COLLECTIVE EXCITATIONS IN SEMIMETALS

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Collective excitations in a two-band electron system with Coulomb interaction are investigated in the presence of bound electron-hole pairs. The case of intersecting bands, which is formally similar to the case of superconductivity, is considered. An acoustic branch, which is not suppressed, by plasma oscillations (as in a superconductor) but exists simultaneously with the plasma branch, is obtained by taking into account only the Coulomb interaction matrix elements (which vary like $|\mathbf{k}|^{-2}$ at low values of the momentum transfer $\mathbf{k}$). If short-range exchange matrix elements are taken into account, a gap appears in the acoustic branch. When $\mathbf{k} = 0$ the energy of the excitations with nonvanishing momenta is found to be close to 2Δ quadratically in the coupling constant.

KELDYSH and Kopaev[1] and the present authors[2] have considered one phenomenon connected with the Coulomb excitation of electrons in close-lying energy bands. The gist of the phenomenon is that particles can be paired with holes in such a way that realignment of the spectrum takes place near the band extrema, whereby a gap appears in the spectrum if the bands had initially overlapped. The situation is similar here to superconductivity, and the production of a pair of single-particle excitations must in particular be regarded as a breaking of the bond of an electron-hole pair. With change of temperature the system experiences a second-order phase transition, the transition temperature being a point of pair condensation in the ground state. The existence of a condensate of bound pairs of quasiparticles allows us to expect superfluid properties to appear below the phase transition point. To check on such a possibility, it is necessary to verify that all the branches of the excitations satisfy the superfluidity criterion. To this end, we consider in this paper the collective oscillations, in which the bound pairs move as a unit. We confine ourselves to the case of strongly overlapping bands, when an appreciable realignment of the spectrum takes place in a narrow energy layer at the Fermi surface. This case is simplest mathematically, in view of the complete formal similarity to the case of superconductivity.

Collective excitations in a superconductor were considered in detail by Vaks, Galitskiĭ, and Larkin[3]. We make use of their method, which is based on the formal similarity between the problem and the one-dimensional relativistic problem, in which the role of the mass is played by the magnitude of the gap Δ, and that of the one-dimensional spatial momentum by the particle energy reckoned from the Fermi level. We confine ourselves to examination of zero-spin excitations for the case when the pairing occurs in a singlet state. It can be easily shown that neither excitations with spin 1 nor excitations in a system with triplet pairs produce new branches. We use the notation in the cited paper[3]. Some of the formulas will turn out to be identical with those of the mentioned paper. Nonetheless, we shall write them out because we are dealing with an entirely different physical phenomenon, and also to preserve the unity of the exposition.

1. RELATIVISTIC FORMULATION OF THE PROBLEM

The Hamiltonian of the two-band system is of the form

$$H = \sum [a^{+}_{\mathbf{p}2} a_{\mathbf{p}2} E_2(\mathbf{p}) + a_{\mathbf{p}1}{}^{+} a_{\mathbf{p}1} E_1(\mathbf{p})] + \frac{1}{2} \sum a^{+}_{\mathbf{p}_1 n_1} a_{\mathbf{p}_1' n_1'} \begin{pmatrix} \mathbf{p}_1 n_1 \, \mathbf{p}_2' n_2' \\ \mathbf{p}_1' n_1' \mathbf{p}_2 n_2 \end{pmatrix} a^{+}_{\mathbf{p}_2' n_2'} a_{\mathbf{p}_2 n_2}. \quad (1)$$

Here

$$a_{\mathbf{p}n} = \begin{pmatrix} a_{\mathbf{p}n, 1/2} \\ a_{\mathbf{p}n, -1/2} \end{pmatrix},$$

$$E_2(\mathbf{p}) = (p^2 - p_F{}^2)/2m_2, \qquad E_1(\mathbf{p}) = (p_F{}^2 - p^2)/2m_1,$$

p_F is the Fermi momentum. The interaction matrix element

$$\begin{pmatrix} \mathbf{p}_1 n_1 & \mathbf{p}_2' n_2' \\ \mathbf{p}_1' n_1' & \mathbf{p}_2 n_2 \end{pmatrix} = \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 \psi^*_{\mathbf{p}_1 n_1}(\mathbf{r}_1) \psi_{\mathbf{p}_1' n_1'}(\mathbf{r}_1) \times \frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|} \psi^*_{\mathbf{p}_2' n_2'}(\mathbf{r}_2) \psi_{\mathbf{p}_2 n_2}(\mathbf{r}_2) \quad (2)$$

is taken with the Bloch functions $\psi_{pn}(\mathbf{r})$; n runs through the values 2 and 1, corresponding to the upper and lower energy bands. The summation over all **p** is in the unit cell of reciprocal space. When the extrema of the bands are at different points of the Brillouin cell, **p** must be measured in each band from its own extremum. The formulas will not differ in this case from those obtained in the case of coinciding extrema. We shall include in the Hamiltonian (1) only the Coulomb matrix elements

