SOUND ABSORPTION IN THE INTERMEDIATE STATE OF SUPERCONDUCTORS

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Propagation of sound in the intermediate state is accompanied by movement of the interface between the phases. As a result, eddy currents appear in the normal layers. The Joule heat released in this case leads to additional absorption of sound. It is shown that at low sound frequencies ($\delta \gg a$, where δ in the skin layer thickness and a is the spacing of the intermediate-state structure) the absorption is proportional to the square of the frequency and at not very small values of a it exceeds the ordinary absorption due to electron viscosity. At high frequencies ($\delta \ll a$) the additional absorption is proportional to the square root of the frequency.

THE volume of a superconductor in the intermediate state is broken up into a system of alternating layers of normal and superconducting phases. In the regions occupied by the normal phase, there exists a magnetic field equal in magnitude to the critical field and directed along the layers (see[1]). In the superconducting regions there is no magnetic field.

A necessary condition for equilibrium between phases is that the magnetic field in the normal phase at the interface with the superconducting phase be equal to the critical field. Since the critical field depends on the pressure and on the temperature, the change produced in the latter by passage of a sound wave leads to violation of this condition. As a result, the boundary between the phases begins to move, and an alternating magnetic field appears and induces eddy currents in the normal phase. In the intermediate state there is thus an additional mechanism, connected with the release of Joule heat, for the absorption of sound-wave energy. In this paper we calculate this additional absorption and show that under certain conditions it can exceed the usual absorption connected with electron viscosity.

We shall assume that the mean free path of the electrons is much smaller than the thickness of the normal layers and the skin-layer depth. In this case we can use the static conductivity of the metal σ. We assume also that the frequency of sound is sufficiently low in order for the length of the sound wave to be large, both compared with the depth of the skin layer and compared with the thickness of the normal layers a_n.

The magnetic field **H** in the normal layers is determined by the well known equations

$$\frac{\partial \mathbf{H}}{\partial t} - \frac{c^2}{4\pi\sigma}\Delta\mathbf{H} = \operatorname{rot}[\mathbf{vH}], \qquad (1)^*$$

$$\operatorname{div}\mathbf{H} = 0, \qquad (2)$$

where $\mathbf{v} = \dot{\mathbf{u}}$ is the velocity of the medium and $\mathbf{u} = \mathbf{u}_0\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ is the displacement vector in the sound wave. All the relations must, of course, be linearized with respect to **u**.

The boundary conditions for (1) and (2) are the usual requirements of the vanishing of the component of **H** and of the tangential component of the electric field on the boundary between phases, in a coordinate system attached to the considered element of the boundary (see[2,3]). On the other hand, the tangential component of the field **H** should equal on the boundary to critical value H_c.

We introduce a system of coordinates such that the z axis is normal to the boundaries between phases, and the xy plane lies in the middle of the normal layer. Let the x axis be directed along the magnetic field H_{c0} in the absence of sound. The boundary conditions can now be written in the form

$$E_x|_{z=\pm a_n/2} = 0, \quad E_y + \frac{U}{c}H_{c0}\Big|_{z=\pm a_n/2} = 0,$$

$$H_x - H_c|_{z=\pm a_n/2} = 0, \quad H_z - H_{c0}\frac{\partial U}{\partial x}\Big|_{z=\pm a_n/2} = 0, \qquad (3)$$

where $\mathbf{E} = (c/4\pi\sigma)\operatorname{curl}\mathbf{H} - (1/c)\mathbf{v}\times\mathbf{H}$ is the elec-

*rot ≡ curl, $[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

tric field, and U is the deviation of the boundary between phases from the equilibrium position. This deviation is represented in the form of a sum $u_z + \zeta$, where $\mathbf{u}$ is the displacement of the lattice in the sound wave, and ζ is the displacement of the boundary between phases, accompanied by a phase transition. It will be shown later that $\zeta \ll u$ in the case considered by us.

The value of the critical field H_c which enters in (3) differs from the critical field H_{c0} in the absence of sound, owing to the presence of a change in the temperature and deformation. We write H_c in the form

$$H_c = H_{c0}(1 + \alpha u_{ii}) + \frac{\partial H_c}{\partial T} T', \tag{4}$$

where $u_{ii} = \operatorname{div} \mathbf{u}$. For simplicity we confine ourselves to an examination of the case of an isotropic metal and longitudinal sound. The second term in the right side of (4) describes the change of the critical field under the influence of deformation and the temperature change caused by it, while the last term corresponds to the change under the influence of the change in temperature T', resulting from the release or absorption of heat when the boundaries between phases move. The constant $\alpha \sim 1$ can be expressed in terms of the derivatives $(\partial H_c/\partial T)_P$ and $(\partial H_c/\partial P)_T$:

$$\alpha = -\frac{\rho}{H_{c0}}\left(s_l^2 - \frac{4}{3}s_t^2\right)\left\{\left(\frac{\partial H_c}{\partial P}\right)_T - \left(\frac{\partial H_c}{\partial T}\right)_P \frac{T}{\rho C}\left(\frac{\partial \rho}{\partial T}\right)_P\right\}, \tag{5}$$

where ρ is the density of the metal, s_l and s_t are the speeds of longitudinal and transverse sound, and C the specific heat per unit volume of the intermediate state. All the thermodynamic quantities are defined as averages over the intermediate state. For example, $C = (a_n C_n + a_s C_s)/(a_n + a_s)$, where C_n and C_s are the specific heat of the normal and superconducting phases, and a_s is the thickness of the superconducting layers.

