ELECTROMAGNETIC RADIATION EMITTED DURING PASSAGE OF A FAST CHARGED PARTICLE THROUGH A SEMICONDUCTOR

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The features of emission of coupled waves excited by a fast charged particle are investigated, with spatial dispersion taken into account. It is shown that the Cerenkov radiation from transverse waves is distributed over the surface of two or three cones, the number of which depends on the sign of the effective exciton mass and the parameters of the semiconductor. The direction of the Cerenkov radiation produced by the supplementary transverse wave emerging to the vacuum depends on the sign of the effective mass of the transverse exciton. The radiation from the transverse waves can enter the vacuum only if the effective mass of the longitudinal exciton is positive and the direction of the radiation coincides with that of the particle motion. Losses due to excitation of the supplementary waves are calculated. A relation is derived for the particle velocity required for excitation of the supplementary waves. It is shown that the width of the spectrum of the radiation from transverse waves can be changed by varying the density of the free carriers. The emission spectrum produced in vacuum by transverse waves consists of a number of narrow bands. The number and width of the bands depend on the parameters of the semiconductor.

It must be recognized in the study of semiconductor properties that along with the charges bound in the lattice they contain also free charges. The inductive interaction between these charges and the optical lattice vibrations leads to appearance of coupled electromagnetic waves. Some properties of such waves were studied earlier,[1] in particular, the dispersion laws in the transparency region.

Since the dispersion of coupled waves is essentially different from the dispersion of electromagnetic waves in a dielectric or a plasma, it is of interest to determine the energy lost by a fast charged particle to the excitation of such waves and also the radiation which will emerge into vacuum when a charge passes through a semiconducting plate. Since a semiconductor has more parameters than a dielectric, there are grounds for assuming that the emission spectrum of the former is more complicated, and that by varying the parameters it is possible to change the frequency and the width of the bands in the spectrum.

Special attention has been paid in this investigation to a study of radiation from the supplementary transverse and longitudinal bound waves due to spatial dispersion.

1. ENERGY LOST BY PARTICLE TO EXCITATION OF BOUND WAVES

The energy lost by fast particles to excitation of transverse and longitudinal electromagnetic waves in a medium, in the range of frequencies for which there is no absorption, is given by the following expressions:[2]

$$W^{\perp} = \frac{2e^2}{c^2}\int \omega\, d\omega \int \frac{\varkappa^3 d\varkappa}{\varkappa^2 + \omega^2/v^2} \times \delta\left\{\varkappa^2 + \omega^2\left[\frac{1}{v^2} - \frac{1}{c^2}\varepsilon^{\perp}\left(\omega, \sqrt{\varkappa^2 + \frac{\omega^2}{v^2}}\right)\right]\right\}, \quad (1)$$

$$W^{\parallel} = \frac{2e^2}{v^2}\int \omega\, d\omega \int \frac{\varkappa d\varkappa}{\varkappa^2 + \omega^2/v^2}\,\delta\left[\varepsilon^{\parallel}\left(\omega, \sqrt{\varkappa^2 + \frac{\omega^2}{v^2}}\right)\right], \quad (2)$$

where v is the particle velocity and ε is the dielectric constant.

We see from (1) that the Cerenkov radiation due to the transverse waves is produced under the condition

$$v \geqslant c / [\varepsilon(\omega, k)]^{1/2}, \quad (3)$$

and if we disregard the spatial dispersion, then this radiation will be distributed over the surface of a cone with angle satisfying the relation

$$\cos\theta = c / vn(\omega),$$

while account of the spatial dispersion transforms (3) into

$$v \geqslant c / n_i(\omega), \tag{3a}$$

where n_i is one of the roots of the dispersion relation for transverse waves in a medium. We see from (3a) that in this case there can be several radiation cones. To determine the regions of integration with respect to ω in (1) and (2), we consider the dispersion relations for the transverse and longitudinal bound waves in an isotropic semiconductor.[1]

