reduce the effect of reflection from the end faces of the neodymium rod and the walls of the solution cell, the amplitude coefficient of reflection from the output mirror, $\sqrt{R_2}$, must be much larger than the amplitude coefficient of reflection from the end faces and cell walls, $\sqrt{r}$, where $r \approx 0.04$. Therefore, we have set $R_2 = 0.5 - 0.6$, as the optimal value satisfying both conditions.

Since the absorption cross section of the molecules of phototropic substances is very high at the generation frequency (usually, $\sigma_f \sim 10^{-15}$–10^{-16} cm^2), being by four orders of magnitude larger than the absorption cross section of the ions of the active medium, only a negligible portion of the total pulse energy is expended on bleaching the filter if the giant-pulse length is shorter than the decay time of the upper level in phototropic molecules. Consequently, in this case the value of N_i in Eq. (2) is close to that of the initial population inversion contained in the corresponding equations in [2]. We have carried out the following experiment to verify the applicability of this assumption to our case. The emission of a Q-switched laser (the transmittance of the cell with the solution in the resonator amounted here to 30–40%) was split by a glass plate into two beams which were detected independently by two receivers (FÉK-09). The beam intensities were equalized to obtain the same amplitudes of both signals at the oscilloscope. A vessel with polymethine dye solution was then placed in the path of one beam, and the intensities of both beams were compared again. This method allowed us to evaluate the energy absorbed by the solution during the bleaching process. Measurements showed that, within the limits of experimental error (< 10%), no change in intensity of the beam which passed through the cell was observed, even when the transmittance of the cell was reduced to very small values (< 0.1).[3)]

It can be assumed, therefore, that the bleaching of the filter used in this work (at sufficiently high power levels) indeed requires a very small portion of the pulse energy, and the filter becomes practically totally transparent.

In order to obtain more reliable data on the length of a giant pulse, oscillographic traces of several pulses were photographed for each value of transmittance τ_f and the resulting data were averaged. As an example, Fig. 2 shows an oscillographic trace of a giant pulse for $\tau_f \sim 0.4$. The drawing clearly shows that the pulse has an asymmetric shape and the leading edge is steeper than the trailing edge. Since the values of pulse length were given in [2] in units of photon lifetime T in the resonator, this quantity was used in the numerical comparison with the theory.

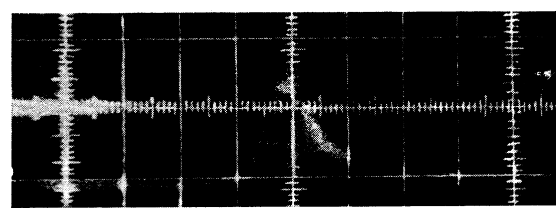

FIG. 2. Oscillographic trace of a giant pulse, for $\tau_f = 0.4$ (specimen 8 × 130 mm). Sweep 10 nsec/cm.

Under the conditions of our experiment, the losses γ were determined mainly by radiation leakage through the mirror R_2, i.e., $T = L/c\gamma = 2L/c(1 - R_2)$. Total pulse length $T_1 + T_2$ and the value of T_2/T_1 at various transmittances τ_f were measured for all four neodymium rods investigated. The results obtained are given in Table I; owing to the lack of any basic difference between the results obtained for various specimens, the table is limited to the results for one of the specimens (8 mm dia × 130 mm) as an example.

In addition, to facilitate the comparison of the results of the experiments with those of theoretical computation, Fig. 3 shows a theoretical curve of the pulse length $t_1 + t_2$ as a function of $\ln(n_i/N_p)$, according to Wagner and Lengyel.[2] The experimental points for all the specimens investigated are marked on the same curve. As can be seen from Fig. 3, the agreement between experiment

[3)]Total bleaching of the passive filter (phthalocyanine solutions) was observed also by Armstrong[7] in the case of a ruby laser.

Table I

	τ_f											
	1.00	0.97	0.93	0.90	0.86	0.83	0.73	0.66	0.53	0.42	0.39	0.34
N_i/N_p	1.00	1.09	1.21	1.35	1.44	1.61	2.04	2.37	3.08	3.52	3.76	4.17
$\ln(N_i/N_p)$	0.00	0.09	0.195	0.30	0.365	0.475	0.71	0.86	1.13	1.26	1.325	1.43
T_1+T_2, msec	430	95	57	57	41	30	20	18.5	17.5	10.5	11	9.5
t_1+t_2	77	17	10.2	9.3	7.3	4.9	3.3	3.04	2.87	1.87	1.96	1.70

Remarks: (1) Computations made with $R_1 = 0.99$, $R_2 = 0.51–0.55$, $L = 41–42$ cm, $T = 5.5 – 6.1$ nsec. (2) The results for $\tau_f = 1.00$ pertain to the case of a pure solvent.