From symmetry considerations it is clear that all the quantities are of the form $f(z)\exp(ik_x x + ik_y y - i\omega t)$. We introduce in place of H a new unknown function $\mathbf{h}$:

$$\mathbf{H} - \mathbf{H}_{c0} = \frac{i}{\omega}\operatorname{rot}[\mathbf{v}\mathbf{H}_{c0}] + H_{c0}\mathbf{h}$$
$$= i(\mathbf{H}_{c0}\mathbf{k})\mathbf{u} - i\mathbf{H}_{c0}(\mathbf{k}\mathbf{u}) + H_{c0}\mathbf{h}. \tag{6}$$

Substituting (6) in (1)—(3) and taking into consideration the fact that only the x component of the vector $\mathbf{h}$ differs noticeably from zero, as can be readily verified, we obtain in our approximation the following conditions for the determination of $h \equiv h_x$:

$$\partial^2 h / \partial z^2 + q^2 h = 0, \tag{7}$$

$$h|_{z=\pm a_n/2} = u_{ii}[\alpha + 1 - (\mathbf{mn})^2] + \frac{1}{H_{c0}}\frac{\partial H_c}{\partial T}T', \tag{8}$$

$$\frac{\partial h}{\partial z} - q^2\zeta|_{z=\pm a_n/2} = 0, \tag{9}$$

where $q = (1 + i)/\delta$, $\delta = c/\sqrt{2\pi\sigma\omega}$ is the depth of the skin layer, and $\mathbf{m}$ and $\mathbf{n}$ are unit vectors along $\mathbf{H}_{c0}$ and $\mathbf{k}$.

To obtain the complete system of equations we must also write out an equation for the determination of the quantity T' in the right side of (8). To this end we consider the ratio of the time of equalization of the temperature in the system $\tau \sim a^2/\chi$ ($a = a_s + a_n$, χ is the coefficient of temperature conductivity) to the reciprocal oscillation frequency. Since $\chi \sim v_F l$, where v_F is the Fermi velocity of the electrons and l is the mean free path, we have

$$\omega\tau \sim \omega a^2 / v_F l \sim (\omega a / s)(sa / v_F l).$$

The first factor in the last expression is small, in accord with our earlier assumptions inasmuch as it represents the ratio of the thickness a of the layers to the wave length of the sound $\lambda \sim s/\omega$. On the other hand, the mean free path of the electrons l should at any rate satisfy the condition $l \gtrsim \xi_0 \sim \hbar v_F/T_c$, for otherwise the superconductor would have negative surface tension on the boundary between the phases and the considered intermediate state would not be realized. We see now that $\omega\tau \ll 1$ for any reasonable value of a. Under these conditions we can assume that the temperature does not depend on the coordinates (more accurately, it varies over distances of the order of the sound wave) and is determined from the heat-balance equation

$$(a_n + a_s)C\frac{\partial T'}{\partial t} = Q(\dot{\zeta}_- - \dot{\zeta}_+), \quad Q = -\frac{T}{4\pi}H_c\frac{\partial H_c}{\partial T}, \tag{10}$$

where Q is the heat of transition from the normal state to the superconducting state and $\zeta_\pm$ is the value of the displacement ζ at $z = \pm a_n/2$.

Relations (7)—(10) form a complete system of equations. From (7) we get

$$h = Ae^{iqz} + Be^{-iqz}, \tag{11}$$

where A and B do not depend on z. Substituting (11) in (8) and (9), and taking (10) into account, we obtain

$$h = \frac{u_{ii}[\alpha + 1 - (\mathbf{mn})^2]\cos qz}{\cos(qa_n/2) - iq\delta D\sin(qa_n/2)},$$
$$D = \frac{T\delta}{4\pi C(a_s + a_n)}\left(\frac{\partial H_c}{\partial T}\right)^2. \tag{12}$$

We are interested in the time-averaged dissipation of the energy per unit volume.

