

*THE EFFECT OF INTERELECTRON COLLISIONS ON THE ELECTRON DISTRIBUTION
FUNCTION IN A STRONG ELECTRIC FIELD*

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When electrons are scattered by optical phonons of limiting frequency ω_0 at temperatures $T \ll \omega_0$, the electron collision frequency is much larger in the active region, $\varepsilon > \omega_0$, than in the passive one, $\varepsilon < \omega_0$. Therefore, one has to distinguish the case when interelectron collisions predominate only in the passive region from predominance in the whole momentum space. Analysis of the kinetic equation with allowance for this fact shows that interelectron collisions open a new channel for the relaxation of energy and momentum gained by passive electrons from the field, via transfer of energy and momentum to the active region and from it to the lattice. Unlike the usual relaxation which takes place upon absorption and instantaneous re-emission, the described energy relaxation is not associated with the dispersion of optical phonons.

1. INTRODUCTION

INTERELECTRON collisions have the peculiarity that they only redistribute the energy and momentum between the electrons without changing the energy and momentum of the whole electron system. Therefore in most cases they do not substantially affect the balance of the energy and momentum acquired by the electrons from the field and given away to the lattice. Allowance for interelectron scattering in the solution of the kinetic equation leads only to a change in shape of the maximum of energy distribution, which does not substantially influence the calculation of average values.^[1] This was shown by a direct calculation of the ohmic mobility μ ^[2-4] and of the non-ohmicity coefficient β .^[5, 4] An exception is the case of electron runaway,^[6] in which the interelectron collisions completely rearrange the energy distribution.^[7]

Scattering of electrons by the lattice is usually described by the effective collision frequencies in energy, $\tilde{\nu}$, and momentum ν ; generally speaking, $\tilde{\nu}$ and ν are unequal, and there are two possibilities: $\tilde{\nu} \approx \nu$ (inelastic scattering), and $\tilde{\nu} \ll \nu$ (almost elastic scattering). Interelectron collisions lead to a redistribution of energy and momentum with the same frequency ν^{ee} . The following limiting cases are considered:^[8]

A. $\nu, \tilde{\nu} \ll \nu^{ee}$ — interelectron collisions completely dominate, and the distribution function is a shifted Maxwellian function

$$f(\mathbf{p}) = C \exp \{ -(\mathbf{p} - \mathbf{p}_E)^2 / 2mT_E \}, \quad (1.1)$$

where \mathbf{p}_E is the drift momentum and T_E the electron temperature; both depend on the electric field \mathbf{E} .

B. $\tilde{\nu} \ll \nu^{ee} \ll \nu$ (for elastic scattering only) — interelectron collisions dominate only over the energy dissipation, therefore only the isotropic part of the distribution is Maxwellian:

$$f_0(\varepsilon) = C \exp \{ -\varepsilon / T_E \}, \quad (1.2)$$

and the anisotropic part is found from the kinetic equation.

C. $\nu^{ee} \ll \nu, \tilde{\nu}$ — the interelectron collisions may be completely disregarded.

The situation should be entirely different for low-temperature scattering by optical phonons with limiting frequency ω_0 . At $T \ll \omega_0$ the number of phonons is small, therefore the spontaneous emission of phonons by electrons in the active region, $\varepsilon > \omega_0$, is more probable than the induced absorption in the passive region, $\varepsilon < \omega_0$. As a result it may appear that interelectron collisions dominate in the passive region but do not dominate in the active one, so that assumptions A, B, and C do not exhaust all cases of the competition between interelectron and lattice scattering.

Another, even more substantial peculiarity of the considered lattice scattering mechanism can be perceived when one compares the results of Stratton^[9] (approximation A) with those of Davydov and Shmushkevich^[10] (approximation C). In

case C the energy transfer to the lattice is possible only because of optical phonon dispersion, while neglect of this dispersion does not cause any difficulties in case A. This means, as will be shown below, that interelectron collisions open a new channel for relaxation of the energy obtained from the field in the passive region, by means of transfer to the active region; from this region the energy is transmitted to the lattice. It will be shown, furthermore, that papers ^[9] and ^[10] do not cover all conditions of energy relaxation on optical phonons.

