STABILITY LIMITS OF THE SUPERCONDUCTING STATE IN A MAGNETIC FIELD FOR SUPERCONDUCTORS OF THE SECOND KIND

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The Gibbs free energy function $\Omega\{\Delta\}$ for a superconductor in a magnetic field is treated on the basis of the BCS theory as a functional of the superconducting ordering parameter. This extends the corresponding expression of the Ginzburg-Landau theory to arbitrary temperatures. The free energy is minimum at equilibrium. This yields the equilibrium electrodynamic equations, which in the case of superconductors of the second kind describe the usual superconducting state with finite penetration of the field as well as the mixed Abrikosov state with vortex lines that pierce the bulk superconductor. The second variation of the free energy is investigated in the vicinity of the superconducting state and it is shown that the state is stable with respect to small fluctuations of the ordering parameter up to a certain critical field $H'_{c1} > H_{c1}$, for which $\delta^2\Omega$ changes sign for the first time. The field strength $H'_{c1}(T)$ is calculated for extremely hard superconductors ($\kappa \gg 1$).

As is well known[1], starting with a critical field $H_{c_1} \sim H_c\kappa^{-1}\ln\kappa$, it becomes thermodynamically convenient in a bulky superconductor of the second kind, placed in a magnetic field, for a transition to occur into a mixed state, in which the magnetic field partially penetrates into the superconductor in the form of quantized vortex filaments. At the same time, the equations of electrodynamics for such superconductors admit formally, up to the thermodynamic critical field H_c, equilibrium solutions of the usual type, describing the superconducting states with penetration of a field to a depth δ[2]. The main mass of the superconductor is in this case in a state with $H = 0$, which is stable relative to the local fluctuations that lead to the formation of the vortices. The latter, obviously, is valid also for the limiting region in the case of a sufficiently small magnetic field. Thus, the possibility arises of delaying the penetration of the vortex into the superconductor for field with $H > H_{c1}$ and, indeed, an elementary analysis of the potential energy of the vortex near the surface[3] shows that there exists a potential barrier for the penetration of the vortex into the superconductor and that this barrier vanishes for a certain field $\sim H_c$. This picture of the delay was observed experimentally[4].

From the thermodynamic point of view, the phenomenon under consideration is "superheating" of the superconducting state, and the maximum field H'_{c1}, up to which this "superheating" is physically possible, is defined as the absolute stability limit of the metastable superconducting states relative to small fluctuations. It is obvious that this instability arises first on the boundary of the superconductor, where the field reaches its maximum value.

To calculate the limiting field $H'_{c1}(T)$ we define, following Landau[5,6], the thermodynamic potential of the superconductor[1)] as a functional of the ordering parameter, the equilibrium value of which is obtained from the condition for the minimum of this potential. According to modern theory of superconductivity[7,8], the superconducting ordering of a metal is manifest in the fact that, owing to the pairing of electrons with opposite spins, the pair production operator $\psi^+_\alpha(\mathbf{r})\psi^+_\beta(\mathbf{r})$ can be replaced (accurate apart from fluctuations) by a macroscopic c-number which differs from zero. Starting from this, we transform identically in the Hamiltonian of the superconductor

$$\mathcal{H} = \int dV\,\{\psi^+\xi(\hat{\mathbf{p}} - e\mathbf{A})\psi + {}^1/_2\, g(\psi^+(\psi^+\psi)\psi)\}$$

(where $\xi(\mathbf{p}) = p^2/2m - \mu$; $\hat{\mathbf{p}} = -i\nabla$ ($\hbar = c = 1$); μ—chemical potential, $g < 0$—coupling constant;

1)In view of the fact that the local fluctuations of the ordering parameter do not change the chemical potential, it is immaterial in what follows whether we are considering the free energy F or the potential $\Omega = F - \mu N$.

