DETERMINATION OF THE SLIP COEFFICIENT OF VORTICES IN ROTATING HELIUM II

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The interaction between vortices and a solid surface is manifest in the fact that a disc freely suspended in rotating liquid helium II lags the head of the suspension, which is set in additional rotational motion. The vortex slip coefficient can be determined on the basis of the magnitude of the lag.

IN a paper delivered to the Canadian Conference on Low Temperature Physics, Osborne reported an unsuccessful attempt to measure the tension of Onsager-Feynman vortex filaments. As noted by Osborne himself, his failure is connected with the slip of the vortices. After cessation of the external short-duration action, the vortices succeeded in restoring their initial configuration during the observation time, so that their stationary deformation could not be recorded.

Bearing this circumstance in mind, we have perfected an instrument in which the vortices were subjected to continuous action, so that restoration of their initial unperturbed configuration became impossible.

The instrument comprises a torsion pendulum (see Fig. 1). Disc 1, made of aluminum foil 20 μ thick and radius R = 2 cm, was suspended with the aid of a straightened glass rod 2 and a phosphor-bronze suspension 3 (7 cm long and 10 μ in diameter) from the minute hand 4 of a stop watch 5. The entire system together with liquid-helium 6 could be rotated uniformly by means of a permanent electromagnet NS coupled to a telechron motor through a friction drive wheel. The speed could be varied from 0.038 to 0.5 sec^{-1} by proper choice of rubber rollers with different radii.

The liquid-helium II together with the suspension system was made to rotate uniformly at a temperature 1.46° K. After the stationary rotation set in (15–20 min), a "Kiev" motion picture camera was used to fix the position of the moving beams on a scale. The reflections of light beams O and O_1 from mirrors 7 and 8 were focused prior to the start of rotation.

When the stop watch was turned on, the minute hand and with it the disc were set in additional rotation relative to the rotating liquid helium, with angular velocity $\omega = 3.5 \times 10^{-3}$ sec^{-1}. This caused

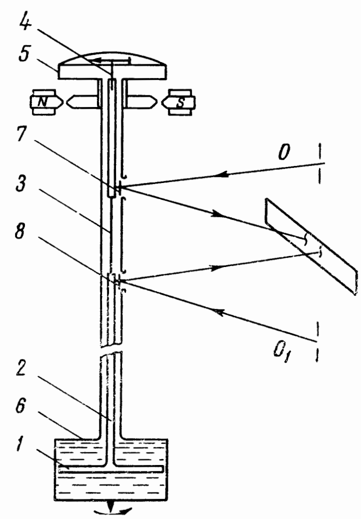

FIG. 1. Diagram of the instrument.

the vortices in the gap between the lower rough surface of the disc and the bottom of the vessel (also rough) to bend, and the disc was acted upon by an elastic decelerating torque $M_v = f_v\varphi_c$. Here φ_c is the angle through which the disc was rotated relative to the bottom of the container in a time $t = t_c$, and f_v is the contribution to the torque by the inclination of the vortices (per unit angle). This caused the disc to lag the minute hand by $\Delta\varphi_c$, which is calculated from the magnitude of the displacement ΔN_{max} observed between the light beams:

$$\Delta\varphi_c = \frac{\Delta N_{max}}{2L} = \frac{\Delta n_c - \Delta n_0}{2L}. \quad (1)$$

Here Δn_0 and Δn_c are the distances between the beams before and after turning on the stopwatch, and L = 115 cm is the distance from the scale to the mirror.

Control experiments carried out in vacuum and in non-rotating helium II have shown that the disc

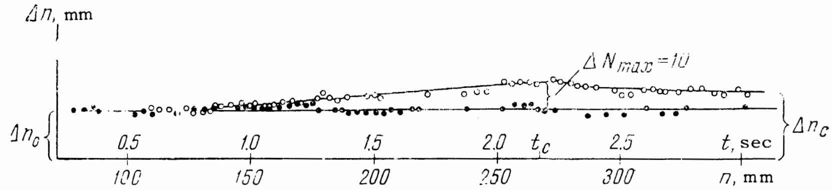

FIG. 2. Lag of disc at ω_{01} = 0.038 sec^{-1} and ω = 3.5 × 10^{-3} sec^{-1}, and T = 1.46°K; the quantity t_c = 2.2 sec on the time axis indicates the initial start of slipping.

follows immediately the rotation of the stopwatch and that the angle between the beams remains unchanged.

In rotating helium II the situation is different (Fig. 2). After turning on the stopwatch, the disc lags the stopwatch by a certain angle which increases from zero to a certain value $\Delta\varphi_c$, which subsequently becomes constant during the time of observation (motion picture photography). This lag amounts to $\Delta\varphi_c = (4.4 \pm 0.4) \times 10^{-3}$ rad at an angular rotation of the liquid $\omega_{01} = 0.038$ sec^{-1} and a temperature 1.46° K.

At the sensitivity of our apparatus, this displacement was maximal. At $\omega_{02} = 0.098$ sec^{-1} the lag amounted to $(2.6 \pm 0.2) \times 10^{-3}$ rad (Fig. 3),[1)] and at larger velocities no lag was observed at all in the limits of the measurement error.

