INHOMOGENEOUS RESONANCE IN ANTIFERROMAGNETS

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Oscillations of the magnetic moments of an antiferromagnetic plate are considered, for the case in which the magnetic moments of the sublattices lie in the plane of the plate. The characteristic frequencies of inhomogeneous antiferromagnetic resonance are calculated, in the magnetostatic approximation, in antiferromagnets with two types of anisotropy: an axis and a plane of easy magnetization.

1. INTRODUCTION

RESONANCE in ferromagnetic specimens in an inhomogeneous magnetic field was studied experimentally by White and Solt [1]. It was discovered that, in contrast to the case of a homogeneous magnetic field, there are not one, but a whole series of characteristic frequencies. In the general case of an arbitrary three-axis ellipsoid, the problem of calculating the characteristic frequencies was solved by Walker [2]; he in fact established the connection between the frequencies of inhomogeneous resonance and the spin-wave spectrum.

The present paper considers the characteristic frequencies of an antiferromagnetic plate when the field and the deviations of the magnetic moments from their equilibrium values are inhomogeneous. It is shown that the characteristic frequencies of oscillation of the magnetic moments in an antiferromagnetic plate (the frequencies of inhomogeneous resonance) lie within a strictly defined interval. The relative size of this interval is of order $1/\delta$ in comparison with the frequency of homogeneous resonance (δ is the constant of homogeneous exchange interaction between the sublattices).

From the point of view of spin-wave theory, the frequencies of inhomogeneous resonance in antiferromagnets are determined by the magnetic dipole interaction. In this paper, the dependence of the limiting frequencies of inhomogeneous resonance on the external magnetic field is found over a wide interval of magnetic fields; that is, it is shown how the frequency interval within which the frequencies of inhomogeneous resonance lie changes with change of the external magnetic field.

2. DISPERSION EQUATIONS

In order to find the spectrum of inhomogeneous oscillations of the magnetic moments in an antiferromagnetic plate, we shall start from the equations of magnetostatics,

$$\mathrm{rot}\,\mathbf{h} = 0, \quad \mathrm{div}\,(\mathbf{h} + 4\pi\hat{\chi}\mathbf{h}) = 0, \qquad (1)^*$$

where $\hat{\chi}(\mathbf{k}, \omega)$ is the high-frequency magnetic susceptibility tensor and $\mathbf{h}$ is the alternating magnetic field. Outside the body the tensor $\hat{\chi}$ is, of course, equal to zero. By introducing, instead of the magnetic field $\mathbf{h}$, the scalar potential φ,

$$\mathbf{h} = \nabla\varphi,$$

we can put Eqs. (1) into the form

$$\Delta\varphi^{(i)} + 4\pi\chi_{ik}\nabla_i\nabla_k\varphi^{(i)} = 0, \qquad \Delta\varphi^{(e)} = 0, \qquad (2)$$

where $\varphi^{(i)}$ is the magnetic potential inside the body and $\varphi^{(e)}$ is the magnetic potential outside the body. We shall suppose that the origin of coordinates is located at the center of the plate, of thickness 2d, with the z axis perpendicular to the plate surface (Fig. 1).

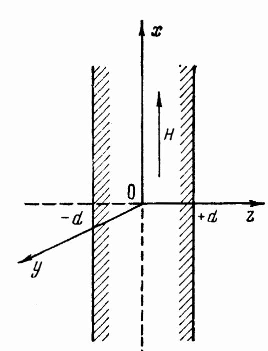

FIG. 1

*rot ≡ curl.

Besides Eqs. (2), determining the magnetic potential inside and outside the body, it is necessary to take account of the boundary conditions: continuity of the tangential components of the magnetic field $\mathbf{h}$ and of the normal component of the magnetic induction $\mathbf{b} = \mathbf{h} + 4\pi\mathbf{m}$ at a surface of the plate ($z = \pm d$); in addition, the magnetic field $\mathbf{h}^{(e)}$ must vanish when $z \to \pm\infty$. This leads to the following boundary conditions for the magnetic potentials $\varphi^{(i)}$ and $\varphi^{(e)}$:

$$\varphi^{(i)}|_{z=\pm d} = \varphi^{(e)}|_{z=\pm d}, \qquad \varphi^{(e)}|_{z\to\pm\infty} = 0,$$

$$(1+4\pi\chi_{zz})\left.\frac{\partial\varphi^{(i)}}{\partial z}\right|_{z=\pm d} + 4\pi\left(\chi_{zx}\frac{\partial\varphi^{(i)}}{\partial x} + \chi_{zy}\frac{\partial\varphi^{(i)}}{\partial y}\right)\Bigg|_{z=\pm d} = \left.\frac{\partial\varphi^{(e)}}{\partial z}\right|_{z=\pm d}. \tag{3}$$