We note that the inhibition of energy relaxation in the passive region leads, in a sense, to runaway, which explains the importance of interelectron collisions from another point of view.

2. THE KINETIC EQUATION

Let us write schematically the kinetic equation for the steady state

$$F + S + C = 0, \quad (2.1)$$

Here F is the field term, and the collision terms S and C correspond, to optical and interelectron scattering, respectively. We consider fields E for which $\langle \mathcal{E} \rangle \ll \omega_0$. However, it is necessary to consider the region $\mathcal{E} > \omega_0$ too, because only there can the electrons transfer energy to the lattice. When estimating the dependence of the S term on the large parameter $u = \omega_0/T$, we separate only the most substantial factor e^{-u} so as not to restrict ourselves to a consideration of a specific type of scattering (deformation or polarization), upon which α of the preexponential factor u^α may depend. Then

$$S(\mathbf{p}) \sim \nu^+ f(\mathbf{p}), \quad p > p_0. \quad (2.2)$$

$$S(\mathbf{p}) \sim \nu^- f(\mathbf{p}), \quad p < p_0. \quad (2.3)$$

Here $p_0 = (2m\omega_0)^{1/2}$, and $\nu^+ \approx \nu^0$ and $\nu^- \approx e^{-u}\nu^0$ are the phonon emission and absorption frequencies. Estimating with the same accuracy, we have

$$C(\mathbf{p}) \sim \nu^{ee} f(\mathbf{p}). \quad (2.4)$$

In the active region the scattering is inelastic, therefore there is a choice between approximations A and C only. We consider A first, for which it is necessary that

$$\nu^{ee} \gg \nu^0. \quad (2.5)$$

Then the interelectron collisions dominate in the passive region, too, so that distribution (1.1) may be used for all \mathbf{p} . The parameters \mathbf{p}_E and T_E of this distribution are obtained from the energy and momentum balance equations

$$\mathbf{j}E = Q(T_E, \mathbf{p}_E), \quad nE = N(T_E, \mathbf{p}_E), \quad \mathbf{j} = n \frac{e}{m} \mathbf{p}_E. \quad (2.6)$$

Here \mathbf{j} and n are the current and the concentration, and Q and N are the rates of loss of energy and directed momentum upon scattering by optical phonons. The characteristics of the test electron for both polarization and deformation scattering, necessary to compute Q and N are given in ^[11].

Let us now consider the criterion of applicability of the shifted Maxwell distribution, ^[9] restricting ourselves for simplicity to the case $T_E - T \approx T$. Representing Q and N in terms of the effective frequencies of energy and momentum relaxation, $\tilde{\nu}$ and ν , and estimating them, we get

$$Q \sim \tilde{\nu} T_E \sim \nu^0 \omega_0 e^{-u}, \quad (2.7)$$

$$N \sim \nu p_E \sim \nu^0 p_E u e^{-u}. \quad (2.8)$$

This makes $\tilde{\nu}$ and ν of the same order as $\nu^0 u e^{-u}$, the presence of the factor e^{-u} in such an integral estimate of the energy and momentum relaxation rate is obvious: in the passive region relaxation is due to absorption, the probability of which is $\sim e^{-u}$, and the active region, where the emission probability is ~ 1 , contains a small fraction e^{-u} of the electrons. (The momentum relaxation rate is estimated in ^[9] somewhat differently and leads to $\nu \approx \nu^0 e^{-u}$, but this does not change the nature of the subsequent considerations.) The criterion for the applicability of the distribution (1.1) is claimed to be $\nu^{ee} \gg \nu^0 u e^{-u}$. Such an integral criterion is weaker than the necessary differential criterion (2.5) found by comparing the relaxation rates in every point of the momentum space, and does not yield a Maxwellian distribution in the active region; it is shown below that such a criterion leads to a Maxwellian distribution only in the passive region.

We note, furthermore, that in ^[9] Q and N are calculated by assuming a small anisotropy of the distribution function $p_E^2/2m \ll T_E$, which is not justified at noticeable heat rise $T_E - T \gtrsim T$, because in this case the scattering is not elastic.

