ON THE DETERMINATION OF THE CRITICAL TEMPERATURE OF A SUPERCONDUCTOR CONTAINING A PARAMAGNETIC IMPURITY

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It is shown that the critical temperature T_c of a superconductor considered in the two-band model is inversely proportional at low impurity concentrations to the sum of the relaxation times for interband scattering by the impurity and the relaxation times for exchange scattering of electrons of each of the bands by the paramagnetic atoms. A critical impurity concentration exists in the high concentration region.

In an earlier paper[1] we have considered the two-band model of a superconductor with nonmagnetic impurity, and have shown that interband scattering by the impurity causes an appreciable change in the temperature of the superconductor. In this communication we generalize the results obtained in the earlier paper to include the case of a paramagnetic impurity. It turns out here that in the region of small impurity concentrations the critical temperature T_c of the superconductor depends on the relaxation time of the interband scattering by the impurity and on the relaxation time of exchange scattering of electrons of each of the two bands by the paramagnetic atoms. Unlike the nonmagnetic case, a critical impurity concentration exists in the region of large impurity concentrations.

To prove these statements we start from an impurity potential

$$\hat{v}(\mathbf{r}) = u_1(\mathbf{r}) + (S\hat{\sigma})u_2(\mathbf{r}) \tag{1}$$

and use the computational technique of[1,2]. It is easy to see that the formulas of[1] are modified as follows: in determining the relaxation times τ_{ij} that enter in the single-particle Green's function[1], it is necessary to replace the quantity $|u_1|^2$ by $|u_1|^2 + \frac{1}{4}S(S+1)|u_2|^2$, whereas in the kernel of the equation for the function K ($K = \overline{GG}$) it is necessary to replace $|u_1|^2$ by $|u_1|^2 - \frac{1}{4}S(S-1)|u_2|^2$. In this connection we introduce besides the times τ_{ij} also the times κ_{ij}, which differ from the former in that their definition contains the combination $|u_1|^2 - \frac{1}{4}S(S+1)|u_2|^2$. Then calculations yield the following dependence of T_c ($\beta_c = (k_V T_c)^{-1}$, β_{c0} is the corresponding quantity for the pure superconductor) on the impurity

$$\ln\frac{\beta_c}{\beta_{c0}} = \varkappa^{\pm}\frac{(p_1+p_2)^2}{l_2-l_1}[I(\beta_c\sqrt{l_2}) - I(\beta_c\sqrt{l_1})] - \frac{[\sqrt{l_1}I(\beta_c\sqrt{l_2}) - \sqrt{l_2}I(\beta_c\sqrt{l_1})]}{\sqrt{l_2}-\sqrt{l_1}} \tag{2}$$

where

$$\varkappa^{\pm} = \mp N_1p_1[V_{11} + V_{22}j - V_{12}(j_1 + jj_2)] - \frac{N_2 + jN_1}{2N_1N_2} \times [b_0 \pm \sqrt{b_0^2 - 4a}][(p_1+p_2)\sqrt{b_0^2-4a}]^{-1};$$

$$p_i = \frac{\hbar}{2\tau_i} - \frac{\hbar}{2\varkappa_{ii}};\quad j = \frac{N_2}{N_1}\frac{p_2}{p_1};\quad j_1 = \frac{\hbar}{2\varkappa_{12}p_1};\quad j_2 = \frac{\hbar}{2\varkappa_{21}p_2};$$

$$l_{1,2} = \frac{(p_1+p_2)^2}{2}(1 - 2a \mp \sqrt{1-4a});$$

$$a = p_1p_2(1 - j_1j_2)/(p_1+p_2)^2. \tag{3}$$

Here I is a function whose definition is given in[1], $N_{1,2}$ the state density on the Fermi surfaces of the two bands, V_{im} the matrix element of the interaction potential for states belonging to different bands, and

$$b_0 = V_{11}N_1 + V_{22}N_2,\quad a = N_1N_2(V_{11}V_{22} - V_{12}V_{21}).$$

In the limiting case of low impurity concentrations $\beta_c\sqrt{l_1} \gg 1$, $\beta_c\sqrt{l_2} \ll 1$, we have

$$T_c \approx T_{c0} - \frac{\pi}{4}\frac{\hbar}{2k_B}\varkappa^{\pm}\left(\frac{1}{\tau_{12}} + \frac{1}{\tau_{21}} + \frac{1}{\tau_{11}^s} + \frac{1}{\tau_{22}^s}\right), \tag{4}$$

where

$$\tau_{ii}^s\quad([\tau_{ii}^s]^{-1} = [\tau_{ii}]^{-1} - [\varkappa_{ii}]^{-1})$$

is the exchange relaxation time. It is easy to see that $\kappa^{\pm}$ is larger than zero and smaller than unity.

Thus, the linear decrease of T_c with increasing impurity concentration is connected with the exchange part of the interaction and with the interband transitions.

With increasing concentration, a second limiting case $\beta_c\sqrt{l_2} \gg 1$ and $\beta_c\sqrt{l_1} \ll 1$ is possible. Both inequalities can be satisfied simultaneously since the parameter l_1 is proportional to the exchange part of the interaction (1), whereas l_2 is proportional to the non-exchange part. As a rule the exchange part of the interaction is much smaller than the non-exchange part. In this case formula (2) becomes

$$(\varkappa^{\pm}-1)\ln\frac{\beta_c}{\beta_{c0}} \approx \frac{\pi}{4}\beta_c(p_1+p_2)\alpha(\varkappa^{\pm}-1)-(\varkappa^{\pm}-\alpha) \times\left[\frac{\pi^2}{6\beta_c^2(p_1+p_2)^2}+\ln\left(\frac{2\gamma\beta_{c0}(p_1+p_2)}{\pi}\right)\right]. \quad (5)$$

Finally, in the limiting case of large impurity concentrations, $\beta_c\sqrt{l_1} \gg 1$ and $\beta_c\sqrt{l_2} \gg 1$, we have

$$(k_BT_c)^2 \approx \frac{6\alpha^2(p_1+p_2)^2}{\pi^2}\left[\ln\frac{1}{\alpha}-\frac{\ln(2\gamma\beta_{c0}(p_1+p_2)/\pi)}{1-\varkappa^{\pm}}\right]. \quad (6)$$

At some critical impurity concentration, defined by the condition

$$2\pi^{-1}\gamma\beta_{c0}(p_1+p_2) = (1/\alpha)^{1-\varkappa\pm}, \quad (7)$$

the critical temperature of the superconductor vanishes. This proves the statements made above.

As expected, the obtained expressions go over in the single-band case into the corresponding formulas of Abrikosov and Gor'kov[2] and in the case of a nonmagnetic impurity into the results of[1].

1 V. A. Moskalenko and M. E. Palistrant, JETP **49**, 770 (1965), Soviet Phys. JETP **22**, 536 (1966).

2 A. A. Abrikosov and L. P. Gor'kov, JETP **39**, 1781 (1960), Soviet Phys. JETP **12**, 1243 (1961).

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