H = curl A—magnetic field) the term describing the interaction

$$\tfrac{1}{2}g(\psi_\lambda^+(\psi_\mu^+\psi_\mu)\psi_\lambda) = \tfrac{1}{2}[\Delta(\psi_\lambda^+\psi_\mu^+ I_{\mu\lambda}) - \Delta^*(I_{\lambda\mu}\psi_\mu\psi_\lambda) - 2\Delta^*\Delta/g] + \tfrac{1}{2}g(\psi_\lambda^+\psi_\mu^+ + I_{\lambda\mu}\Delta^*/g) \times (\psi_\mu\psi_\lambda - I_{\mu\lambda}\Delta/g); \quad I_{\alpha\beta} = -I_{\beta\alpha}, \quad I^2 = -1.$$

Inasmuch as the main contribution to the thermodynamic potential $\Omega = -T \ln \mathrm{Sp}\, e^{-\mathcal{H}/T}$ is made by the microscopic states that realize the equilibrium macroscopic state, the last fluctuation term in the interaction can be neglected and we can consider the Hamiltonian

$$\mathcal{H}_0 = \int dV\, \{\psi^+\xi(\hat{\mathbf{p}} - e\mathbf{A})\psi + \tfrac{1}{2}[\Delta(\psi_\lambda^+\psi_\mu^+ I_{\mu\lambda}) - \Delta^*(I_{\lambda\mu}\psi_\mu\psi_\lambda) - 2\Delta^*\Delta/g]\}.$$

The diagonalization of this Hamiltonian and the calculation of the thermodynamic potential as a functional of Δ and A reduce, in final analysis, to a solution of the Gor'kov equation[9] for the Green's function

$$\begin{pmatrix} -i\omega + \xi(\hat{\mathbf{p}}_1 - e\mathbf{A}(\mathbf{r}_1)), & -\Delta(\mathbf{r}_1) \\ -\Delta^*(\mathbf{r}_1), & -i\omega - \xi(\hat{\mathbf{p}}_1 + e\mathbf{A}(\mathbf{r}_1)) \end{pmatrix} \times \begin{pmatrix} G(\mathbf{r}_1\mathbf{r}_2;\ \omega), & F(\mathbf{r}_1\mathbf{r}_2;\ \omega) \\ F^+(\mathbf{r}_1\mathbf{r}_2;\ \omega), & \tilde{G}(\mathbf{r}_1\mathbf{r}_2;\ \omega) \end{pmatrix} = \begin{pmatrix} \delta(\mathbf{r}_1 - \mathbf{r}_2), & 0 \\ 0, & \delta(\mathbf{r}_1 - \mathbf{r}_2) \end{pmatrix}. \tag{1}$$

The first variation of the potential Ω is of the form[2)]

$$\delta\Omega = \int dV \Big\{ T\sum_{\omega_n} [F^+(\mathbf{rr};\ \omega_n)\delta\Delta(\mathbf{r}) + F(\mathbf{rr};\ \omega_n)\delta\Delta^*(\mathbf{r})] - \frac{\Delta^*(\mathbf{r})}{g}\delta\Delta(\mathbf{r}) - \frac{\Delta(\mathbf{r})}{g}\delta\Delta^*(\mathbf{r}) \Big\}, \quad \omega_n = \pi T(2n+1),$$

$$n = 0, \pm 1, \pm 2, \ldots$$

It follows therefore that at equilibrium

$$\Delta(\mathbf{r}) = gT\sum_{\omega_n} F(\mathbf{rr};\ \omega_n), \quad \Delta^*(\mathbf{r}) = gT\sum_{\omega_n} F^+(\mathbf{rr};\ \omega_n). \tag{2}$$

The superconducting current is determined by the expression

$$\mathbf{j}(\mathbf{r}) = -\left(\frac{\delta\Omega}{\delta\mathbf{A}(\mathbf{r})}\right)_\Delta = -\frac{e}{m}T\sum_{\omega_n}[\hat{\mathbf{p}} - \hat{\mathbf{p}}' - 2e\mathbf{A}(\mathbf{r})]G(\mathbf{rr}';\ \omega_n)\,|_{\mathbf{r}'=\mathbf{r}}. \tag{3}$$