We shall seek solutions of Eqs. (2) in the form of plane waves inside the specimen and waves attenuated along the z axis outside the specimen:

$$\varphi^{(i)} = (Ae^{ik_z z} + Be^{-ik_z z})\, e^{i(k_x x + k_y y)},$$

$$\varphi^{(e)} = [C\theta(z-d)\,e^{-fz} + D\theta(-z-d)\,e^{fz}]\, e^{i(k_x x + k_y y)}, \tag{4}$$

where

$$\theta(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \end{cases}.$$

On substitution of (4) in (2), it is easily found that

$$f^2 = k_x^2 + k_y^2, \tag{5}$$

and that the components k_x, k_y, and k_z of the wave vector are connected with the frequency ω by the relation

$$k^2 + 4\pi[\chi_{xx}(\mathbf{k},\omega)k_x^2 + \chi_{yy}(\mathbf{k},\omega)k_y^2 + \chi_{zz}(\mathbf{k},\omega)k_z^2] = 0. \tag{6}$$

We remark that if we take account of spatial dispersion of the tensor $\hat{\chi}$, then Eq. (6) determines the spin-wave spectrum with allowance for magnetic dipole interaction.

On eliminating the arbitrary constants A, B, C, and D in the expressions for the magnetic potentials $\varphi^{(i)}$ and $\varphi^{(e)}$ by means of the boundary conditions (3), we get one additional equation relating to each other the frequencies and the wave vectors of inhomogeneous magnetic oscillations in the plate:

$$\operatorname{ctg} 2k_z d = \frac{k_z^2\mu_{zz}^2 - (k_y\mu_{zy} + k_x\mu_{zx})^2 - (k_x^2+k_y^2)}{2k_z(k_x^2+k_y^2)^{1/2}\mu_{zz}}, \tag{7}^*$$

where $\hat{\mu} = 1 + 4\pi\hat{\chi}$. Equations (6) and (7) determine the frequencies of the characteristic oscillations of an antiferromagnetic plate.

*ctg ≡ cot.

In a consideration of long-wavelength oscillations of the magnetic moments, it is possible to neglect spatial dispersion of the high-frequency magnetic susceptibility tensor $\hat{\chi}(\mathbf{k},\omega)$; then Eqs. (6) and (7) will determine the frequencies of inhomogeneous antiferromagnetic resonance. Since the spatial dispersion of the tensor $\hat{\chi}(\mathbf{k},\omega)$ is determined by inhomogeneous exchange interaction in antiferromagnets, it may be neglected only if

$$\Theta_c(ak)^2 \ll \beta(\mu M_0),$$

where Θ_c is a quantity of the order of the Néel temperature, a is the lattice constant, β is the magnetic anisotropy constant ($\beta \sim 1$), μ is the Bohr magneton, and M_0 is the magnetic moment of a sublattice.

3. FREQUENCIES OF CHARACTERISTIC INHOMOGENEOUS OSCILLATIONS IN ANTIFERROMAGNETS WITH MAGNETIC ANISOTROPY OF THE "EASY AXIS" TYPE

In order to find explicit expressions for the frequencies of inhomogeneous resonance from Eqs. (6) and (7), it is necessary to know the high-frequency magnetic susceptibility tensor of the antiferromagnet. If the external magnetic field is parallel to the plane of the plate and is less than the turning-over field H_{AE}, then the high-frequency magnetic susceptibility tensor of an antiferromagnet with magnetic anisotropy of the "easy axis" type has the form [3]

$$\hat{\chi} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \chi_{yy} & \chi_{yz} \\ 0 & \chi_{zy} & \chi_{zz} \end{pmatrix},$$

$$\chi_{yy} = \chi_{zz} = \Omega_1^2\frac{\Omega^2 - \omega^2 - (\mu H)^2}{[\Omega^2 - (\omega-\mu H)^2][\Omega^2 - (\omega+\mu H)^2]}$$

$$\chi_{zy} = -\chi_{yz} = -\frac{2i\Omega_1^2\omega\mu H}{[\Omega^2 - (\omega-\mu H)^2][\Omega^2 - (\omega+\mu H)^2]},$$

$$\Omega_1^2 = 2(\beta-\beta')(\mu M_0)^2, \qquad \Omega^2 = 2\delta(\beta-\beta')(\mu M_0)^2. \tag{8}$$