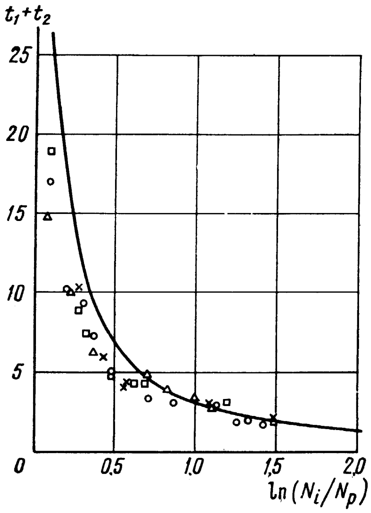

FIG. 3. Relative giant pulse length, $t_1 + t_2$, as a function of the population inverse coefficient. ○ – specimen 8 mm dia × 120 mm; □ – 9.3 mm dia × 108 mm; × – 12 mm dia × 115 mm; △ – 15 mm dia × 120 mm.

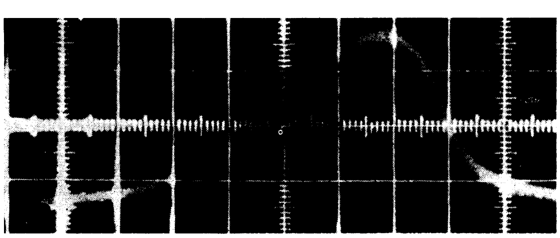

FIG. 4. Oscillographic trace of a giant pulse, for $\tau_f = 0.92$ (specimen 12 mm dia × 115 mm). Sweep 20 nsec/cm.

and theory is sufficiently good in the range of N_i/N_p values from ~1.8 to 4.5 (the interval in $\ln(N_i/N_p)$ from 0.6 to 1.5). On the other hand, in the region of lower N_i/N_p values (1.8–1.1), the experimental points are displaced downwards from the theoretical curve by amounts exceeding the possible experimental error.

The deviation of experimental pulse-length values from theoretical values[2] in the region where $N_i/N_p < 1.4$ is also accompanied by a change in the shape of the giant pulse (Fig. 4), caused by the tendency for the trailing edge to increase its steepness in relation to the leading edge. A clearer quantitative picture of this effect is given in Table II, which shows averaged experimental values of trailing-to-leading edge length ratios, $(T_2/T_1)_{exp}$; the averaging process was applied to data for various specimens within an interval of τ_f values. Theoretical values $(T_2/T_1)_{theor}$, taken from [2], are included in the table for comparison.

The above experiment thus showed a good agreement between the values of giant-pulse length calculated from the theory and obtained by experiment, for the region $N_i/N_p \gtrsim 1.8$, i.e., precisely for the area which is most interesting from the viewpoint of the production of short high-peak-power pulses.

The observed disagreement between theory and experiment in the region of low values of N_i/N_p is apparently due mainly to the fact that the theory in [2] was based on a model of an ideal, instantaneously actuated shutter. However, this model can be applied to a passive shutter only if certain conditions are satisfied. These call for a sufficiently rapid (in comparison with processes whose effect is neglected in the derivation of the equation in [2]) and sufficiently complete bleaching of the filter. This, in turn, is equivalent to the requirement for a sufficiently large absorption cross-section for the filter molecules in relation to the cross-sec-

Table II

τ_f	N_i/N_p	T_2/T_1	
		experiment	theory
0.98 — 0.97	1.07 — 1.09	0.97 ± 0.02	1.02 ± 0.005
0.94 — 0.92	1.21 — 1.31	0.77 ± 0.07	1.07 ± 0.01
0.91 — 0.89	1.31 — 1.38	0.86 ± 0.03	1.09 ± 0.01
0.87 — 0.85	1.42 — 1.53	1.10 ± 0.05	1.12 ± 0.01
0.83 — 0.79	1.61 — 1.78	1.11 ± 0.05	1.16 ± 0.02
0.74 — 0.72	1.99 — 2.04	1.20 ± 0.07	1.22 ± 0.02
0.69 — 0.66	2.09 — 2.37	1.35 ± 0.10	1.27 ± 0.03
0.56 — 0.50	2.70 — 3.28	1.45 ± 0.10	1.43 ± 0.05
0.42 — 0.34	3.52 — 4.45	1.44 ± 0.20	1.60 ± 0.10

tion for active particles, a sufficiently long lifetime of the filter molecules at the upper level in relation to the giant pulse length, etc. None of these conditions are met when the values of N_i/N_p are small; $T_1 + T_2$ is then increased by an order of magnitude (in comparison with its value at high N_i/N_p), and photon density decreases by 3–4 orders of magnitude. Therefore, it would be necessary in this case to consider a more complete system of equations, taking account of the kinetics of the processes occurring in the passive filter (see, for example, [8]).

In conclusion, the authors thank Yu. S. Ivanov for his assistance in carrying out the experiment.

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Translated by S. Kassel
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