In conjunction with Maxwell's equations (curl $\mathbf{H} = 4\pi\mathbf{j}$, div $\mathbf{H} = 0$) Eqs. (1), (2), and (3) define the penetration of the field into the superconductor. For extremely hard superconductors ($\delta_0(0) \gg \xi_0 \sim v_0/T_c$; ξ_0—electron correlation radius, v_0—Fermi velocity, $\delta_0(0)$—depth of penetration of the weak field when T = 0) the problem can be solved under the assumption that the velocity of the superconducting condensate

$$\mathbf{v}_s = (\nabla\chi - 2e\mathbf{A})/2m$$

(χ—phase of Δ; $\Delta = |\Delta|e^{i\chi}$) is constant in (1), (2), and (3). This leads[10,11,2] to the following relations (for the Green's functions G, F, and F^+ we took their Fourier transforms):

$$\begin{pmatrix} G(\mathbf{p};\ \omega), & F(\mathbf{p};\ \omega) \\ F^+(\mathbf{p};\ \omega) & \tilde{G}(\mathbf{p};\ \omega) \end{pmatrix} = \frac{1}{(\omega + i\mathbf{p}_0\mathbf{v}_s)^2 + \xi^2 + \Delta^2}\begin{pmatrix} i\omega - \mathbf{p}_0\mathbf{v}_s + \xi, & -\Delta \\ -\Delta, & i\omega - \mathbf{p}_0\mathbf{v}_s - \xi \end{pmatrix}; \tag{4}$$

$$\frac{1}{|g|} = \frac{mp_0}{(2\pi)^2}\int d\xi \frac{\mathrm{th}(\varepsilon_0/2T)}{\varepsilon_0} = \frac{mp_0}{(2\pi)^2}\int\int \frac{d\xi\, dO}{4\pi}\frac{1}{\varepsilon} \times \left[1 - f\left(\frac{\varepsilon - \mathbf{p}_0\mathbf{v}_s}{T}\right) - f\left(\frac{\varepsilon + \mathbf{p}_0\mathbf{v}_s}{T}\right)\right],$$

$$\mathbf{j} = Ne\mathbf{v}_s - \frac{p_0 e}{2\pi^2}\int\int \frac{d\xi\, dO}{4\pi}\mathbf{p}_0\left[f\left(\frac{\varepsilon - \mathbf{p}_0\mathbf{v}_s}{T}\right) - f\left(\frac{\varepsilon + \mathbf{p}_0\mathbf{v}_s}{T}\right)\right],$$

$$\varepsilon = \sqrt{\xi^2 + \Delta^2}, \quad \varepsilon_0 = \sqrt{\xi^2 + \Delta_0^2}, \quad f(x) = (e^x + 1)^{-1},$$

$$N = p_0^3/3\pi^2 \tag{5}$$

(Δ_0 is the gap in the BCS theory[7], dO—solid-angle element in the direction of $\mathbf{p}_0/p_0$).

The integration of Maxwell's equations for a superconducting half-space bounded by a plane gives the following result[2]:

$$H^2 = 8mp_0T\sum_{\omega_n>0}\Big\{(\omega_n^2 + \Delta_0^2)^{1/2} + \frac{\omega_n^2}{(\omega_n^2 + \Delta_0^2)^{1/2}} - \frac{i}{2p_0v_s}[(\omega_n - ip_0v_s)((\omega_n - ip_0v_s)^2 + \Delta^2)^{1/2} - (\mathrm{conj.})]\Big\}. \tag{6}$$

The condition for the stability of this solution is the positiveness of the second variation of the potential Ω in the vicinity of the given solution. This value of the field H'_{c1} on the surface of the superconductor, at which $\delta^2\Omega$ first reverses sign, is indeed the limit of existence of the superconducting state. The second variation of Ω is

$$\delta^2\Omega = \int dV \Big\{ T\sum_{\omega_n}\tfrac{1}{2}[\delta F^+(\mathbf{rr};\ \omega_n)\delta\Delta(\mathbf{r}) + \delta F(\mathbf{rr};\ \omega_n)\delta\Delta^*(\mathbf{r})] - g^{-1}\delta\Delta^*(\mathbf{r})\delta\Delta(\mathbf{r}) \Big\}, \tag{7}$$

where δF and δF^+ are defined in accordance with (1), by means of the following formulas