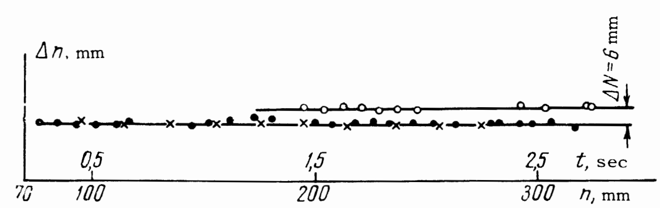

FIG. 3. Lag of disc at ω_{02} = 0.038 sec^{-1} and ω = 3.5 × 10^{-3} sec^{-1}, T = 1.46°K.

The absence of a shift of the non-rotating helium II shows that the viscous deceleration of the additional rotation of the disc is quite insignificant and lies outside the limits of the sensitivity of the instrument. Therefore, when interpreting the results obtained in rotating helium it is necessary to take into account only the elastic moments of the suspension and of the vortex filaments. Then the equation of motion of the disc takes the form

$$f_f(\omega t - \varphi) - f_v\varphi = I d^2\varphi / dt^2, \qquad (2)$$

where f_f is the torque of the suspension filament, ω is the angular velocity of the minute hand of the stopwatch, φ is the angle of rotation of the disc, and I is the moment of inertia of the disc.

It follows therefore that after damping of the initial oscillations, the disc will move with angular velocity $f_f\omega/(f_f + f_v)$, which is smaller than the velocity of the minute hand. Thus, the lag of the disc will increase. However, the unavoidable slipping of the vortices stops the growth of this lag. After the angle of rotation of the disc reaches a certain value φ_c (at the instant of time $t = t_c = 2.2$ sec), the upper ends of the vortices cease to follow the disc, and the decelerating torque acquires a constant value $M_v = f_v\varphi_c$. Then a permanent shift is established $\Delta\varphi_c = \varphi_c f_v/f_f$, which indeed was observed by us experimentally (Figs. 2 and 3).

Thus, the results of our experiments show that an interaction is produced between the disc and the vortices, which does not vanish in spite of the slipping. The slip only limits the magnitude of deformation of the vortices.

From the obtained data we can also determine the slip coefficient. To this end we make use of the condition for the fastening of the vortices to a solid surface[1, 2]

$$(\mathbf{V}_L - \mathbf{V}_\sigma)_\tau = a(\boldsymbol{\omega} / \omega)_\tau. \qquad (3)$$

Condition (3) is tantamount to assuming that the difference between the tangential components of the velocities $\mathbf{V}_L$ and $\mathbf{V}_\sigma$ of the vortex and of the surface is proportional to the tangential component of the force (see Fig. 4).

$$\varepsilon_\tau = \varepsilon(\boldsymbol{\omega} / \omega)_\tau = \varepsilon \sin\alpha, \qquad (4)$$

with which the vortex with stress ϵ acts on the

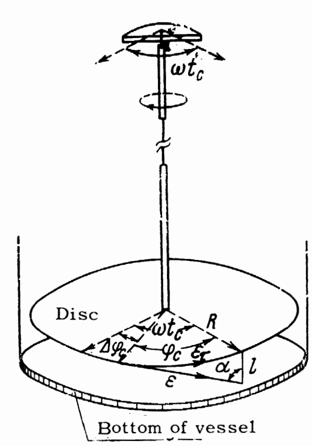

FIG. 4. Diagram showing the arrangement of the surfaces and of the vortex. ωt_c – angle of rotation of the minute hand at $t = t_c$, $\Delta\varphi_c$ – lag of disc at $t = t_c$, φ_c – angle of rotation of disc at $t = t_c$, l – height of gap between the disc and bottom of the container.

[1)] Figures 2 and 3 have been plotted in a coordinate system which rotates at the same angle of velocity as the motor.

solid surface. A value of $a = 0$ of the slip coefficient corresponds to complete fastening of the vortices (absolutely rough surface). In our experiments the disc surface was covered with sand particles of $50\,\mu$ size.

The unit vector of the tangent to the vortex line $(\boldsymbol{\omega}/\omega)_\tau$ for small α can be quite simply determined from Fig. 4. Indeed,

$$(\omega / \omega)_\tau = \sin\alpha = \tan\alpha = \varphi_c R / l. \qquad (5)$$

Here l is the height of the gap between the disc and the bottom of the container ($l = 0.1$ cm). It should be noted that we are neglecting the effect of the vortices located above the disc because of the slight bending.

Substituting in (4) the values $(\mathbf{V}_L - \mathbf{V}_\sigma)_\tau = \omega R$ and the value of $(\boldsymbol{\omega}/\omega)_\tau$ from (5), we obtain for a the formula

$$a = \frac{\omega l}{\varphi_c} = \frac{\omega l}{\omega t_c - \Delta\varphi_c}. \qquad (6)$$

The experimental data for helium II rotating at $\omega_{01} = 0.038\ \text{sec}^{-1}$ yield $\varphi_c = 3.2 \times 10^{-3}$ rad and $a = 0.10 \pm 0.01$ cm/sec. Comparing this result with the values of a calculated from the relation $a/\sqrt{\Omega} = f(\omega_0)$ obtained in [1] (Ω = angular frequency of disc oscillation) points to a fully satisfactory agreement between the two.

The lack of noticeable lag $\Delta\varphi_c$ at large ω_0 is obviously connected with strong increase in the slip coefficient with increasing rotation velocity. For an angular velocity $\omega_0 = 0.098\ \text{sec}^{-1}$ we were unable to calculate the slip coefficient because of the difficulty in determining the angle φ_c.

In conclusion we consider it our pleasant duty to thank É. L. Andronikashvili for suggesting the topic and valuable advice, Yu. G. Mamaladze for participating in the discussions of the results, and V. G. Tartinskikh for technical help.

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