(We recall that the z axis is perpendicular to the plane of the plate and that the x axis is chosen along the magnetic field (Fig. 1).)

By using formulas (8), we get from (6) the following expressions for the frequencies of inhomogeneous oscillations:

$$\omega_{1,2}^2 = \Omega^2 + \mu^2H^2 + 2\pi(1 - k_x^2/k^2)\Omega_1^2 \pm \{(2\pi)^2(1-k_x^2/k^2)^2\Omega_1^4 + 4\mu^2H^2[2\pi(1-k_x^2/k^2)\Omega_1^2 + \Omega^2]\}^{1/2}. \tag{9}$$

From (9) it is clear that the values of the resonance frequencies depend on the ratio k_x^2/k^2. The frequencies $\omega_{1,2}(\mathbf{k})$ attain their largest values when $k_x = 0$:

$$\omega_{1,2max}^2 = \Omega^2 + \mu^2 H^2 + 2\pi\Omega_1^2 \pm \{4\pi^2\Omega_1^4 + 4\mu^2 H^2(\Omega^2 + 2\pi\Omega_1^2)\}^{1/2}. \quad (10)$$

They attain their smallest values when $k_x = k$:

$$\omega_{1,2min} = \Omega \pm \mu H. \quad (11)$$

The frequencies of homogeneous resonance $\omega_{01,2}$, coincide in this case with the frequencies $\omega_{1,2max}$.

To convince ourselves that formulas (10) and (11) determine an interval of inhomogeneous resonance frequencies, it is necessary to verify that Eq. (7) has solutions when

$$k_x^2 \ll k_y^2 + k_z^2, \qquad k_x^2 \gg k_y^2 + k_z^2.$$

On substituting the expressions (9) for the frequencies $\omega_{1,2}(\mathbf{k})$ in Eq. (7), we find that when $k_x^2 \ll k_y^2 + k_z^2$,

$$\operatorname{ctg} 2k_z d \approx \frac{1}{2}\left(\frac{k_z}{|k_y|}\mu_{zz} - \frac{|k_y|}{k_z}\frac{1+\mu_{zy}^2}{\mu_{zz}}\right),$$

and that when $k_x^2 \gg k_y^2 + k_z^2$,

$$\operatorname{ctg} 2k_z d = -\frac{|k_x|}{2k_z}\frac{1}{\mu_{zz}}.$$

If the external magnetic field is larger than the turning-over field, the tensor $\hat{\chi}(\omega)$ has the following nonvanishing components [3]:

$$\chi_{zz} = \chi_{yy} = \chi_0\varepsilon_1^2 / (\varepsilon_1^2 - \omega^2),$$

$$\chi_{zy} = -\chi_{yz} = -i\chi_0\varepsilon_1\omega / (\varepsilon_1^2 - \omega^2), \quad (12)$$

where $\chi_0 = 1/\delta$ is the static magnetic susceptibility, and where the frequency $\epsilon_1 = 2\delta\mu M_0 \cos\theta$, θ being the angle between the magnetic moments $\mathbf{M}_1$ and $\mathbf{M}_2$ of the sublattices and the external magnetic field $\mathbf{H}$:

$$\cos\vartheta = H / H_E.$$

By use of formula (12), it is easy to find from (6) expressions for the frequencies of inhomogeneous resonance:

$$\omega^2 = \varepsilon_1^2[1 + 4\pi\chi_0(1 - k_x^2/k^2)]. \quad (13)$$

Hence it is clear that the resonance frequencies lie in the interval

$$\varepsilon_1^2 \leqslant \omega^2 \leqslant \varepsilon_1^2(1 + 4\pi\chi_0).$$