2)At a temperature close to the critical value T_c, the potential Ω goes over into the corresponding expression of the Ginzburg-Landau theory[6].

$$\delta F(\mathbf{rr};\omega_n)=\int dV'[G(\mathbf{rr}';\omega_n)\delta\Delta(\mathbf{r}')\bar{G}(\mathbf{r}'\mathbf{r};\omega_n)$$
$$+F(\mathbf{rr}';\omega_n)\delta\Delta^*(\mathbf{r}')F(\mathbf{r}'\mathbf{r};\omega_n)],$$
$$\delta F^+(\mathbf{rr};\omega_n)=\int dV'[F^+(\mathbf{rr}';\omega_n)\delta\Delta(\mathbf{r}')F^+(\mathbf{r}'\mathbf{r};\omega_n)$$
$$+\bar{G}(\mathbf{rr}';\omega_n)\delta\Delta^*(\mathbf{r}')G(\mathbf{r}'\mathbf{r};\omega_n)]. \quad (8)$$

Since the characteristic dimension of the "dangerous" fluctuations of Δ, which lead to the formation of vortices, is of the order of $\xi_0\delta(T)/\delta_0(0)\sim\delta/\kappa \ll \delta$, we can as before assume that v_s = const and Δ = const in (7) and (8) and we can use formulas (4) for the equilibrium Green's functions[3]. Substituting (4) in (7) and (8) and going over to the Fourier transform of all the quantities, we get

$$\delta^2\Omega=\frac{1}{2}\int\frac{d^3\mathbf{q}}{(2\pi)^3}[a_\mathbf{q}\delta\Delta_\mathbf{q}^*\delta\Delta_\mathbf{q}+a_{-\mathbf{q}}\delta\Delta_{-\mathbf{q}}^*\delta\Delta_{-\mathbf{q}}$$
$$+b_\mathbf{q}(\delta\Delta_\mathbf{q}^*\delta\Delta_{-\mathbf{q}}^*+\delta\Delta_\mathbf{q}\delta\Delta_{-\mathbf{q}})],$$

where

$$\delta\Delta(\mathbf{r})=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\delta\Delta_\mathbf{q}e^{i\mathbf{qr}},\qquad \delta\Delta^*(\mathbf{r})=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\delta\Delta_\mathbf{q}^*e^{-i\mathbf{qr}},$$

$$a_\mathbf{q}=\frac{1}{|g|}+\frac{mp_0}{2\pi^2}T\sum_{\omega_n}\int\int\frac{d\xi\, dO}{4\pi}$$
$$\times\frac{(i\omega_n-\mathbf{p}_0\mathbf{v}_s+\xi_+)(i\omega_n-\mathbf{p}_0\mathbf{v}_s-\xi_-)}{[(\omega_n+i\mathbf{p}_0\mathbf{v}_s)^2+\xi_+^2+\Delta^2][(\omega_n+i\mathbf{p}_0\mathbf{v}_s)^2+\xi_-^2+\Delta^2]},$$

$$b_\mathbf{q}=\frac{mp_0}{2\pi^2}T\sum_{\omega_n}\int\int\frac{d\xi\, dO}{4\pi}$$
$$\times\frac{\Delta^2}{[(\omega_n+i\mathbf{p}_0\mathbf{v}_s)^2+\xi_+^2+\Delta^2][(\omega_n+i\mathbf{p}_0\mathbf{v}_s)^2+\xi_-^2+\Delta^2]}.$$

$$\xi_\pm=\xi\pm\mathbf{v}_0\mathbf{q}/2. \quad (9)$$

Let us diagonalize the expression for $\delta^2\Omega$ with the aid of the substitution

$$\delta\Delta_\mathbf{q}=\alpha_\mathbf{q}u_\mathbf{q}+\beta_\mathbf{q}u_{-\mathbf{q}}^*,\quad \alpha_\mathbf{q}=\alpha_{-\mathbf{q}}=\alpha_\mathbf{q}^*,$$
$$\delta\Delta_\mathbf{q}^*=\alpha_\mathbf{q}u_\mathbf{q}^*+\beta_\mathbf{q}u_{-\mathbf{q}},\quad \beta_\mathbf{q}=\beta_{-\mathbf{q}}=\beta_\mathbf{q}^*,\quad \alpha^2+\beta^2=1.$$