For the transverse waves the dispersion relation is of the form

$$(\omega_0^2 - \omega^2 + \alpha_1 k^2)(\omega^2 - k^2c^2 - \omega_{0e}^2) + 4\pi\gamma\omega^2 = 0, \tag{4}$$

where ω_0 is the frequency of exciton absorption, ω_{0e} the Langmuir frequency of the free carriers, α_1 the ratio of the transverse-exciton activation energy to its effective mass, and γ a constant describing the structure of the exciton bands. We see from this relation that two transverse waves can exist in a semiconductor, viz., a fundamental wave with the dispersion law

$$n_1^2 = \frac{(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2)}{\omega^2(\omega^2 - \omega_3^2)}, \tag{5}$$

where

$$\omega_{1,2}^2 = {}^1/_2(\omega_0^2 + \omega_{0e}^2 + 4\pi\gamma)$$

$$\pm [{}^1/_4(\omega_0^2 + \omega_{0e}^2 + 4\pi\gamma)^2 - \omega_0^2\omega_{0e}^2]^{1/2},$$

$$\omega_3^2 = \omega_0^2 + \alpha_1\omega_{0e}^2 / c^2, \tag{6}$$

and a supplementary wave with the dispersion law

$$n_2^2 = \frac{c^2}{\alpha_1}\left(1 - \frac{\omega_0^2}{\omega^2} - \frac{\alpha_1}{c^2}\frac{\omega_{0e}^2}{\omega^2}\right). \tag{7}$$

Upon examining the relation $n^2 = n^2(\omega)$ in (5) we conclude that the fundamental wave produces a radiation that can be distributed in the following frequency intervals: when $\alpha_1 > 0$ in the intervals

$$\omega_3 < \omega < \omega_2, \quad \omega > \omega_1 \text{ for } \omega_3 < \omega_2, \tag{8}$$

$$\omega_2 < \omega < \omega_3, \quad \omega > \omega_1 \text{ for } \omega_2 < \omega_3 \tag{9}$$

and $\alpha_1 < 0$ in the intervals

$$\omega_2 < \omega < \omega_3, \quad \omega > \omega_1 \text{ for } \omega_0^2 > |\alpha_1|\omega_{0e}^2 / c^2, \tag{10}$$

$$\omega > \omega_1 \qquad \text{for } \omega_0^2 < |\alpha_1|\omega_{0e}^2 / c^2. \tag{11}$$

The radiation from the supplementary wave (7) can occur only in a semiconductor with definite parameters: for $\alpha_1 > 0$ in the interval

$$\omega^2 > \omega_0^2 + \alpha_1\omega_{0e}^2 / c^2, \tag{12}$$

and for $\alpha_1 < 0$ and $|\alpha_1|\,\omega_{0e}^2/c^2 < \omega_0^2$ in the interval

$$\omega^2 < \omega_0^2 - |\alpha_1|\omega_{0e}^2 / c^2. \tag{13}$$

Thus, according to (8)–(13), we reach the conclusion that in an isotropic semiconductor the Cerenkov radiation due to transverse waves is distributed on the surface of three or two cones, depending on the parameters of the semiconductor and on the sign of the effective mass of the transverse exciton.

Investigating the dispersion relation for longitudinal coupled waves

$$(\omega_0^2 + \alpha k^2 - \omega^2)(\omega^2 - \omega_{0e}^2) + 4\pi\gamma\omega^2 = 0, \tag{14}$$

where α is the ratio of the longitudinal exciton activation energy to its effective mass, we conclude that the loss to excitation of longitudinal waves in semiconductors with parameters under which the condition

$$\varepsilon^{\parallel}(\omega, k) = 0 \tag{15}$$

is satisfied will be accompanied by emission of waves in two frequency bands.

