DOUBLE NN SCATTERING WITH A POLARIZED BEAM AND A POLARIZED TARGET

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Experiments are discussed on measuring nucleon polarization produced as a result of the scattering of a polarized beam by a polarized proton target. In such experiments it becomes possible for the first time to determine the components of the third rank polarization tensor M_{ikq}. The correlation of polarizations is discussed also in the case when one of the initial particles is polarized. Relativistic formulas for the quantities being measured are obtained. The problem of reconstructing the scattering matrix is discussed.

1. INTRODUCTION

THE use of polarized proton targets considerably extends the possibilities of investigations in the physics of elementary particles. In experiments utilizing a polarized target the parities of particles and of resonances can be uniquely determined [1-6], the parameters R and A of meson-nucleon scattering can be measured [7,1], the angular interval for the measurement of polarization can be appreciably extended [8,1] etc.

The use of polarized targets essentially simplifies the measurements of the polarization characteristics of nucleon-nucleon scattering. Measurement of the scattering cross section for nucleons on a polarized proton target enables us to determine the polarization of the recoil protons. The tensor C_{ik} describing the correlation of polarizations can be obtained from the cross section for the scattering of a polarized beam by a polarized target. A measurement of nucleon polarization in the scattering of an unpolarized beam by a polarized target enables us to determine the depolarization tensor D_{ik} and the polarization transfer tensor K_{ik}. Here we shall discuss possible experiments on the measurement of nucleon polarization arising as a result of the scattering of a polarized beam by a polarized target. In these experiments it becomes possible for the first time to determine the components of the third rank polarization tensor M_{ikq}. Measurement of such complicated polarization characteristics will help to remove the remaining ambiguities of phase analysis. These measurements will be particularly important in that energy region where the cross sections for inelastic processes are comparable to the cross section for elastic scattering. In order to carry out a phase analysis in this region it is necessary to utilize the results of the phenomenological analysis of inelastic processes. Moreover, in the high energy domain there is a considerable increase in the number of states the interaction in which must be taken into account. As is well known, the amplitude for elastic scattering can also be determined directly from the experimental data on elastic scattering without making use of information from inelastic processes [8]. However, this method requires a large number of polarization measurements.

2. THIRD RANK POLARIZATION TENSOR

We consider the polarization of the final particle arising as the result of the scattering of a polarized beam by a polarized target. The subscript 1 in the spin matrices will refer to the incident and the scattered particles, and the subscript 2 to the target particles and the recoil particles. In the case of identical particles we shall define the scattered particle to be the one emerging into the angular integral $0 \leq \theta \leq \pi/2$ in the c.m.s.

In the rest system obtained by a Lorentz transformation from the c.m.s. the polarization of the scattered particle is equal to

$$P_{1i}' = \frac{\mathrm{Sp}\, \sigma_{1i} M^1/_2(1+(\boldsymbol{\sigma}_1\mathbf{P}_1))^1/_2(1+(\boldsymbol{\sigma}_2\mathbf{P}_2))M^+}{\mathrm{Sp}\, M^1/_2(1+(\boldsymbol{\sigma}_1\mathbf{P}_1))^1/_2(1+(\boldsymbol{\sigma}_2\mathbf{P}_2))M^+}, \quad (1)$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are the polarizations of the initial particles in the corresponding rest systems, while M is the scattering matrix in the c.m.s.

As is well known, the nucleon-nucleon scattering matrix $M(\mathbf{k}', \mathbf{k})$ is characterized by five complex functions of the angle and the energy and has the following general form [9,10]:

$$M(\mathbf{k}', \mathbf{k}) = (u+v) + (u-v)(\sigma_1\mathbf{n})(\sigma_2\mathbf{n}) + c[(\sigma_1\mathbf{n}) + (\sigma_2\mathbf{n})] + (g-h)(\sigma_1\mathbf{m})(\sigma_2\mathbf{m}) + (g+h)(\sigma_1\mathbf{l})(\sigma_2\mathbf{l}). \quad (2)$$

Here $\mathbf{k}$ and $\mathbf{k}'$ are unit vectors in the directions of the initial and final momenta in the c.m.s., while $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{n}$ form an orthonormal triad of vectors:

$$\mathbf{l} = \frac{\mathbf{k}'+\mathbf{k}}{|\mathbf{k}'+\mathbf{k}|}, \quad \mathbf{m} = \frac{\mathbf{k}'-\mathbf{k}}{|\mathbf{k}'-\mathbf{k}|}, \quad \mathbf{n} = [\mathbf{lm}] = \frac{[\mathbf{kk}']}{|[\mathbf{kk}']|}. \quad (3)^*$$

The matrix (2) satisfies the requirements of invariance under rotations, reflections and time reversal, and is also symmetric with respect to the simultaneous interchange of initial and final spin indices. The latter condition is a consequence of identity in the case of pp scattering, while for np-scattering it is only true within the framework of isotopic invariance.