$$\begin{pmatrix} \mathbf{p}_1 n_1 & \mathbf{p}_2' n_2' \\ \mathbf{p}_1' n_1' & \mathbf{p}_2 n_2 \end{pmatrix} = V(\mathbf{p}_1 - \mathbf{p}_1'), \tag{3}$$

which behave like $|\mathbf{k}|^{-2}$ for small $\mathbf{p}_1 - \mathbf{p}_1' = \mathbf{k}$. The remaining interactions, if small compared with (3), can lead to an appreciable change only in the acoustic branches of the spectrum of the collective excitations. We shall show that such interactions give rise to a gap in the acoustic branch.

We introduce, following[3], the four-rowed matrices

$$\gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \gamma_4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_3 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4}$$

and the four-component operators

$$a_\mathbf{p} = \begin{pmatrix} a_{\mathbf{p}2} \\ a_{\mathbf{p}1} \end{pmatrix}, \qquad \bar{a}_\mathbf{p} = a_\mathbf{p}^+ \gamma_1. \tag{5}$$

The dispersion law in both bands is rewritten in the form

$$E_2(\mathbf{p}) = (1-\xi)E(\mathbf{p}), \qquad E_1(\mathbf{p}) = -(1+\xi)E(\mathbf{p}),$$
$$E(\mathbf{p}) = (p^2 - p_F^2)/2m, \quad m_2^{-1} + m_1^{-1} = 2m^{-1},$$
$$-m_2^{-1} + m_1^{-1} = 2\xi m^{-1}. \tag{6}$$

In the notation of (4)—(6), the Hamiltonian takes the form

$$H = \sum \bar{a}_\mathbf{p}(-i\gamma_3 E(\mathbf{p}) + \gamma_4 \xi E(\mathbf{p}))\, a_\mathbf{p} + {}^1/_2 \sum \bar{a}_{\mathbf{p}_1}\gamma_4 a_{\mathbf{p}_1'} V(\mathbf{p}_1 - \mathbf{p}_1')\, \bar{a}_{\mathbf{p}_2'}\gamma_4 a_{\mathbf{p}_2}. \tag{7}$$

We define the Green's function by means of

$$G(\mathbf{p}, t) = i\langle T a_\mathbf{p}(t)\bar{a}_\mathbf{p}(0)\rangle. \tag{8}$$

We introduce a one-dimensional relativistic momentum operator

$$\hat{p} = \gamma_3 p_3 + \gamma_4 p_4, \quad p_3 = E, \quad p_4 = ip_0 = i(\varepsilon + \xi E). \tag{9}$$

For noninteracting particles, the Green's function takes the form

$$G_0(p) = G_0(\mathbf{p}, \varepsilon) = -i\hat{p}/p^2, \tag{10}$$

where $p^2 = p_3^2 + p_4^2$. For interacting particles, the matrix G has diagonal terms that correspond to interband transitions. The function G is expressed in terms of the self-energy part:

$$G^{-1}(p) = G_0^{-1}(p) + \Sigma = i\hat{p} + \Sigma, \tag{11}$$

where, in the first approximation

$$\Sigma(p) = -i\int d^4p'\gamma_4 G(p')\gamma_4 \mathscr{V}(\mathbf{p} - \mathbf{p}'). \tag{12}$$

In formula (12) there has been carried out partial summation of the Coulomb chains of components $\mathscr{V}(\mathbf{p} - \mathbf{p}')$:

$$\mathscr{V}(\mathbf{p} - \mathbf{p}') = \frac{4\pi e^2}{(\mathbf{p} - \mathbf{p}')^2 + \varkappa_D^2}. \tag{13}$$

The nondiagonal elements in Σ can be referred to the term $i\hat{p}$. By the same token, they reduce to a renormalization of the zero-order spectrum constants; we shall assume that this normalization has been effected. Thus, the matrix Σ is diagonal in the band index and is of the form

$$\Sigma = \hat{\Delta} = \begin{pmatrix} \Delta & 0 \\ 0 & \Delta^* \end{pmatrix}. \tag{14}$$

Choosing the phases of the Bloch functions $\psi_{pn}(\mathbf{r})$ in suitable fashion, we can make Δ real, so that $\hat{\Delta} = \Delta$. The Green's function G is then of the form

$$G(p) = \frac{\Delta - i\hat{p}}{p^2 + \Delta^2}. \tag{15}$$

The spectrum of the elementary excitations is determined by the pole of G(p):

$$\varepsilon = -\xi E \pm (E^2 + \Delta^2)^{1/2}.$$

Substituting (15) and (14) in (12) we obtain for the gap Δ the equation

$$\Delta = -i\int \mathscr{V}(\mathbf{p} - \mathbf{p}')\frac{\Delta d^4p'}{p'^2 + \Delta^2}. \tag{16}$$