We proceed now to the case C in the active region, where

$$\nu^{ee} \ll \nu^0. \quad (2.9)$$

Following ^[10], we will not, however, assume that the anisotropic part of the distribution function is small, because there is no justification for this as yet. We develop the collision term

$$S = B - A, \quad B = B^+ + B^-, \quad A = A^+ + A^-, \quad (2.10)$$

where A and B designate the departure and arrival, and the signs \pm pertain to processes with

emission and absorption of phonons. The estimate of these terms in the active region is

$$A^\pm \sim v^\pm f(\epsilon), \quad B^\pm \sim v^\pm f(\epsilon \pm \omega_0). \quad (2.11)$$

The dominant term is A^+ , which is large in comparison with A^- (obviously) and B^+ (in virtue of $\langle \epsilon \rangle \ll \omega_0$). This does not apply to B^- , for which we have $B^- = A^+$ when $E = 0$ (detailed balance). The field term is estimated as

$$F \sim eE f(\epsilon) / \langle p \rangle \lesssim eE (mT)^{-1/2} f(\epsilon). \quad (2.12)$$

Confining ourselves to fields $E \ll E_0$, where $E_0 = \nu^0 (mT)^{1/2}$ is the characteristic field in the active region, we may neglect F in comparison with A^+ . Thus the kinetic equation for the active region assumes the form

$$B^- - A^+ = 0, \quad (2.13)$$

whence the distribution at $p > p_0$ is expressed through the distribution at $p < p_0$:

$$f(\mathbf{p}) = \tau_0^+(\mathbf{p}) \int_{p < p_0} (d\mathbf{p}') W^-(\mathbf{p}', \mathbf{p}) f(\mathbf{p}'). \quad (2.14)$$

Here τ_0^+ is the emission lifetime^[11]

$$1/\tau_0^+(\mathbf{p}) = \int (d\mathbf{p}') W^+(\mathbf{p}, \mathbf{p}'). \quad (2.15)$$

Substitution of (2.14) closes the kinetic equation for the passive region, eliminating the active region from consideration. However S and C then undergo an essential alteration, because they have to account implicitly for the presence of electrons in the active region.

Substituting (2.14) in S , we can easily show that this amounts to the replacement of $W(\mathbf{p}, \mathbf{p}')$ by

$$\hat{W}(\mathbf{p}, \mathbf{p}') = \int_{p'' > p_0} (d\mathbf{p}'') \tau_0^+(\mathbf{p}'') W^-(\mathbf{p}, \mathbf{p}'') W^+(\mathbf{p}'', \mathbf{p}') \quad (2.16)$$

and to the confinement of the integration range to the passive region. The S term written in this form will be denoted by \hat{S} . \hat{W} may be interpreted as the probability of a combined transition from \mathbf{p} to \mathbf{p}' through \mathbf{p}'' . W^- yields the probability of the absorption transition $\mathbf{p} \rightarrow \mathbf{p}''$, and $\tau_0^+ W^+$ gives the conditional probability that the instantaneous emissive transition from \mathbf{p}'' will occur just into \mathbf{p} . The combined transition probability formulates mathematically the idea of absorption and instantaneous reemission as a single scattering event.^[12, 10]

The order of magnitude of \hat{W} is determined by the factor W^- in the integral (2.16). Therefore all effective frequencies of the combined scattering contain the factor e^{-u} . If one disregards the dis-

persion of the optical phonons then, unlike the usual scattering W , the combined scattering \hat{W} is elastic. Owing to dispersion,

$$\omega(\mathbf{q}) = \omega_0 - q^2 / 2(Mm)^{1/2}, \quad (2.17)$$

where M is on the order of the nuclear mass, a small inelasticity arises with a parameter $\delta \sim u(m/M)^2 \ll 1$. This circumstance leads to a difference in the estimate of the isotropic part \hat{S}_0 and of the anisotropic parts \hat{S}_l (the index l corresponds to the expansion of S in Legendre polynomials):

$$\hat{S}_0 \sim \tilde{v} f_0, \quad \hat{S}_l \sim \hat{v} f_l, \quad (2.18)$$

where the effective frequencies of energy and momentum relaxation resulting from combined scattering are

$$\hat{v} \sim v^0 e^{-u}, \quad \tilde{v} \sim \delta \hat{v} \sim e^{-u} \delta v^0. \quad (2.19)$$

Comparison of (2.19) with (2.3) shows that the replacement of S by \hat{S} leads to a sharp overestimate of the symmetrical part.