As can be seen from expression (1), the polarization $P_{1i'}$ is determined by the following tensors:

$$P_i^0 = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}MM^+ = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{2i}MM^+ = \frac{1}{4\sigma_0}\operatorname{Sp}M\sigma_{1i}M^+ = \frac{1}{4\sigma_0}\operatorname{Sp}M\sigma_{2i}M^+ = A_i, \quad (4)$$

$$D_{ik} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}M\sigma_{1k}M^+, \quad (5)$$

$$K_{ik} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}M\sigma_{2k}M^+ = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{2i}M\sigma_{1k}M^+, \quad (6)$$

$$P_{ik} = \frac{1}{4\sigma_0}\operatorname{Sp}M\sigma_{1i}\sigma_{2k}M^+, \quad (7)$$

$$M_{ikq} = \frac{1}{4\sigma_0}\operatorname{Sp}\sigma_{1i}M\sigma_{1k}\sigma_{2q}M^+. \quad (8)$$

In these formulas σ_0 is the differential cross section for the scattering of an unpolarized beam by an unpolarized target in the c.m.s., while P_i^0 is the polarization occurring in the scattering of unpolarized nucleons. In virtue of the principles of invariance the polarization P_i^0 coincides with the asymmetry A_i arising in the scattering of a polarized beam by an unpolarized target (or of an unpolarized beam by a polarized target)[9,10].

The depolarization tensor D_{ik} and the polarization transfer tensor K_{ik} can be determined from experiments on the measurement of polarization in the case when either the incident beam or the target is polarized. By studying the cross section for the scattering of a polarized beam by a polarized target it is possible to determine the components of the tensor P_{ik}. We note that the tensor P_{ik} is related to the tensor describing the correlation of polarizations

$$C_{ik} = (4\sigma_0)^{-1}\operatorname{Sp}\sigma_{1i}\sigma_{2k}MM^+$$

by the relation [1,8]

$$P_{ik}(\mathbf{k}', \mathbf{k}) = C_{ik}(-\mathbf{k}, -\mathbf{k}'), \quad (9)$$

which follows from the invariance with respect to time reversal. Relation (9) means that the components of the tensor P_{ik} can be determined also from experiments on the measurement of the correlation of nucleon polarization arising in the scattering of unpolarized particles.

A new quantity measured in the experiments under consideration is the third rank pseudotensor M_{ikq}. We investigate the structure of this tensor. From considerations of invariance with respect to rotations and reflections we obtain

$$\begin{aligned} M_{ikq} = {} & M_{lln}l_il_kn_q + M_{lmn}l_im_kn_q + M_{lnl}l_in_kl_q \\ & + M_{lnm}l_in_km_q + M_{mln}m_il_kn_q + M_{mmn}m_im_kn_q \\ & + M_{mnm}m_in_km_q + M_{mnl}m_in_kl_q \\ & + M_{nml}n_im_kl_q + M_{nmm}n_im_km_q + M_{nlm}n_il_km_q \\ & + M_{nll}n_il_kl_q + M_{nnn}n_in_kn_q. \end{aligned} \quad (10)$$

Not all the components of this tensor are independent. In order to verify this we utilize the relation [1,6]

$$M(\mathbf{k}', \mathbf{k}) = (\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M(\mathbf{k}', \mathbf{k})(\sigma_1\mathbf{n})(\sigma_2\mathbf{n}), \quad (11)$$

which follows from the requirements of invariance with respect to reflections in the scattering plane. We show first of all that the component M_{nnn} coincides with the polarization $P^0 = (\mathbf{P}^0\mathbf{n})$ which arises in the scattering of unpolarized particles. Indeed, we have

$$M_{nnn} = \frac{1}{4\sigma_0}\operatorname{Sp}(\sigma_1\mathbf{n})M(\sigma_1\mathbf{n})(\sigma_2\mathbf{n})M^+ = \frac{1}{4\sigma_0}\operatorname{Sp}(\sigma_2\mathbf{n})MM^+ = P^0. \quad (12)$$

Further, utilizing (11) and the symmetry requirements we obtain

$$M_{nmm} = -M_{nll}, \quad M_{mnm} = M_{lnl}, \quad M_{mmn} = M_{lln}. \quad (13)$$

Thus, the tensor M_{ikq} is characterized by nine components. In subsequent discussion we shall use half the sums and half the differences of components with different indices:

$$M_{nlm}^{\pm} = {}^1/_2(M_{nlm} \pm M_{nml}), \quad M_{lnm}^{\pm} = {}^1/_2(M_{lnm} \pm M_{mnl}), \quad M_{lmn}^{\pm} = {}^1/_2(M_{lmn} \pm M_{mln}). \quad (14)$$

*$[\mathbf{kk}'] \equiv \mathbf{k} \times \mathbf{k}'$.