When choosing the interval of integration with respect to ω, we must also take into account the condition for existence of macroscopic supplementary waves[3]

$$\lambda_0 / n > a, \tag{16}$$

where λ_0 is the wavelength in vacuum and a is the lattice constant. Indeed, from (14) we have

$$n^2 = \frac{c^2}{\alpha}\left[1 - \frac{\omega_0^2}{\omega^2} - \frac{4\pi\gamma}{\omega^2}\left(1 - \frac{\omega_{0e}^2}{\omega^2}\right)^{-1}\right], \tag{17}$$

and since the ratio c^2/α is large, the condition (16) can be satisfied only in the case when the quantity in the square bracket will be sufficiently small. Analyzing this expression, we find that relation (16) will be satisfied for longitudinal waves if the following relations hold between the semiconductor parameters: when

$$\omega_{0e}^2 < \omega_0^2 < 4\pi\gamma, \quad \omega_0^2 > \omega_2^2 4\pi\gamma / \omega_{0e}^2$$

the emission spectrum will lie in the frequency interval

$$\omega_0^2\left(1 + \frac{4\pi\gamma}{\omega_{0e}^2} - \frac{\alpha n_0}{c^2}\right)^{-1} < \omega^2 < \omega_0^2\left(1 - \frac{4\pi\gamma}{\omega_{0e}^2}\right)^{-1}; \tag{18}$$

and when $4\pi\gamma < \omega_{0e}^2 < \omega_0^2$ the emission spectrum will lie in the following two frequency intervals ($n_0 = \lambda_0/a$):

$$\omega_{0e}^2\left(1 - \frac{4\pi\gamma}{\omega_0^2 - \omega_{0e}^2 - \alpha n_0/c^2}\right) < \omega^2 < \omega_{0e}^2\left(1 - \frac{4\pi\gamma}{\omega_0^2 - \omega_{0e}^2}\right), \tag{19}$$

$$(\omega_0^2+4\pi\gamma)\left(1-\frac{an_0}{c^2}\right)^{-1}<\omega^2<\omega_0^2+4\pi\gamma. \quad (20)$$

Now integrating (2) with respect to κ and choosing the lower limit of integration with respect of ω from the condition $\kappa^2 \geq 0$, and the upper limit from (18)–(20) we obtain the following value for the loss due to excitation of longitudinal coupled waves:

$$W^{\parallel}=\frac{2\pi\gamma e^2}{v^2}\sum_i\left[\frac{\omega_1^4}{(\omega_1^2-\omega_2^2)(\omega_1^2-\omega_{0e}^2)}\ln\frac{\tau_{i1}-\omega_1^2}{\tau_{i2}-\omega_1^2}\right.$$
$$+\frac{\omega_2^4}{(\omega_2^2-\omega_1^2)(\omega_2^2-\omega_{0e}^2)}\ln\frac{\tau_{i1}-\omega_2^2}{\tau_{i2}-\omega_2^2}$$
$$\left.+\frac{\omega_{0e}^4}{(\omega_{0e}^2-\omega_1^2)(\omega_{0e}^2-\omega_2^2)}\ln\frac{\tau_{i1}-\omega_{0e}^2}{\tau_{i2}-\omega_{0e}^2}\right]. \quad (21)$$

For a semiconductor with parameters $\omega_{0e}^2<\omega_0^2<4\pi\gamma$ and $\omega_0^2>\omega_2^2 4\pi\gamma/\omega_{0e}^2$

$$i=1,\quad \tau_{11}=\omega_0^2\left(\frac{4\pi\gamma}{\omega_{0e}^2}+1-\frac{an_0}{c^2}\right)^{-1},$$
$$\tau_{12}=\omega_0^2\left(\frac{4\pi\gamma}{\omega_{0e}^2}+1-\frac{\alpha}{v^2}\right)^{-1};$$

for a semiconductor with parameters $4\pi\gamma<\omega_{0e}^2<\omega_0^2$

$$i=1,2,\quad \tau_{11}=\omega_{0e}^2\left[1-4\pi\gamma\left(\omega_0^2-\omega_{0e}^2-\frac{an_0}{c^2}\right)^{-1}\right],$$
$$\tau_{12}=\omega_{0e}^2\left[1-4\pi\gamma\left(\omega_0^2-\omega_{0e}^2-\frac{\alpha}{v^2}\right)^{-1}\right],$$
$$\tau_{21}=(\omega_0^2+4\pi\gamma)\left(1-\frac{an_0}{c^2}\right)^{-1},$$
$$\tau_{22}=(\omega_0^2+4\pi\gamma)\left(1-\frac{\alpha}{v^2}\right)^{-1}.$$