Because the combined scattering is almost elastic, all three cases, A, B, and C determined by the comparison of \hat{S} with C can be present in the passive region. However, for A and B the distributions (1.1) or (1.2) will be valid only when $p \ll p_0$; for $p \gg p_0$ they should be found from (2.14). Another important peculiarity is that the use of such Maxwellian distributions which are deformed in the active region causes C or C_0 to vanish only with accuracy to terms of the order of e^{-u} . This causes the appearance of additional terms in the balance equations for \mathbf{p}_E and T_E .

Case A takes place in the active region when

$$v^0 e^{-u} \ll v^{ee} \ll v^0. \quad (2.20)$$

The balance equations for the passive region are

$$\mathbf{j}_E = \hat{Q}(T_E, \mathbf{p}_E) + Q^{ee}(T_E, \mathbf{p}_E), \quad (2.21)$$

$$nE = \hat{N}(T_E, \mathbf{p}_E) + N^{ee}(T_E, \mathbf{p}_E). \quad (2.22)$$

Here \hat{Q} and \hat{N} are the energy and momentum losses for the combined scattering of passive electrons; Q^{ee} and N^{ee} are the above-mentioned additional terms describing the energy and momentum transfer from the passive region to the active one, which occurs when passive electrons are scattered by active electrons. The energy and momentum acquired by the active electrons from the field should have been subtracted from the left members, but in these terms their contribution to current and concentration contains the small factor e^{-u} .

The balance terms \hat{Q} and \hat{N} are estimated by means of the effective frequencies (2.19), and Q^{ee} and N^{ee} —with the help of the frequency $\nu^{ee}e^{-u}$. Therefore, by virtue of (2.20), the momentum relaxation N^{ee} via transfer to the active region is insignificant in comparison with the direct relaxation \hat{N} . This is not the case with energy relaxation; if $\nu^{ee} \gg \delta\nu^0$ then the term Q^{ee} is significant, and if $\nu^{ee} \ll \delta\nu^0$, then the significant term is \hat{Q} . The effective frequency of energy relaxation, $\nu^{ee}e^{-u}$ or $e^{-u}\delta\nu^0$, is in either case smaller than the effective frequency of momentum relaxation ν^0e^{-u} (it is assumed here that $\delta \gg e^{-u}$). The scattering is therefore “elastic,” and if (2.20) holds then the distribution (1.1) has a small anisotropy in the passive region. It follows from (2.14) that $f(\mathbf{p})$ has a small anisotropy in the active region, too.

Case B takes place in the passive region when

$$e^{-u}\delta\nu^0 \ll \nu^{ee} \ll \nu^0e^{-u}. \quad (2.23)$$

The balance equation in the passive region is

$$\mathbf{jE} = \hat{Q}(T_E) + Q^{ee}(T_E). \quad (2.24)$$

The meaning of the balance terms is the same as in (2.21). They are estimated by means of the same frequencies $e^{-u}\delta\nu^0$ and $\nu^{ee}e^{-u}$, so that Q^{ee} is always insignificant.

Case C occurs in the passive region when

$$\nu^{ee} \ll e^{-u}\delta\nu^0. \quad (2.25)$$

In cases B and C the anisotropy of the distribution function is also small.

3. MEASUREMENTS OF ENERGY AND MOMENTUM RELAXATION

The analysis of the kinetic equation in Sec. 2 shows that there exist in reality two mechanisms by which energy gained by the passive electron from the field can be transferred to the lattice:

I. Having absorbed a phonon, the passive electron becomes active and transfers energy to the lattice; however, the loss to activation is not compensated by the possibilities of energy return if phonon dispersion is absent. The effective frequency of this mechanism is

$$\tilde{\nu}_I \sim e^{-u}\delta\nu^0. \quad (3.1)$$

II. The passive electron transmits energy to the active electron, which transfers it to the lattice. From the estimates of Q^{ee} in (2.21) and (2.24) and of Q in (2.6) it is seen that the effective frequency of this mechanism is