We present the expressions for the components of the tensor M_{ikq} and for the cross section σ_0 calculated with the aid of the matrix (2):

$$\sigma_0 M_{nll} = 4\,\mathrm{Re}\, c^* h, \qquad \sigma_0 M_{mmn} = 4\,\mathrm{Re}\, c^* v,$$
$$\sigma_0 M_{mnm} = 4\,\mathrm{Re}\, c^* g, \qquad \sigma_0 M_{nnn} = \sigma_0 P^0 = 4\,\mathrm{Re}\, c^* u,$$
$$\sigma_0 M^+_{nlm} = 4\,\mathrm{Im}\, uh^*, \qquad \sigma_0 M^-_{nlm} = 4\,\mathrm{Im}\, gv^*,$$
$$\sigma_0 M^+_{lnm} = 4\,\mathrm{Im}\, hv^*, \qquad \sigma_0 M^-_{lnm} = 4\,\mathrm{Im}\, ug^*,$$
$$\sigma_0 M^+_{lmn} = 4\,\mathrm{Im}\, hg^*, \qquad \sigma_0 M^-_{lmn} = 4\,\mathrm{Im}\, uv^*,$$
$$\sigma_0 = 2(|u|^2 + |v|^2 + |c|^2 + |g|^2 + |h|^2). \qquad (15)$$

It can be shown that there exist algebraic relations between these quantities. In order to do this we utilize the identity

$$x\,\mathrm{Im}\, yz^* + y\,\mathrm{Im}\, zx^* + z\,\mathrm{Im}\, xy^* = 0, \qquad (16)$$

which is valid for any three complex numbers x, y, z. As a result of the invariance of observable quantities with respect to multiplication of the scattering matrix by a phase factor, the amplitude c in the expansion (2) can be assumed to be real. The remaining four complex functions u, v, g and h satisfy the relations

$$u\,\mathrm{Im}\, gh^* + g\,\mathrm{Im}\, hu^* + h\,\mathrm{Im}\, ug^* = 0,$$
$$v\,\mathrm{Im}\, gh^* + g\,\mathrm{Im}\, hv^* + h\,\mathrm{Im}\, vg^* = 0. \qquad (17)$$

From here with the aid of expressions (15) we obtain three relations between the components of the tensor under consideration:

$$-M^-_{lnm} M_{nll} + M^+_{lmn} P^0 + M^+_{nlm} M_{mnm} = 0,$$
$$M^-_{nlm} M_{nll} + M^+_{lmn} M_{mmn} - M^+_{lnm} M_{mnm} = 0,$$
$$M^+_{lmn} M^-_{lmn} - M^+_{lnm} M^-_{lnm} + M^+_{nlm} M^-_{nlm} = 0. \qquad (18)$$

It is evident that relations (18) enable us to express three components in terms of the remaining ones.

In addition to the tensor M_{ikq} we can construct the following three third rank polarization tensors [1]:

$$N_{ikq} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\, \sigma_{2i} M \sigma_{1k} \sigma_{2q} M^+, \qquad (19)$$
$$C_{ikq} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\, \sigma_{1i} \sigma_{2k} M \sigma_{1q} M^+, \qquad (20)$$
$$P_{ikq} = \frac{1}{4\sigma_0}\,\mathrm{Sp}\, \sigma_{1i} \sigma_{2k} M \sigma_{2q} M^+. \qquad (21)$$

The tensor N_{ikq} appears in the expression for the polarization of the recoil particle in the case when the beam and the target are polarized, while C_{ikq} (P_{ikq}) appears in the expression for the correlation of polarizations in the case of the scattering of a polarized beam by an unpolarized target (or of an unpolarized beam by a polarized target). From invariance with respect to time reversal and $\sigma_1 \rightleftarrows \sigma_2$ symmetry it can be easily seen that

$$N_{ikq}(\mathbf{k}', \mathbf{k}) = M_{iqk}(\mathbf{k}', \mathbf{k}), \qquad (22)$$
$$C_{ikq}(\mathbf{k}', \mathbf{k}) = P_{kiq}(\mathbf{k}', \mathbf{k}) = -M_{qik}(-\mathbf{k}, -\mathbf{k}'). \qquad (23)$$

Thus, the components of the tensor M_{ikq} also define all the other third rank tensors.

As is well known, the measurement of nucleon polarization in the energy range from 20 to 100 MeV is difficult because no target analyzers with sufficient analyzing power are available. Relations (22) and (23) can be utilized to determine the components of M_{ikq} over the whole angular interval. For example, in scattering through large angles it can turn out that a measurement of N_{ikq}, i.e., of the polarization of recoil particles, is simpler than the measurement of M_{ikq}.