This yields

$$\delta^2\Omega=\int\frac{d^3\mathbf{q}}{(2\pi)^3}\left[1-\left(\frac{2b_\mathbf{q}}{a_\mathbf{q}+a_{-\mathbf{q}}}\right)^2\right]^{1/2}$$
$$\times\left\{\left[\left(\frac{a_\mathbf{q}+a_{-\mathbf{q}}}{2}\right)^2-b_\mathbf{q}^2\right]^{1/2}+\frac{a_\mathbf{q}-a_{-\mathbf{q}}}{2}\right\}u_\mathbf{q}^*u_\mathbf{q}. \quad (10)$$

Equating to zero the integrand in the curly brackets of (10), we obtain the formula $v_s = v_s(q, \cos\theta)$ ($\mathbf{v}_s\cdot\mathbf{q} = v_s q\cos\theta$), the smallest value of which determines the critical value of the velocity v_s. Substitution of this value into (6) gives the value of the limiting field H'_{c1}. In the general case of arbitrary temperatures, this problem should be solved by numerical means. In the limiting cases $T\sim T_c$ and $T = 0$ the calculation can also be carried out in explicit form.

Let us consider first temperatures close to critical T_c. Then

$$\Delta/T_c\ll 1,\quad p_0v_s/T_c\ll 1,\quad v_0q/T_c\ll 1$$

and in all formulas we confine ourselves to the first nonvanishing terms of the expansion in the small quantities:

$$\frac{a_\mathbf{q}+a_{-\mathbf{q}}}{2}-b_\mathbf{q}=\rho\frac{(v_0q)^2}{6},$$

$$\frac{a_\mathbf{q}-a_{-\mathbf{q}}}{2}=\rho\cdot\frac{2}{3}(p_0v_0)(qv_s)\cos\theta,$$

$$\frac{a_\mathbf{q}+a_{-\mathbf{q}}}{2}+b_\mathbf{q}=\rho\Delta^2,\qquad \rho=\frac{mp_0}{2\pi^2}\frac{7}{8}\frac{\zeta(3)}{(\pi T_c)^2},$$

$$\Delta^2=\Delta_0^2-\frac{2}{3}(p_0v_s)^2.$$

From the condition that (10) must be positive, it follows that[4]:

$$p_0v_s<\left[\frac{3}{2}\frac{\Delta_0^2+(v_0q)^2/12}{1+2\cos^2\theta}\right]^{1/2},\qquad v_{cr}'=\frac{\Delta_0}{\sqrt{2}p_0}.$$

Substituting the critical value of the velocity v'_{cr} in (6), taken in the same approximation ($\Delta/T_c \ll 1$), we get

$$\frac{H_{c1}'}{H_c}(T_c)=\frac{\sqrt{5}}{3},\qquad H_c=\left(\frac{7mp_0}{4\pi}\zeta(3)\right)^{1/2}\frac{\Delta_0^2}{\pi T_c}.$$

It is clear from general considerations that in the case of arbitrary temperatures the first to be excited is the long-wave ($q = 0$) part of the fluctuations, and then by virtue of symmetry we have $\cos^2\theta = 1$. When $T = 0$, replacing the sum

$$T\sum_{\omega_n}\to\int\frac{d\omega}{2\pi}$$

and going over in the limit at $q\to 0$ ($\cos^2\theta = 1$) we obtain (for $p_0v_s > \Delta_0 > \Delta$)

$$\frac{a_\mathbf{q}+a_{-\mathbf{q}}}{2}-b_\mathbf{q}=\frac{Nq^2}{8m\Delta^2}[1-(1+2\lambda^2)\sqrt{1-\lambda^2}],\quad \lambda=\frac{\Delta}{p_0v_s},$$

$$\frac{a_\mathbf{q}+a_{-\mathbf{q}}}{2}+b_\mathbf{q}=\frac{3}{2}\frac{N}{p_0v_0}[1-\sqrt{1-\lambda^2}],$$

[3]For the same reason we can neglect the small variation connected with the local changes of Δ, which should be taken into account in the general case, since the external field is specified in our case only at infinity.