$$\tilde{\nu}_{II} \sim (1/\nu^0 + 1/\nu^{ee})^{-1}e^{-u}. \quad (3.2)$$

This can also be verified by the following simple consideration. During the time τ^{ee} the excess energy $\Delta\mathcal{E}$ obtained in the passive region is Maxwellized; part of it, $e^{-u}\Delta\mathcal{E}$, enters the active region. This part is transferred to the lattice during the time τ^0 . The relaxation time of the energy $e^{-u}\Delta\mathcal{E}$ is $\tau^0 + \tau^{ee}$, and the relaxation time of the total energy $\Delta\mathcal{E}$ is $(\tau^0 + \tau^{ee})e^u$, yielding (3.2). Both mechanisms are equally effective when $\nu^{ee} \sim \delta\nu^0$.

Reference 9 is implicitly based on the mechanism II, because no allowance is made there for phonon dispersion, which means $\delta = 0$. This is actually true only under condition (2.5), therefore the slowest limiting step in mechanism II is the transfer of Maxwellized energy to the lattice; as a result it turns out that $\tilde{\nu}_{II} \sim \nu^0e^{-u}$, and ν^{ee} drops out of consideration. If $\nu^{ee} \ll \nu^0$ the limiting step is the maxwellization, and $\tilde{\nu}_{II} \sim \nu^{ee}e^{-u}$.

Unlike energy relaxation, the relaxation of the directed momentum is possible in the passive region, too; however it proceeds with difficulty in this region, making the contribution of the active region of the same order of magnitude, in spite of the small number of electrons in the active region. Therefore the momentum transfer in the active region can also play a role. The effective frequencies of momentum relaxation in mechanisms I and II are obtained from (3.1) and (3.2) at $\delta \sim 1$, i.e.,

$$\nu_I \sim \nu^0e^{-u}, \quad \nu_{II} \sim \tilde{\nu}_{II}. \quad (3.3)$$

However, the frequency ν^{ee} drops always out of consideration here because when $\nu^{ee} \gg \nu^0$ it drops out of ν_{II} , and for $\nu^{ee} \ll \nu^0$ the mechanism II is insignificant in comparison with I.

It is easy to trace the effect of interelectron collisions by observing the ohmic mobility μ and the nonohmicity factor β of weakly heated electrons, for which

$$\mu(E) = \mu(1 + \beta E^2). \quad (3.4)$$

In order of magnitude we have^[4]

$$\mu \sim \frac{e}{m} \nu^{-1}, \quad \beta \sim \frac{e^2}{mT} (\tilde{\nu}\nu)^{-1}. \quad (3.5)$$

By changing the concentration n one can vary the relation between ν^{ee} and ν^0 . The three cases occurring at $\nu^{ee} \ll \delta\nu^0$ do not differ in their relaxation mechanisms, the dependence of μ and β on n must therefore be weak in the corresponding concentration range. But the change from $\nu^{ee} \sim \delta\nu^0$ to $\nu^{ee} \sim \nu^0$ causes β to decrease δ times, because a new channel of energy relaxation is opened, which hinders the heating. A relation $\beta \sim n^{-1/2}$ should be observed in the interval. A change in μ takes

place, too: we see from the applicability conditions of the approximations A and C, as well as from the calculation made in ^[9] for polar scattering, that the mobility μ decreases u times on going from $\nu^{ee} \sim e^{-u\delta}\nu^0$ to $\nu^{ee} \sim \nu^0$. We cannot trace the variation of μ without a direct calculation of the collision terms, because we did not estimate the power-law preexponential factors.

The form of the distribution function should strongly influence the microscopic parameters connected with the "tail" of the function such as electron emission and impact ionization. Thus, on going from $\nu^{ee} \gg \nu^0$ to $\nu^{ee} \ll \nu^0$, the distribution "tail" in the active region decreases abruptly, and both these effects should sharply diminish. A noticeable decrease in the distribution tail was discovered in an experimental measurement of the distribution function of hot holes in germanium. ^[13, 14]

We note that the peculiarities of low-temperature scattering by optical phonons will also be observed for acoustic phonons at "infralow" temperatures $T \lesssim ms^2$ (s is the speed of sound), because electrons with $\varepsilon < \frac{1}{2}ms^2$ cannot emit acoustic phonons. However, $\delta \sim 1$ for acoustic phonons.

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