In concluding this section we make two remarks. The first refers to proton-proton scattering. As a result of the Pauli exclusion principle, the tensor M_{ikq} satisfies in this case the relation

$$M_{ikq}(\mathbf{k}', \mathbf{k}) = M_{iqk}(-\mathbf{k}', \mathbf{k}). \qquad (24)$$

From this we obtain for the components of the tensor

$$M_{mnl}(\pi - \theta) = -M_{lmn}(\theta), \qquad M_{mln}(\pi - \theta) = -M_{lnm}(\theta),$$
$$M_{mnm}(\pi - \theta) = -M_{mmn}(\theta), \qquad M_{nll}(\pi - \theta) = M_{nll}(\theta),$$
$$M_{nml}(\pi - \theta) = -M_{nml}(\theta), \qquad M_{nlm}(\pi - \theta) = -M_{nlm}(\theta), \qquad (24')$$

where θ is the scattering angle in the c.m.s. Thus, in the case of identical particles it is sufficient to measure the polarization in the range $0 \le \theta \le \pi/2$ for all possible orientations of the polarizations of the initial particles.

The second remark refers to the low energy domain, where it is sufficient to take into account the interaction only in the S-state. The scattering amplitude in this case has the form

$$M = \alpha + \beta(\sigma_1\sigma_2), \qquad (25)$$

where the coefficients α and β are related to the scattering amplitudes in the $^3S_1$- and the $^1S_0$-states b_T and b_S by the following relations:

$$\alpha = {}^1\!/_4(3b_T + b_S), \qquad \beta = {}^1\!/_4(b_T - b_S). \qquad (26)$$

The tensor M_{ikq} is equal to

$$M_{ikq} = M_{nlm}\varepsilon_{ikq}, \qquad (27)$$
$$\sigma_0 M_{nlm} = 2\,\mathrm{Im}\, \alpha^*\beta. \qquad (28)$$

In the case of pp-scattering $\alpha = -\beta$ and $M_{ikq} = 0$. Restricting ourselves to the main term in the expansion with respect to the momentum p we obtain for the scattering of neutrons by protons

$$\sigma_0 M_{nlm} = {}^1\!/_2 p a_S a_T (a_S - a_T). \quad (29)$$

Here a_S and a_T are the singlet and the triplet scattering lengths.

3. MEASURED QUANTITIES

For the determination of quantities measured experimentally we introduce in the laboratory system two orthonormal vector triads

$$\mathbf{k}_l', \quad \mathbf{n}_l = [\mathbf{k}_l\mathbf{k}_l'] / |[\mathbf{k}_l\mathbf{k}_l']|, \quad \mathbf{s}_l' = [\mathbf{n}_l\mathbf{k}_l']; \quad (30)$$

$$\mathbf{k}_l, \quad \mathbf{n}_l. \quad \mathbf{s}_l = [\mathbf{n}_l\mathbf{k}_l], \quad (31)$$

where $\mathbf{k}_l'$ ($\mathbf{k}_l$) is the unit vector in the direction of the momentum of the scattered (incident) nucleon. It is obvious $\mathbf{k}_l = \mathbf{k}$, $\mathbf{n}_l = \mathbf{n}$. The polarization of the scattered particles can be naturally characterized by its components in the system (30), while the initial polarizations can be naturally characterized by the components in the system (31). However, we should keep in mind that in the relativistic domain in comparing the results of the calculation of the average values of the spin operators with the results of polarization measurements in addition to the usual kinematic corrections it is necessary to take into account the characteristic relativistic rotation [11-14]. The component of the polarization vector of the scattered particle along the direction $\mathbf{a}_l$ measured in the l.s. taking this effect into account is equal to

$$(\langle\sigma_1\rangle_l \mathbf{a}_l) = \sigma^{-1}\,\mathrm{Sp}\,(\sigma_1(\mathbf{a}_l)_R) M\, {}^1\!/_2(1 + (\sigma_1\mathbf{P}_1)) \times {}^1\!/_2(1 + (\sigma_2\mathbf{P}_2)) M^+,$$

$$\sigma = \mathrm{Sp}\, M\, {}^1\!/_2(1 + (\sigma_1\mathbf{P}_1))\, {}^1\!/_2(1 + (\sigma_2\mathbf{P}_2)) M^+. \quad (32)$$

Here σ is the differential scattering cross section, while

$$(\mathbf{a}_l)_R = R_n\,(\Omega_1)\mathbf{a}_l. \quad (33)$$

In (33) $R_n(\Omega_1)$ is the operator for a rotation about the normal n through an angle $\Omega_1 = \theta - 2\theta_l$ (θ_l is the scattering angle in the l.s.).