$$\dot{E} = \frac{1}{a_s + a_n} \int\limits_{-a_n/2}^{a_n/2} \frac{|j|^2}{2\sigma} dz, \tag{13}$$

where j is the density of the electric current, expressed in terms of the field h by means of the formula

$$j = \frac{c}{4\pi} H_{c0} \frac{\partial h}{\partial z}. \tag{14}$$

From (13), with allowance for (14) and (12) we have

$$\dot{E} = \left(\frac{cH_c}{8\pi}\right)^2 \frac{2|u_{ii}|^2[\alpha + 1 - (\mathbf{mn})^2]^2 [\mathrm{sh}(a_n/\delta) - \sin(a_n/\delta)]}{\sigma(a_s + a_n)\delta}$$
$$\times \left\{\cos^2\left(\frac{a_n}{2\delta}\right) + \mathrm{sh}^2\left(\frac{a_n}{2\delta}\right) + D\left[\mathrm{sh}\left(\frac{a_n}{\delta}\right) + \sin\left(\frac{a_n}{\delta}\right)\right]\right.$$
$$\left. + 2D^2\left[\sin^2\left(\frac{a_n}{2\delta}\right) + \mathrm{sh}^2\left(\frac{a_n}{2\delta}\right)\right]\right\}^{-1}. \tag{15*}$$

Taking the ratio of $\dot{E}$ to the energy flux in the sound wave, we obtain the sound absorption coefficient

$$\Gamma = \left(\frac{cH_c}{8\pi}\right)^2 \frac{2[\alpha + 1 - (\mathbf{mn})^2]^2 [\mathrm{sh}(a_n/\delta) - \sin(a_n/\delta)]}{\rho\sigma(a_s + a_n)\delta s_l^3}$$
$$\times \left\{\cos^2\left(\frac{a_n}{2\delta}\right) + \mathrm{sh}^2\left(\frac{a_n}{2\delta}\right) + D\left[\mathrm{sh}\left(\frac{a_n}{\delta}\right) + \sin\left(\frac{a_n}{\delta}\right)\right]\right.$$
$$\left. + 2D^2\left[\sin^2\left(\frac{a_n}{2\delta}\right) + \mathrm{sh}^2\left(\frac{a_n}{2\delta}\right)\right]\right\}^{-1}. \tag{16}$$

We note that in the isotropic model considered by us Γ depends only on one angle—between the direction of the magnetic field $\mathbf{H}_{c0}$ and the sound wave vector $\mathbf{k}$.

Before we consider different limiting cases, let us estimate the value of D, which, as seen from (12), determines the influence of the heat released during the phase transition on the sound absorption. Since

$$C \sim \frac{p_F{}^2 T}{v_F \hbar^3}, \quad \frac{\partial H_c}{\partial T} \sim \frac{H_c(0)}{T_c}\frac{T}{T_c},$$

where $H_c(0) \sim T_c p_F (v_F \hbar^3)^{-1/2}$ is the critical field at T = 0, we get $D \sim (\delta/a_n)(T/T_c)^2$.

Let $a_n \ll \delta$. Then for $T/T_c \ll 1$ we obtain from (16)

$$\Gamma = \frac{a_n^3[\alpha + 1 - (\mathbf{mn})^2]^2}{24(a_s + a_n)\rho\sigma s_l^3\delta^4}\left(\frac{cH_c}{2\pi}\right)^2, \tag{17}$$

*sh ≡ sinh.

that is, the absorption is proportional to the square of the frequency of sound and does not depend in practice on the temperature. If $T/T_c \sim 1$, then

$$\Gamma = \frac{a_n^3[\alpha + 1 - (\mathbf{mn})^2]^2}{24(a_n + a_s)\rho\sigma s_l^3\delta^4}\left(\frac{cH_c}{2\pi}\right)^2\left\{1 + D\frac{a_n}{\delta}\right\}^{-2}, \tag{18}$$

We see thus that in this region Γ is a rather complicated function of the temperature.

Let now $a_n \gg \delta$. Then automatically $D \ll 1$, and (16) yield

$$\Gamma = \left(\frac{cH_c}{8\pi}\right)^2 \frac{4[\alpha + 1 - (\mathbf{mn})^2]^2}{\rho\sigma(a_n + a_s)s_l^3\delta}, \tag{19}$$

that is, $\Gamma \sim \sqrt{\omega}$, which is perfectly natural, for in this case the absorption occurs in a narrow region of the order of δ near the boundaries between phases.

Finally, let us estimate the ratio of the coefficient of sound absorption by the mechanism in question to the ordinary absorption coefficient, the order of magnitude of which is $p_F^4 l\omega^2/\rho s^3\hbar^3$ [4]. Recognizing that $\sigma \sim e^2 p_F^2 l/\hbar^3$ (e = electron charge), we obtain with the aid of (17) for the sought ratio a value of the order of $(e^2/\hbar c)(v_F/c)(a/\xi_0)^2$, which, owing to the last large factor, can be large compared with unity. If this is so, then the dependence of the absorption on the frequency has the following form. At very small ω ($a_n \ll \delta$) the absorption is proportional to ω^2, in the region $a_n \gg \delta$ it is proportional to $\omega^{1/2}$, and, finally, at still higher frequencies, when the usual absorption predominates, it is again proportional to ω^2.

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