We obtain analogously the loss connected with excitation of a transverse supplementary wave

$$W^{\perp}=\frac{e^2}{2c^2}\left(\omega_0^2+\alpha_1\frac{\omega_{0e}^2}{c^2}\right)\frac{\alpha_1}{v^2}\left[\frac{n_0^2\beta^2-1}{1-n_0\alpha_1/c^2}\right.$$
$$\left.-\ln\frac{n_0\beta^2(1-\alpha_1/v^2)}{1-n_0\alpha_1/c^2}\right]. \quad (22)$$

It must be noted that the supplementary waves will be excited by a moving charge only if its velocity satisfies the condition

$$n_0\beta^2>1. \quad (23)$$

This relation follows from the condition for the existence in (1) and (2) of an interval of integration with respect to the frequency.

2. RADIATION INTO VACUUM FROM A SEMI-INFINITE CONDUCTOR

Let a charge e move along the z axis and cross the boundary between the semiconductor and the vacuum (z = 0) at the instant t = 0. The semiconductor fills the space z < 0 (medium 1). The solution of the inhomogeneous Maxwell equations for such a case was obtained by Sitenko.[4] To find the general solution, we must add to it the solution of the system of homogeneous equations[1]

$$\operatorname{rot}\mathbf{E}=\frac{i\omega}{c}\mathbf{H}, \quad (24)^*$$

$$\operatorname{rot}\mathbf{H}=-\frac{i\omega}{c}\left(\varepsilon'\mathbf{E}-\frac{4\pi}{c}\mathbf{P}\right), \quad (25)$$

$$\left[-\alpha_1\operatorname{rot}\operatorname{rot}\Delta+(\omega_0^2-\omega^2)\operatorname{rot}\operatorname{rot}+\frac{\omega^2}{c^2}\varepsilon'(\alpha_1\Delta+\alpha_2\operatorname{grad}\operatorname{div})\right.$$
$$\left.-\frac{\omega^2}{c^2}(\omega_0^2-\omega^2)\varepsilon_1\right]P=0, \quad (26)$$

where

$$\varepsilon'=1-\frac{\omega_{0e}^2}{\omega^2},\quad \varepsilon_1=1-\frac{4\pi\gamma}{\omega^2-\omega_0^2}-\frac{\omega_{0e}^2}{\omega^2}$$

We seek the Fourier components of the field and of the polarization factor in the form

$$H_\varphi^{(1)}=\int H_\varphi^{(1)\prime}e^{i(\varkappa\rho+\lambda_1 z)}\,d\mathbf{k}, \quad (27)$$

$$\mathbf{P}=\int(\mathbf{P}_1'e^{i(\varkappa\rho+\lambda_1 z)}+\mathbf{P}_2'e^{i(\varkappa\rho+\lambda_2 z)})\,d\mathbf{k}, \quad (28)$$

$$H_\varphi^{(2)}=\int H_\varphi^{(2)\prime}e^{i(\varkappa\rho+\lambda z)}\,d\mathbf{k}, \quad (29)$$

where $\kappa^2+\lambda^2=\omega^2/c^2$.

From (26) and

$$(\omega_0^2-\omega^2)\mathbf{P}-\alpha_1\Delta\mathbf{P}-\alpha_2\operatorname{grad}\operatorname{div}\mathbf{P}=\frac{\gamma}{\varepsilon'}\left(\frac{ic}{\omega}\operatorname{rot}\mathbf{H}-4\pi\mathbf{P}\right) \quad (30)$$

we obtain

$$\lambda_1^2+\varkappa^2=\frac{\omega^2}{c^2}\varepsilon_1,\quad \lambda_2^2+\varkappa^2=\frac{\omega^2-\omega_0^2}{\alpha}\varepsilon_1. \quad (31)$$