Utilizing considerations of invariance we obtain from (32) the following expressions for the components of the polarization of the scattered particle:

$$\sigma(\langle\sigma_1\rangle_l \mathbf{n}_l) = \sigma_0[P^0 + D_{nn}(\mathbf{P}_1\mathbf{n}_l) + K_{nn}(\mathbf{P}_2\mathbf{n}_l) + P^0(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{n}_l) + M_{nkk}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{k}_l) + M_{nks}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{s}_l) + M_{nsk}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{k}_l) + M_{nss}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{s}_l)], \quad (34)$$

$$\sigma(\langle\sigma_1\rangle_l \mathbf{k}_l') = \sigma_0[D_{k'k}(\mathbf{P}_1\mathbf{k}_l) + D_{k's}(\mathbf{P}_1\mathbf{s}_l) + K_{k'k}(\mathbf{P}_2\mathbf{k}_l) + K_{k's}(\mathbf{P}_2\mathbf{s}_l) + M_{k'kn}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{n}_l) + M_{k'sn}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{n}_l) + M_{k'nk}(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{k}_l) + M_{k'ns}(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{s}_l)], \quad (35)$$

$$\sigma(\langle\sigma_1\rangle_l \mathbf{s}_l') = \sigma_0[D_{s'k}(\mathbf{P}_1\mathbf{k}_l) + D_{s's}(\mathbf{P}_1\mathbf{s}_l) + K_{s'k}(\mathbf{P}_2\mathbf{k}_l) + K_{s's}(\mathbf{P}_2\mathbf{s}_l) + M_{s'kn}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{n}_l) + M_{s'sn}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{n}_l) + M_{s'nk}(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{k}_l) + M_{s'ns}(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{s}_l)]. \quad (36)$$

Here we have utilized the following notation:

$$D_{ab} = (\mathbf{a}_l)_{Ri} D_{ik} (\mathbf{b}_l)_k, \quad K_{ab} = (\mathbf{a}_l)_{Ri} K_{ik} (\mathbf{b}_l)_k,$$

$$M_{abc} = (\mathbf{a}_l)_{Ri} M_{ikq} (\mathbf{b}_l)_k (\mathbf{c}_l)_q. \quad (37)$$

We note that the well known Wolfenstein parameters for triple scattering [15] determined taking the relativistic rotation into account are given in the present notation by

$$D = D_{nn}, \quad R = D_{s's}, \quad A = D_{s'k},$$
$$R' = D_{k's}, \quad A' = D_{k'k}. \quad (38)$$

The differential cross sections for the scattering of a polarized beam by a polarized target can be written in the form

$$\sigma = \sigma_0[1 + P^0(\mathbf{P}_1\mathbf{n}_l) + P^0(\mathbf{P}_2\mathbf{n}_l) + P_{nn}(\mathbf{P}_1\mathbf{n}_l)(\mathbf{P}_2\mathbf{n}_l) + P_{kk}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{k}_l) + P_{ks}(\mathbf{P}_1\mathbf{k}_l)(\mathbf{P}_2\mathbf{s}_l) + P_{ks}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{k}_l) + P_{ss}(\mathbf{P}_1\mathbf{s}_l)(\mathbf{P}_2\mathbf{s}_l)],$$

$$P_{ab} = (\mathbf{a}_l)_i P_{ik} (\mathbf{b}_l)_k. \quad (39)$$

As can be seen from the formulas given above the nucleon polarization arising in the collision of a polarized beam with a polarized proton target is characterized in addition to the quantities determined in simpler polarization experiments also by the twelve quantities M_{abc}. These quantities are related by linear expressions to the nine components of the tensor M_{ikq} and it is therefore obvious that between them there must be three relations. Indeed, it can be shown that

$$M_{nkk} = -M_{nss}, \quad \frac{M_{s'sn} - M_{k'kn}}{M_{s'kn} + M_{k'sn}} = \mathrm{tg}\,\theta_l,$$

$$\frac{M_{s'ns} - M_{k'nk}}{M_{s'nk} + M_{k'ns}} = \mathrm{tg}\,\theta_l. \quad (40)^*$$

Below we give formulas relating the measured parameters M_{abc} to the components of the tensor M_{ikq}:

$$M_{nkk} = M_{nll}\cos\theta - M^+_{nlm}\sin\theta,$$

$$M_{nks} = M_{nll}\sin\theta + M^+_{nlm}\cos\theta + M^-_{nlm},$$

$$M_{nsk} = M_{nll}\sin\theta + M^+_{nlm}\cos\theta - M^-_{nlm},$$

*tg ≡ tan.

