QUANTUM THEORY OF A PARTICLE WITH ELECTRIC AND MAGNETIC CHARGES

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The quantum theory of a spinor particle possessing simultaneously an electric charge e and a scalar magnetic charge g is investigated. In this case the particle has an intrinsic magnetic moment e/2m and an intrinsic electric moment g/2m, and the magnitudes of the electric and magnetic charges are arbitrary. Because the particle has a scalar magnetic charge there is nonconservation of the P and T parities.

It has been shown in a paper by Dirac[1] that quantum theory allows the existence of particles with a magnetic charge, and that then the product of the electric and magnetic charges must be $2\pi n$. Later, Cabibbo and Ferrari[2] considered the possibility of constructing a quantum electrodynamics with the Dirac monopole. This latter work is erroneous, however, as we shall show. In the present paper we construct the quantum dynamics of spinor particles carrying simultaneously an electric and a magnetic charge. We assume that the magnetic charge, like the electric, is a scalar.

In this case the Maxwell equations are

$$\partial^\nu F_{\mu\nu} = eI_\mu, \quad \partial^\nu \tilde{F}_{\mu\nu} = gI_\mu. \tag{1}$$

The electromagnetic field intensities can be expressed by means of two potentials[2] A and B:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \varepsilon_{\mu\nu\rho\sigma}\partial^\rho B^\sigma,$$

$$\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - \varepsilon_{\mu\nu\rho\sigma}\partial^\rho A^\sigma, \tag{2}$$

where the dual tensor is defined by

$$\tilde{F}_{\mu\nu} = -{}^1\!/_2\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}, \quad \tilde{\tilde{F}}_{\mu\nu} = -F_{\mu\nu}, \tag{3}$$

and $\epsilon_{\mu\nu\rho\sigma}$ is the completely antisymmetric tensor, $\epsilon_{0123} = -1$.

To describe the particles we apply a method of Mandelstam,[3] by means of which quantum electrodynamics has been formulated without potentials. For a spinor particle we introduce a new wave function $\Psi(x, P)$, which depends on the coordinate x and on a spatial path P extending from $-\infty$ to the point x. If we introduce the gauge-invariant derivative by the definition

$$\partial_\mu\Psi(x,P) = \lim_{\delta x_\mu\to 0}\frac{\Psi(x+\delta x_\mu, P') - \Psi(x,P)}{\delta x_\mu}, \tag{4}$$

(where P′ is obtained from P by adding a displacement δx_μ in the μ direction), then the equations for the particles take the form

$$i\gamma^\mu\partial_\mu\Psi - m\Psi = 0, \quad i\partial_\mu\overline{\Psi}\gamma^\mu + m\overline{\Psi} = 0, \tag{5}$$

and the current appearing in (1) is given by

$$I_\mu = \overline{\Psi}\gamma_\mu\Psi. \tag{6}$$

The dependence of the wave functions on the path is defined by means of the relations

$$\delta_z\Psi(x,P) = -i[eF_{\mu\nu}(z) + g\tilde{F}_{\mu\nu}(z)]\cdot\Psi(x,P)\sigma^{\mu\nu},$$

$$\delta_z\overline{\Psi}(x,P) = i[eF_{\mu\nu}(z) + g\tilde{F}_{\mu\nu}(z)]\cdot\overline{\Psi}(x,P)\sigma^{\mu\nu}, \tag{7}$$

where the dot denotes the symmetrized product, δ_z is a variation of the path in the neighborhood of the point z, and $\sigma^{\mu\nu}$ is a small surface element. It follows from (7) that

$$[\partial_\mu, \partial_\nu]\Psi(x,P) = -i(eF_{\mu\nu} + g\tilde{F}_{\mu\nu})\cdot\Psi(x,P),$$

$$[\partial_\alpha, [\partial_\mu, \partial_\nu]]\Psi(x,P) = -i[\partial_\alpha(eF_{\mu\nu} + g\tilde{F}_{\mu\nu})]\cdot\Psi(x,P). \tag{8}$$

The Jacobi identity is satisfied, since

$$\partial^\nu(e\tilde{F}_{\mu\nu} - gF_{\mu\nu}) = 0. \tag{9}$$

For a finite variation of the path we have

$$\Psi(x,P') = \exp\Big[-i\int(eF_{\mu\nu} + g\tilde{F}_{\mu\nu})\,d\sigma^{\mu\nu}\Big]\cdot\Psi(x,P), \tag{10}$$

where the integration is taken over an arbitrary surface bounded by the contour P′ − P, and consequently

$$\exp\Big[-i\oint_S(eF_{\mu\nu} + g\tilde{F}_{\mu\nu})\,d\sigma^{\mu\nu}\Big] = 1. \tag{11}$$

with integration over a closed surface S. Equation (11) is satisfied for arbitrary e and g because of the identity (9). Accordingly in our case the magnitudes of the electric and magnetic charges are arbitrary.