The frequencies of inhomogeneous resonance attain their smallest values when $k_y^2 + k_z^2 \ll k_x^2$, and their largest when $k_y^2 + k_z^2 \gg k_x^2$. It is easy to verify that each of these cases is compatible with Eq. (7). Figure 2 represents schematically the frequency bands for inhomogeneous resonance as they depend on the external magnetic field. The frequency of homogeneous resonance, in fields larger than the turning-over field, is

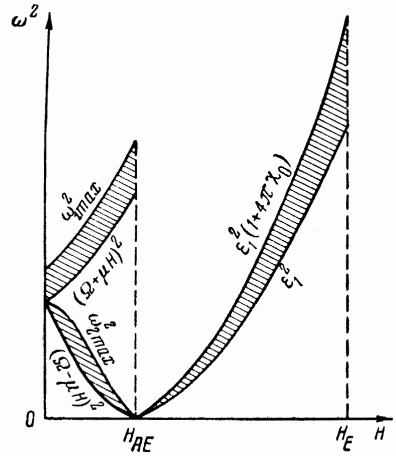

FIG. 2

$$\omega_0 = \varepsilon_1(1 + 4\pi\chi_0)^{1/2}.$$

4. FREQUENCIES OF CHARACTERISTIC INHOMOGENEOUS OSCILLATIONS IN ANTIFERROMAGNETS WITH MAGNETIC ANISOTROPY OF THE "EASY PLANE" TYPE

The frequencies of inhomogeneous resonance in an antiferromagnetic plate with magnetic anisotropy of the "easy plane" type, when the external magnetic field is perpendicular to the plane of the plate and the symmetry axis of the crystal is parallel to that plane, were calculated in [4].1) We consider the case in which the external magnetic field lies in the plane of the plate and the axis of anisotropy is perpendicular to that plane. The high-frequency magnetic susceptibility tensor of an antiferromagnet with magnetic anisotropy of the "easy plane" type has the form [5]

$$\hat{\chi} = \begin{pmatrix} \chi_{xx} & 0 & 0 \\ 0 & \chi_{yy} & \chi_{yz} \\ 0 & \chi_{zy} & \chi_{zz} \end{pmatrix};$$

$$\chi_{xx} = \chi_0\frac{\varepsilon_2^2}{\varepsilon_2^2 - \omega^2}, \quad \chi_{yy} = \chi_0\frac{\mu^2 H^2}{\varepsilon_1^2 - \omega^2},$$

$$\chi_{zz} = \chi_0\frac{\varepsilon_1^2}{\varepsilon_1^2 - \omega^2}, \quad \chi_{yz} = -\chi_{zy} = i\chi_0\frac{\mu\omega H}{\varepsilon_1^2 - \omega^2},$$

$$\chi_0 = 1/\delta, \quad \varepsilon_1 = \mu H, \quad \varepsilon_2^2 = \mu^2 H_A H_E(1 - H^2/H_E^2),$$

$$H_A = (\beta - \beta')M_0, \quad H_E = 2\delta M_0, \quad (14)$$

*Note: Figure 3 in [4] is drawn incorrectly.

where β and β' are the magnetic anisotropy constants.

By using formulas (14), we get the equation for the frequencies of inhomogeneous magnetic resonance:

$$(\omega^2-\varepsilon_1^2)(\omega^2-\varepsilon_2^2)-4\pi\chi_0\{[\varepsilon_1^2-(\varepsilon_1^2-\varepsilon_2^2)k_x^2/k^2]\omega^2-\varepsilon_1^2\varepsilon_2^2\}=0. \quad (15)$$

In the region of weak magnetic fields,

$$0<H<H_{k1}=H_k(1-2\pi\chi_0),$$

$$H_k=\sqrt{H_AH_E}/(1+H_A/H_E)$$

the frequencies $\omega_{1,2}(\mathbf{k})$ have the form (Fig. 3)

$$\omega_1^2\approx\varepsilon_2^2[1+4\pi\chi_0k_x^2/k^2],$$
$$\omega_2^2\approx\varepsilon_1^2[1+4\pi\chi_0(1-k_x^2/k^2)]. \quad (16)$$

In magnetic fields $H = H_k$, the frequencies $\omega_{1,2}^2$ are equal to

$$\omega_1^2=(\mu H_k)^2(1+4\pi\chi_0), \qquad \omega_2^2=(\mu H_k)^2. \quad (17)$$

