SYMMETRY IN THE NONLINEAR THEORY OF THE SPINOR FIELD

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Some restrictions are proposed to remove the arbitrariness in the choice of the nonlinear term in the nonlinear quantum theory of the spinor field. The quantization of the spinor field is based on the invariance of the Lagrangian with respect to strong (or Schwinger) space-time reversal (CPT theorem), strong (or Schwinger) time reversal, and charge conjugation.

It was shown in [1] that the lowest order interaction of the spinor field with itself which leads to nonlinear equations with solutions corresponding to the two signs of the energy is the six-fermion interaction. Thus, for example, the equation

$$\left[i\gamma^\mu \frac{\partial}{\partial x^\mu} - m + g(\bar{\psi}\psi)^{n-1}\right]\psi = 0 \qquad (1)$$

has solutions corresponding to the two signs of the energy only if n is odd. Only for odd n is Eq. (1) invariant under time reversal[1)] (unitary transformation):[2]

$$x \to x, \quad t \to -t,$$
$$\psi(x,t) \to \rho_2\psi(x,-t), \quad \bar{\psi}(x,t) \to -\bar{\psi}(x,-t)\rho_2^{-1} \qquad (2)$$

$(\rho_2 = \gamma_1\gamma_2\gamma_3)$.

Taking into account that any relativistic theory admits solutions corresponding to the two signs of the energy, we consider the restrictions which lead to nonlinear equations having two types of solutions ($\psi^{(+)}$ and $\psi^{(-)}$). To this end we consider the invariance of the interaction of the spinor field under strong reversal (s.r.) of space-time,[4, 5] i.e., the transformation

$$x \to -x, \quad t \to -t,$$
$$\psi(x,t) \to \gamma_5\psi(-x,-t), \quad \bar{\psi}(x,t) \to -\bar{\psi}(-x,-t)\gamma_5,$$

effected by reversing the order of all factors in the expressions for the operators with account of the commutation rules. If the interaction of the spinor field with itself is of the four-fermion type $(\bar{\psi}O\psi)(\bar{\psi}O\psi)$, it is (in contrast to the universal electromagnetic and Yukawa interactions[4, 5]) invariant under s. r. of space-time irrespective of whether it is antisymmetric, $(\bar{\psi}O\psi - \psi O^T\bar{\psi}) \times (\bar{\psi}O\psi - \psi O^T\bar{\psi})$ (the spinor fields $\bar{\psi}$ and ψ anticommute), or symmetric, $(\bar{\psi}O\psi + \psi O^T\bar{\psi})(\bar{\psi}O\psi + \psi O^T\bar{\psi})$ (the spinor fields $\bar{\psi}$ and ψ commute); that is, the connection between spin and statistics does not follow from the invariance of the four-fermion interaction under s. r. of space-time.[2)] If the interaction is of the six-fermion type, it is invariant under s. r. of space-time only if the quantization is done with the help of anticommutators.

Thus it is sufficient to require that the invariance of the interaction of the spinor field under s. r. of space-time imply the connection between spin and statistics, and we are led to nonlinear equations which admit solutions corresponding to the two signs of the energy; that is, in this case the interaction with the lowest order in the spinor field is not the four-fermion, but the six-fermion interaction. If we further require that the currents in the interaction of the spinor field with itself have identical form and that the interaction be invariant under weak time reversal, then we arrive uniquely at Eq. (1) with odd n, or to the simplest equation

$$\left[i\gamma^\mu \frac{\partial}{\partial x^\mu} - m + g(\bar{\psi}\psi)^2\right]\psi = 0.$$

[1] G. K. Chepurnykh, Vestnik MGU (Moscow State Univ.), ser. Phys. and Astron. **5**, 54 (1964).

[1)]The invariance under the transformation (2) is evidently connected with the existence of two types of solutions ($\psi^{(+)}$ and $\psi^{(-)}$), since it can be shown that the solutions (43.18) of [3], corresponding to negative energy, can be obtained by multiplying the solutions (43.17), corresponding to positive energy, by ρ_2.

[2)]The hypothesis of a universal weak four-fermion interaction is not contradicted by any experimental fact, but the weak interaction is not invariant under strong time reversal (T_s) and charge conjugation and hence we cannot in these cases determine whether the spinor field is to be quantized with the help of anticommutators or commutators.

[2] J. Schwinger, Phys. Rev. **82**, 914 (1951).

[3] L. I. Schiff, Quantum Mechanics, McGraw-Hill, N. Y., (1955), Russ. Transl. IIL, 1959.

[4] R. E. Marshak and E. C. G. Sudarshan, Introduction to Elementary Particle Physics, John Wiley & Sons, Inc., N. Y., 1961.

[5] P. E. Matthews, Relativistic Quantum Theory of Elementary Particle Interactions, Rochester Lecture Notes, U. S. Atomic Energy Commission Document NYO-2097, 1957.

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