If, in accordance with the results of the paper by Cabibbo and Ferrari,[2] we were to construct an expression analogous to the second of the equations (8) by permuting the indices cyclically and summing the resulting expressions, we would arrive at a contradiction; the left member would be identically zero, and the right would be a quantity different from zero.

Let us introduce new fields

$$h\mathscr{F}_{\mu\nu} = eF_{\mu\nu} + g\tilde{F}_{\mu\nu}, \qquad h\tilde{\mathscr{F}}_{\mu\nu} = e\tilde{F}_{\mu\nu} - gF_{\mu\nu}, \quad (12)$$

where $e^2 + g^2 = h^2$, so that

$$e = h\cos\theta, \quad g = h\sin\theta. \quad (13)$$

In these notations Eq. (1) takes the form

$$\partial^\nu \mathscr{F}_{\mu\nu} = hI_\mu, \quad \partial^\nu \tilde{\mathscr{F}}_{\mu\nu} = 0 \quad (14)$$

and consequently we can write

$$\mathscr{F}_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu. \quad (15)$$

The change from the field $F_{\mu\nu}$ to $\mathscr{F}_{\mu\nu}$ is obtained as the result of a rotation by the angle θ in the dual space.

In this theory there is nonconservation of the P and T parities; in fact, the right member of the second of the equations (1) is a vector, and the left member is an axial vector. If we introduce an operation M, magnetic charge conjugation, the theory will be invariant with respect to the transformation MCPT. It is not hard to verify that the spinor particle will have a magnetic dipole moment e/2m and an electric dipole moment g/2m.

The equations of motion can be obtained from the Lagrangian

$$L = \frac{i}{2}[\overline{\Psi}\gamma^\mu(\partial_\mu\Psi) - (\partial_\mu\overline{\Psi})\gamma^\mu\Psi] - m\overline{\Psi}\Psi - \frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu}, \quad (16)$$

where the potentials C_μ are to be varied.

Having the Lagrangian, we can easily obtain by Peierls' method the commutation relations for equal times:

$$\{\Psi(x,P), \Psi(x',P)\} = \{\overline{\Psi}(x,P), \overline{\Psi}(x',P)\} = 0,$$
$$\{\Psi(x,P), \overline{\Psi}(x',P)\} = \gamma_4\delta^3(x-x'),$$
$$[\Psi(x,P), F_{0i}(x')] = -e\int_{-\infty}^{x} d\xi_i\delta^3(x-x')\Psi(x,P),$$
$$[\overline{\Psi}(x,P), F_{0i}(x')] = e\int_{-\infty}^{0} d\xi_i\delta^3(x-x')\overline{\Psi}(x,P),$$
$$[\Psi(x,P), \tilde{F}_{0i}(x')] = -g\int_{-\infty}^{x} d\xi_i\delta^3(x-x')\Psi(x,P),$$
$$[\overline{\Psi}(x,P), \tilde{F}_{0i}(x')] = g\int_{-\infty}^{x} d\xi_i\delta^3(x-x')\overline{\Psi}(x,P),$$
$$[F_{ij}(x), F_{i'j'}(x')] = [F_{0i}(x), F_{0i'}(x')] = 0,$$
$$[F_{0i}(x), F_{jk}(x')] = i\left(\delta_{ij}\frac{\partial}{\partial x^k} - \delta_{ik}\frac{\partial}{\partial x^j}\right)\delta^3(x-x'). \quad (17)$$

The equations of motion (1), (5), and (6), the commutation relations (17), and Eq. (7), which defines the dependence of the operators on the path, describe the motion of a spinor particle which has electric and magnetic charges and is acted on by an electromagnetic field.

Because of the magnetic charge on the electron there will, for example, be a difference in the expression for the cross section for electron-electron scattering, the e^2 in the usual expression being replaced by $e^2 + g^2$. If we regard g as a pseudoscalar, the P and T parities will be conserved.

The existing experimental data do not exclude the possibility that all particles have a small magnetic charge along with the electric charge.

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