We use in the solution, besides the ordinary boundary conditions, also the condition[3,7]

$$P_z|_{z=0}=0. \quad (32)$$

By determining the field amplitudes from (27), (29), and (30) and the boundary conditions, we get

$$H_\varphi^{(2)}(R,\theta,t)=\frac{ee^{-3\pi i/4}}{\pi v(2\pi R\sin\theta)^{1/2}}\int I(\omega)e^{-i\omega t}\,d\omega, \quad (33)$$

$$I(\omega)=\int_0^\infty F(\varkappa)e^{f(\varkappa)R}\,d\varkappa,\quad F(\varkappa)=\frac{\omega}{c}\varkappa^{3/2}\frac{\eta(\varkappa)}{\zeta(\varkappa)}, \quad (34)$$

$$\eta(\varkappa)=\frac{\varepsilon-1}{\mu\mu_0\varepsilon}\left\langle\left\{\left[\left(k^2-\frac{\omega^2}{c^2}(\varepsilon_1+1)\right)\varepsilon'+\beta\frac{\omega}{c}\varepsilon_1\lambda_1\right]\gamma_1\right.\right.$$
$$\left.-4\pi\gamma\lambda_1\beta\frac{\omega}{c}\varepsilon_1\right\}[(\omega_0^2-\omega^2)\varepsilon_1$$
$$\left.+\varkappa^2\varepsilon'\alpha]+4\pi\gamma\alpha\beta\varkappa^2\lambda_2\frac{\omega}{c}\varepsilon_1\varepsilon'\right\rangle, \quad (35)$$

*rot ≡ curl.

$$\zeta(\varkappa) = [(\varepsilon_1\lambda - \lambda_1)\gamma_1 + 4\pi\gamma\lambda_1][(\omega_0^2 - \omega^2)\varepsilon_1 + \varkappa^2\varepsilon'\alpha] - 4\pi\gamma\varkappa^2\varepsilon'\alpha\lambda_2, \quad (36)$$

where ε is the dielectric constant with allowance for spatial dispersion,

$$\gamma_1 = \varepsilon_1(\omega_0^2 - \omega^2 + \omega^2\alpha_1/c^2),$$
$$k^2 = \varkappa^2 + \omega^2/v^2, \ \mu_0 = k^2 + \omega^2/c^2,$$
$$\mu = k^2 + \omega^2\varepsilon/c^2, \quad f(\varkappa) = i\varkappa\sin\theta + i\lambda\cos\theta,$$

R is the distance from the point of emergence of the charge from the medium to the point of observation, and θ is the angle between R and the z axis.

The radiation into a unit solid angle over the entire time of flight of the particle is determined by the formula

$$\frac{dW}{d\Omega} = \frac{cR^2}{4\pi}\int_{-\infty}^{\infty} |H_\varphi^{(2)}|^2\, dt. \quad (37)$$

It has been shown in several papers[5, 7] that at large distances from the point of emergence of the particle from the medium the value of $H_\varphi^{(2)}$ is determined by integrating (33) over the steepest descent through the saddle point $\kappa = \omega \sin\theta/c$ without account of the closeness of the poles to the saddle point, while the expressions

$$\operatorname{Re}\mu(\omega, \sin\theta^\perp) = 0, \qquad \operatorname{Re}\varepsilon(\omega\sin\theta^\parallel) = 0 \quad (38)$$

determine the directions of the maximum intensity of the Cerenkov radiation produced by the transverse and longitudinal waves emerging to the vacuum. From (38) we obtain for the directions of maximum radiation from the supplementary waves

$$\beta^2\sin^2\theta^\perp = \frac{v^2}{\alpha_1}\left(1 - \frac{\omega_0^2}{\omega^2} - \frac{\alpha_1}{c^2}\frac{\omega_{0e}^2}{\omega^2}\right) - 1,$$
$$\beta^2\sin^2\theta^\parallel = \frac{v^2}{\alpha}\left[1 - \frac{\omega_0^2}{\omega^2} - \frac{4\pi\gamma}{\omega^2}\left(1 - \frac{\omega_{0e}^2}{\omega^2}\right)^{-1}\right] - 1. \quad (39)$$

At small distances from the point of emergence (distances of the order of several wavelengths) the residues in (33) with respect to the poles $\mu = 0$ and $\varepsilon = 0$ will make the contributions to the radiation from the transverse and longitudinal waves respectively.[6, 7]

The question of the radiation emerging to the vacuum from a dielectric or a plasma, with allowance for spatial dispersion, was dealt with in several papers,[7, 8] and we therefore note only the features distinguishing the radiation generated by coupled supplementary waves. As indicated above, for macroscopic waves to exist in a medium, it is necessary to satisfy the condition (16). Therefore it follows from the dispersion law (7) for the supplementary transverse waves that in a dielectric ($\omega_{0e} = 0$) the condition (16) with $c^2\alpha_1 \gg 1$ is satisfied only for frequencies close to the resonant frequency ω_0, where the attenuation begins to manifest itself strongly, so that the supplementary wave is difficult to observe.