[4]As can be seen from this formula, the wave vector of the fluctuation is measured in terms of the characteristic length δ_0/κ ($\kappa^2 = 6\Delta_0^2\delta_0^2/v_0^2$ – the parameter of the Ginzburg-Landau theory[6]), thus confirming the assumption of the characteristic dimensions of the "dangerous" fluctuation.

$$\left|\frac{a_{\mathbf{q}}-a_{-\mathbf{q}}}{2}\right| = \frac{3Nq}{4p_0}\left(\frac{1}{p_0 v_s}\right)\sqrt{1-\lambda^2}.$$

The condition for the positiveness of (10) is

$$[1-\sqrt{1-\lambda^2}][1-(1+2\lambda^2)\sqrt{1-\lambda^2}] > 3\lambda^2(1-\lambda^2),$$

hence

$$\lambda_{cr}' = [1-(2^{1/3}-1^2]^{1/2}.$$

Expressions (5) and (6) with T = 0 have the form (for $p_0 v_S > \Delta_0$)

$$\Delta_0 / p_0 v_s = (1+\sqrt{1-\lambda^2})\exp(-\sqrt{1-\lambda^2}),$$

$$H^2 = \frac{2mp_0}{\pi}\Delta_0^2\left\{1-\left(\frac{p_0 v_s}{\Delta_0}\right)^2\left[\lambda^2-\frac{2}{3}+\frac{2}{3}(1-\lambda^2)^{3/2}\right]\right\}.$$

Substituting in these formulas the value of λ'_{cr} we obtain ultimately

$$\frac{H_{c1}'}{H_c}(0) = \left\{1-\frac{1}{3}(2^{2/3}+1)(2^{2/3}-1)^3\exp[2(2^{1/3}-1)]\right\}^{1/2}$$

$$\approx 0.8, \quad H_c(0) = \left(\frac{2mp_0}{\pi}\right)^{1/2}\Delta_0(0).$$

The estimate made in[3] of the quantity H'_{c1} gives a value close to that obtained here. In this connection it is necessary to make the following remarks. The field $H'_{c1}(T)$ calculated above is the exact (for $\kappa \gg 1$) stability limit of the superconducting state relative to arbitrary local fluctuations of the parameter Δ. Bean and Livingston[3] specified concretely the form of the fluctuation, this being the vortex penetrating from the surface into the superconductor, and calculated the field at which the potential barrier that is overcome by the vortex during the process of penetration first vanishes. Any such calculation, carried out rigorously on the basis of the equations of electrodynamics with account of the boundary condition, should lead to values of the field H'_{c1} not smaller than those obtained above. The problem itself, that of determining the form of the "dangerous" fluctuations (nucleus) and calculating the energy barrier separating the superconducting state from the mixed state, is essentially the minimax problem for the functional $\Omega\{\Delta\}$. This means that we must find an unstable state, intermediate between the superconducting and the mixed states, whose energy would exceed in minimum fashion the energy of the superconducting state. By virtue of the quantum nature of the vortex, it is clear from energy considerations that the solution should contain only one vortex, but the concrete form of the solution[5)] is not evident beforehand. The energy barrier between the superconducting and the mixed states, calculated in this manner, should, of course, vanish precisely for the field H'_{c1} obtained above.

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[5)]It is obvious that the vortex cannot be too long, for the self-energy of the vortex is proportional to its length; in addition, the magnetic flux of the vortex should go over on the surface into the external flux.

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