$$M_{k'kn} = M_{mmn}\cos\left(\frac{\theta}{2}+\alpha\right) - M^{+}_{lmn}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\sin\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{k'sn} = M_{mmn}\sin\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lmn}\cos\left(\frac{\theta}{2}-\alpha\right) + M^{-}_{lmn}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{k'nk} = M_{mnm}\cos\left(\frac{\theta}{2}+\alpha\right) - M^{+}_{lnm}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lnm}\sin\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{k'ns} = M_{mnm}\sin\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lnm}\cos\left(\frac{\theta}{2}-\alpha\right) + M^{-}_{lnm}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{s'kn} = -M_{mmn}\sin\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lmn}\cos\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{s'sn} = M_{mmn}\cos\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lmn}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\sin\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{s'nk} = -M_{mnm}\sin\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lnm}\cos\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lnm}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$M_{s'ns} = M_{mnm}\cos\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lnm}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lnm}\sin\left(\frac{\theta}{2}+\alpha\right). \quad (41)$$

The angle α is defined in the following manner:

$$\alpha = \theta/2 - \theta_l. \quad (42)$$

It is obvious that in order to obtain nonrelativistic relations one must set $\alpha = 0$. In order to measure the transverse components ($\langle\sigma_{1l}\rangle \mathbf{n}_l$) and ($\langle\sigma_{1l}\rangle \mathbf{s}_{l'}$) of the polarization it is necessary to choose the plane for the analyzing scattering to be orthogonal with respect to $\mathbf{n}_l$ and $\mathbf{s}_{l'}$. The longitudinal component ($\langle\sigma_{1l}\rangle \mathbf{k}_{l'}$) of the polarization can be measured if the nucleons prior to the analyzing scattering are made to go through a magnetic field directed along $\mathbf{n}_l$.

For example, in order to determine the parameter $M_{s'sn}$ it is necessary to scatter the beam with polarization $\mathbf{P}_1 = P_1\mathbf{s}_l$ on a target polarized orthogonally to the scattering plane ($\mathbf{P}_2 = P_2\mathbf{n}_l$). It follows from (34)—(36) and (39) that the polarization arising as a result of this is equal to

$$\langle\sigma_1\rangle_l = [\mathbf{k}_l'(D_{k's}P_1 + M_{k'sn}P_1P_2) + \mathbf{s}_l'(D_{s's}P_1 + M_{s'sn}P_1P_2) + \mathbf{n}_l(K_{nn}P_2 + P^0)] / (1 + P^0P_2). \quad (43)$$

From this it follows that if the normal to the plane of the analyzing scattering is parallel to $\mathbf{s}_{l'}$, then the left-right asymmetry is equal to

$$e_{s'}^{LR} = P_a \frac{P_1(R + M_{s'sn}P_2)}{1 + P^0P_2}, \quad (44)$$

where P_a is the analyzing power of the target-analyzer. If the parameters R and P^0 are known, then from (44) one can obtain $M_{s'sn}$. The remaining quantities M_{abc} can be found in an analogous manner.

The components of the tensor M_{ikq} can also be determined by means of measuring the polarization of the recoil particle arising in the scattering of a polarized beam by a polarized target. Experimentally one measures the components of the polarization vector of the recoil particle in the system associated with the momentum of the recoil particle in the l.s.:

$$\mathbf{n}_l, \mathbf{k}_l'', \mathbf{s}_l'' = [\mathbf{n}_l\mathbf{k}_l'']. \quad (45)$$

Here $\mathbf{k}_l''$ is the unit vector in the direction of the momentum of the recoil particle. Taking the relativistic rotation into account we obtain in analogy with (34)—(36) that from the measurements of the polarization of the recoil particle the following quantities can be determined:

$$N_{nbc} = (\mathbf{n}_l)_i N_{ikq}(\mathbf{b}_l)_k(\mathbf{c}_l)_q,$$
$$N_{k''bc} = (\mathbf{k}_l'')_{Ri} N_{ikq}(\mathbf{b}_l)_k(\mathbf{c}_l)_q,$$
$$N_{s''bc} = (\mathbf{s}_l'')_{Ri} N_{ikq}(\mathbf{b}_l)_k(\mathbf{c}_l)_q. \quad (46)$$

Here $\mathbf{b}_l$ and $\mathbf{c}_l$ are vectors from the triad $\mathbf{n}_l$, $\mathbf{k}_l$, $\mathbf{s}_l$, while $(\mathbf{k}_l'')_R$ and $(\mathbf{s}_l'')_R$ are equal to

$$(\mathbf{k}_l'')_R = R_{\mathbf{n}}(\Omega_2)\mathbf{k}_l'', \qquad (\mathbf{s}_l'')_R = R_{\mathbf{n}}(\Omega_2)\mathbf{s}_l'', \quad (47)$$

where $R_{\mathbf{n}}(\Omega_2)$ is the operator for the rotation about the normal $\mathbf{n}$ by an angle

$$\Omega_2 = 2\Phi_l - \Phi. \quad (48)$$

In this relation Φ_l is the recoil angle in the l.s., while Φ is the recoil angle in the c.m.s. Formulas for relating the quantities (46) determined experimentally with the components of M_{ikq} can be found with the aid of formula (41). It can be easily shown that

$$N_{nbc} = M_{ncb}, \qquad N_{k''bc} = -M_{s'cb} \quad (\alpha \to -\alpha'),$$
$$N_{s''bc} = M_{k'cb} \quad (\alpha \to -\alpha'), \quad (49)$$

where ($\alpha \to -\alpha'$) means that in the corresponding expressions in (41) the angle α must be replaced by the angle $-\alpha'$, with

$$\alpha' = \Phi/2 - \Phi_l. \quad (50)$$

In the non-relativistic limit $\alpha' = 0$.