It is seen for (7) that in a semiconductor with

$$\alpha_1\omega_{0e}^2/c^2 > \omega_0^2 \quad (\text{for } \alpha_1 < 0, \ c^2\omega_0^2/|\alpha_1| > \omega_{0e}^2 > \omega_0^2) \quad (40)$$

Eq. (16) can be satisfied at frequencies that are much farther from exciton resonance, and, in addition, it follows from (40) that by changing the carrier density it is possible to change the width of the spectrum of the coupled transverse waves.

Radiation from longitudinal coupled waves is determined by the pole at $\varepsilon = 0$, while the radiation intervals at the corresponding parameters are determined by expressions (18), (19), and (20). We see from (18) that for semiconductors with parameters satisfying the condition $\omega_{0e}^2 < \omega_0^2 < 4\pi\gamma$, the radiation spectrum lies in a narrow frequency interval, the width of which can be decreased or increased by increasing or decreasing the ratio $4\pi\gamma/\omega_{0e}^2$.

3. RADIATION FROM A PLATE INTO VACUUM

We know that to observe a wave in vacuum it is necessary to satisfy simultaneously the Cerenkov-radiation condition and the condition for emergence of the wave from the medium:[6]

$$c^2/v^2 \leqslant n^2 \leqslant c^2/v^2 + \sin^2\theta \quad (41)$$

and in addition, the flux of the radiative energy whose direction coincides with that of the group velocity must be directed into the vacuum. Since the longitudinal component of the wave vector coincides with the particle-velocity direction, it is clear that the waves radiated into vacuum will have negative group velocity when the particle enters the semiconductor and positive when it leaves the semiconductor.

By determining from (4) the sections with negative group velocity and $n^2 > 0$ we find that when charged particles move from the vacuum into the semiconductor only radiation due to transverse waves will be observed in the vacuum: from the fundamental wave with dispersion law (5) when $\alpha_1 > 0$ and $\omega_3 < \omega_2$ in the frequency interval $\omega_3 < \omega < \omega_2$ (no radiation will emerge into the vacuum when $\omega_2 < \omega_3$), and from the longitudinal wave with dispersion (7) when $\alpha_1 < 0$ and $|\alpha_1|\omega_{0e}^2/c^2 < \omega_2$ in the interval $\omega^2 < \omega_0^2 - |\alpha_1|\omega_{0e}^2/c^2$.

When the particle moves from the semiconductor to the vacuum, we can observe in the latter Cerenkov radiation which is transformed from either transverse or longitudinal coupled waves. When $\alpha_1 > 0$ the fundamental wave will produce radiation in vacuum for $\omega_3 < \omega_2$ in the interval $\omega > \omega_1$ and for $\omega_2 < \omega_3$ in the intervals $\omega_2 < \omega < \omega_3$, and $\omega > \omega_1$. When $\alpha_1 < 0$ the radiation will be in the intervals (10) and (11), and the radiation from the supplementary wave for $\alpha_1 > 0$ will be in the interval $\omega^2 > \omega_0^2 + \alpha_1\omega_{0e}^2/c^2$. The radiation from the coupled longitudinal wave (14) will emerge only when $\alpha > 0$ in the intervals $\omega_2 < \omega < \omega_{0e}$ and $\omega > \omega_1$.

We conclude from these results that when a charged particle passes through a semiconductor plate, the direction of the Cerenkov radiation observed in vacuum and produced by the supplementary transverse wave emerging from the plate depends on the sign of the exciton effective mass. The Cerenkov radiation from the longitudinal wave is directed along the particle motion and can be observed in vacuum only when the longitudinal exciton has positive effective mass.

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