In the case of strong magnetic fields,

$$H_E>H>H_{k2}=H_k(1+2\pi\chi_0)$$

we have the following expressions for the frequencies:

$$\omega_2^2\approx\varepsilon_2^2(1+4\pi\chi_0k_x^2/k^2),$$
$$\omega_1^2\approx\varepsilon_1^2[1+4\pi\chi_0(1-k_x^2/k^2)]. \quad (18)$$

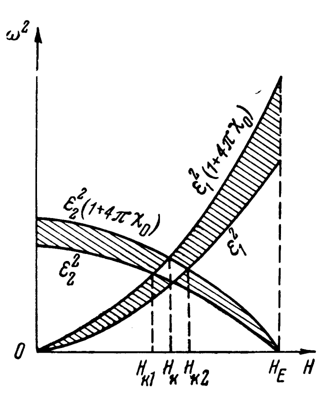

FIG. 3

From formulas (16) to (18) it is clear that the relative width of the interval is of order $2\pi\chi_0 \sim 1/\delta$ for both branches $\omega_{1,2}(\mathbf{k})$. This interval, as a function of the magnetic field, is shown schematically in Fig. 3. The frequencies of homogeneous resonance are here

$$\omega_{01}=\varepsilon_1(1+4\pi\chi_0)^{1/2}, \quad \omega_{02}=\varepsilon_2.$$

In a whole series of antiferromagnets with magnetic anisotropy of the "easy plane" type ($MnCO_3$, $CoCO_3$, α-Fe_2O_3, NiF_2), the magnetic moments of the sublattices in the absence of an external magnetic field are not strictly antiparallel (antiferromagnets with weak ferromagnetism). The high-frequency magnetic susceptibility tensor of such antiferromagnets is determined by formulas (14) with

$$\varepsilon_1^2=\mu^2H(H+H_d), \quad \varepsilon_2^2=\mu^2[H_AH_E+H_d(H+H_d)], \quad (19)$$

and with $H \to H + H_d$, where $H_d = dM_0$ is the Dzyaloshinskiĭ field [5,6]. By use of relations (6), (7), (14), and (19), the frequencies of inhomogeneous resonance can be determined in this case. Just as for ordinary antiferromagnets with magnetic anisotropy of the "easy plane" type, two branches of the resonance frequencies are possible:

$$\omega_2^2=\varepsilon_2^2(1+4\pi\chi_0k_x^2/k^2),$$
$$\omega_1^2=\varepsilon_1^2+4\pi\chi_0[\varepsilon_1^2k_z^2/k^2+\mu^2(H+H_d)^2k_y^2/k^2]. \quad (20)$$

From this it is clear that the frequencies $\omega_{1,2}$ lie in the intervals (Fig. 4).

$$\varepsilon_2^2\leqslant\omega_2^2\leqslant\varepsilon_2^2(1+4\pi\chi_0),$$
$$\varepsilon_1^2\leqslant\omega_1^2\leqslant\varepsilon_1^2+4\pi\chi_0\mu^2(H+H_d)^2.$$

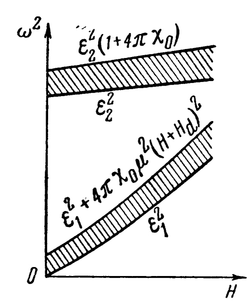

FIG. 4

The frequencies ω_1 reach their smallest values when $k_x^2 \gg k_y^2 + k_z^2$; the wave vector k_z is then determined by the equation

$$\operatorname{ctg}2k_zd=-\frac{|k_x|}{2k_z}\frac{1}{\mu_{zz}}.$$

To the largest values of the frequencies ω_1 correspond wave vectors $k_y^2 \gg k_x^2 + k_z^2$; the wave vector k_z then satisfies the equation

$$\operatorname{ctg}2k_zd=-\frac{|k_y|}{2k_z}\frac{H_d}{H+H_d}$$

It can easily be verified that wave vectors for which the largest and smallest values of the frequency ω_2 are attained are also compatible with equation (7). We note, finally, that the frequencies of homogeneous resonance for spin waves in a plate, in the case of an antiferromagnet with weak ferromagnetism, are determined by the formulas

$$\omega_{01} = \varepsilon_1 (1 + 4\pi\chi_0)^{1/2}, \qquad \omega_{02} = \varepsilon_2.$$

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