The components of the tensor under consideration can also be determined from measurements of the correlation of polarizations arising as a result of the scattering of a polarized beam by an unpolarized target.

Below we reproduce expressions for the measured quantities[1]:

$$C_{nk''k} = M_{mnm}\sin\left(\frac{\theta}{2}-\alpha'\right) + M^{+}_{lnm}\cos\left(\frac{\theta}{2}+\alpha'\right) + M^{-}_{lnm}\cos\left(\frac{\theta}{2}-\alpha'\right),$$

$$C_{nk''s} = -M_{mnm}\cos\left(\frac{\theta}{2}-\alpha'\right) + M^{+}_{lnm}\sin\left(\frac{\theta}{2}+\alpha'\right) + M^{-}_{lnm}\sin\left(\frac{\theta}{2}-\alpha'\right),$$

$$C_{ns''k} = M_{mnm}\cos\left(\frac{\theta}{2}-\alpha'\right) + M^{+}_{lnm}\sin\left(\frac{\theta}{2}+\alpha'\right) - M^{-}_{lnm}\sin\left(\frac{\theta}{2}-\alpha'\right),$$

$$C_{ns''s} = M_{mnm}\sin\left(\frac{\theta}{2}-\alpha'\right) - M^{+}_{lnm}\cos\left(\frac{\theta}{2}+\alpha'\right) + M^{-}_{lnm}\cos\left(\frac{\theta}{2}-\alpha'\right),$$

$$C_{k'k''n} = M_{nll}\sin(\alpha-\alpha') + M^{+}_{nlm}\cos(\alpha-\alpha') + M^{-}_{nlm}\cos(\alpha+\alpha'),$$

$$C_{k's''n} = M_{nll}\cos(\alpha-\alpha') - M^{+}_{nlm}\sin(\alpha-\alpha') + M^{-}_{nlm}\sin(\alpha+\alpha'),$$

$$C_{s'k''n} = M_{nll}\cos(\alpha-\alpha') - M^{+}_{nlm}\sin(\alpha-\alpha') - M^{-}_{nlm}\sin(\alpha+\alpha'),$$

$$C_{s's''n} = -M_{nll}\sin(\alpha-\alpha') - M^{+}_{nlm}\cos(\alpha-\alpha') + M^{-}_{nlm}\cos(\alpha+\alpha'),$$

$$C_{k'nk} = M_{mmn}\cos\left(\frac{\theta}{2}+\alpha\right) + M^{+}_{lmn}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\sin\left(\frac{\theta}{2}+\alpha\right),$$

$$C_{k'ns} = M_{mmn}\sin\left(\frac{\theta}{2}+\alpha\right) - M^{+}_{lmn}\cos\left(\frac{\theta}{2}-\alpha\right) + M^{-}_{lmn}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$C_{s'nk} = -M_{mmn}\sin\left(\frac{\theta}{2}+\alpha\right) - M^{+}_{lmn}\cos\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\cos\left(\frac{\theta}{2}+\alpha\right),$$

$$C_{s'ns} = M_{mmn}\cos\left(\frac{\theta}{2}+\alpha\right) - M^{+}_{lmn}\sin\left(\frac{\theta}{2}-\alpha\right) - M^{-}_{lmn}\sin\left(\frac{\theta}{2}+\alpha\right). \tag{51}$$

The notation is obvious:

$$C_{k's''n} = (\mathbf{k}\,l')_{Ri}(\mathbf{s}\,l'')_{Rk}C_{ikq}(\mathbf{n}\,l)_q$$

etc. From equations (13) it follows that the following relations hold:

$$\frac{C_{ns''k}+C_{nk''s}}{C_{nk''k}-C_{ns''s}} = \operatorname{tg}\left(\alpha'+\frac{\theta}{2}\right),$$

$$\frac{C_{k's''n}-C_{s'k''n}}{C_{s's''n}+C_{k'k''n}} = \operatorname{tg}(\alpha+\alpha'),$$

$$\frac{C_{k'nk}-C_{s'ns}}{C_{s'nk}+C_{k'ns}} = \operatorname{tg}\left(\alpha-\frac{\theta}{2}\right). \tag{52}$$

Finally, information about the components of M_{ikq} can also be obtained in measuring the correlation of polarizations in the case when the target is polarized. The quantities measured in this case can be found from expressions (51) in the following manner:

$$P_{k'k''n} = -C_{s's''n}\ (\alpha \rightleftarrows -\alpha'), \quad P_{s'k''n} = C_{s'k''n}(\alpha \rightleftarrows -\alpha'),$$
$$P_{k's''n} = C_{k's''n}\ (\alpha \rightleftarrows -\alpha'), \quad P_{s'nb} = -C_{nk''b}\ (\alpha' \to -\alpha),$$
$$P_{k'nb} = C_{ns''b}\ (\alpha' \to -\alpha), \quad P_{ns''b} = C_{k'nb}\ (\alpha \to -\alpha'),$$
$$P_{s's''n} = -C_{k'k''n}\ (\alpha \rightleftarrows -\alpha'),$$
$$P_{nk''b} = -C_{s'nb}\ (\alpha \to -\alpha'). \tag{53}$$

The required replacement of angles is shown in brackets.

In conclusion, we consider the problem of the direct reconstruction of the amplitude for nucleon-nucleon scattering. As Schumacher and Bethe have shown[8], for the reconstruction of the amplitude it is sufficient to measure the cross section, the polarization, the depolarization tensor D_{ik}, and also certain components of C_{ik} and K_{ik} (a total of eleven quantities). In doing this the corresponding quantities must be measured with a high degree of accuracy. Utilizing relations (15) and Table II from the review article [1], we obtain[2]

$$\operatorname{Re} h = \frac{\sigma_0}{4c}M_{nll}, \quad \operatorname{Im} v = \frac{\sigma_0}{4c}D_{ml}, \quad \operatorname{Re} v = \frac{\sigma_0}{4c}M_{mmn},$$

$$\operatorname{Im} h = \frac{\sigma_0}{4c}C_{ml}, \quad \operatorname{Re} g = \frac{\sigma_0}{4c}M_{mnm}, \quad \operatorname{Im} g = \frac{\sigma_0}{4c}K_{ml},$$

$$\operatorname{Re} u = \frac{\sigma_0}{4c}P^0. \tag{54}$$

The amplitude c is taken to be real and positive. It is related to the observed quantities by relation (7.46) from [1]. With the aid of (54) other expressions for c can be obtained:

$$c^2 = \frac{\sigma_0(M^2_{mmn}+D^2_{ml})}{2(1+D_{nn}-K_{nn}-C_{nn})} = \frac{\sigma_0(M^2_{mnm}+K^2_{ml})}{2(1-D_{nn}+K_{nn}-C_{nn})} = \frac{\sigma_0(M^2_{nll}+C^2_{ml})}{2(1-D_{nn}-K_{nn}+C_{nn})}. \tag{55}$$

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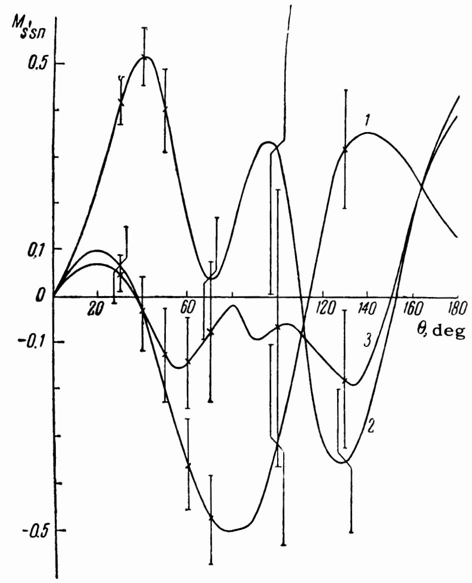
[1]The formula for $C_{s's''n}$ taking relativistic rotations into account was obtained earlier by V. I. Nikanorov.

[2]In Table II of the review article[1] there are some misprints. In formulas (5) and (8) of this table one should replace $l \rightleftarrows m$. The same replacement should also be made in (7.43), (7.44) and (7.50′).



The angular dependence of the parameter $M_{s'sn}$ for np-scattering at an energy of 630 MeV. The graphs 1-3 have been calculated with the aid of sets of phases from[17]. The errors shown are due to the errors in the determination of the elements of the scattering matrix in the course of the phase analysis.

Relations (54) and (55) show to what extent the procedure for reconstructing the scattering amplitude is simplified if the tensor M_{ikq} is known.

The measured parameters M_{abc} are calculated in [16] with the aid of the available sets of phases over a wide range of energies. The diagram shows results of the calculations of the component $M_{s'sn}$ for np-scattering at an energy of 630 MeV. It can be seen from the graphs that a measurement of this quantity would enable us to significantly reduce the ambiguity of the phase analysis.

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212