It was shown earlier[2] that if the bands overlap sufficiently strongly (when the Debye radius κ_D^{-1} is large compared with the reciprocal Fermi momentum p_F^{-1}) the realignment of the spectrum takes place predominantly in an energy layer of thickness $2\omega_0 = \kappa_D p_F/m$ near the Fermi surface. In this region $\mathscr{V}(\mathbf{p} - \mathbf{p}')$ depends only on the angle between **p** and **p**′:

$$\mathscr{V}(\mathbf{p} - \mathbf{p}') = \mathscr{V}(\mathbf{n}\mathbf{n}'), \quad \mathbf{n} = \mathbf{p}/|\mathbf{p}|, \quad \mathbf{n}' = \mathbf{p}'/|\mathbf{p}'|.$$

Confining ourselves in $\mathscr{V}$ to the zeroth harmonic

$$g_0 = \rho\int \mathscr{V}(\mathbf{n}\mathbf{n}')\,d\mathbf{n}'/4\pi,$$

we find that Δ does not depend on the angles and satisfies the equation

$$1 = -ig_0\int d^2p/(p^2 + \Delta^2). \tag{17}$$

The element of integration d^4p is written here in the form

$$d^4p = \rho \frac{d\mathbf{n}}{4\pi} d^2p = \frac{mp_F}{2\pi^2} \frac{d\mathbf{n}}{4\pi} \frac{dp_3 dp_0}{2\pi}. \quad (18)$$

Cutting off the integral (17) at the end-point frequency ω_0, we obtain for Δ the expression

$$1 = g_0 \ln (2\omega_0 / \Delta) \equiv g_0 L. \quad (19)$$

The small parameter in the problem is the ratio κ_D/p_F.

2. COLLECTIVE EXCITATIONS

The spectrum of the collective excitations is determined by the poles of the two-particle Green's function

$$K(1, 2; 3, 4) = i\langle Ta(1)\bar{a}(2)a(3)\bar{a}(4)\rangle. \quad (20)$$

The poles of the two-particle Green's function are obtained from the conditions for the solvability of the homogeneous equation obtained from Dyson's equation by discarding the inhomogeneous part. The variables 3 and 4 enter this equation as parameters and can be omitted. The homogeneous Dyson equation has in first order in the interaction $\mathscr{V}$ the form

$$K(p, k) = iG(p + k/2)\gamma_4 \int \mathscr{V}(p - p') \times K(p', k)\gamma_4 G(p - k/2) d^4p' - iG(p + k/2)\gamma_4 \times G(p - k/2) V(k) \int \mathrm{Sp}\, \gamma_4 K(p', k) d^4p'. \quad (21)$$

Here

$$p = (p_1 + p_2) / 2, \quad k = (\mathbf{k}, \omega) = p_1 - p_2.$$

In expression (21), the significant region of integration is that near the Fermi surface. In this region

$$\mathscr{V}(\mathbf{p} - \mathbf{p}') = \mathscr{V}(\mathbf{nn}'), \quad d^4p = \rho d^2p d\mathbf{n} / 4\pi.$$

Integrating (21) with respect to d^2p we obtain for the functions

$$K_i(\mathbf{n}, k) = {}^1/_4\, \mathrm{Sp}\, \gamma_i \int d^2p K(p, k)$$

the equations (see [3], (27))

$$K_i(\mathbf{n}, k) = \pi_{ir}(\omega, \mathbf{nk}) \rho \int \mathscr{V}(\mathbf{nn}') K_r(\mathbf{n}', k) d\mathbf{n}'/4\pi - \pi_{i4}(\omega, \mathbf{nk}) \cdot 4\rho V(k) \int K_4(\mathbf{n}', k) d\mathbf{n}'/4\pi, \quad (22)$$

where

$$\pi_{ir}(\omega, \mathbf{nk}) = \frac{i}{4} \mathrm{Sp} \int \gamma_i G(p_1) \gamma_4 \gamma_r \gamma_4 G(p_2) d^2p.$$

The quantities π_{ir} are connected with the quantities Π_{ir} calculated in [3] (formula (28′)) by the simple relations

$$\pi_{i1} = \Pi_{i1}, \quad \pi_{i4} = -\Pi_{i4}, \quad \pi_{i3} = -\Pi_{i3}, \quad \pi_{i5} = \Pi_{i5}. \quad (23)$$

Rewriting formulas (A.9) of [3] and taking (23) into account, we obtain

$$\pi_{11} = L - f + \beta^2 f, \quad \pi_{55} = L + \beta^2 f,$$
$$\pi_{53} = -\pi_{35} = -q_4 f / 2\Delta, \quad \pi_{54} = \pi_{45} = -q_3 f / 2\Delta,$$
$$\pi_{33} = f + (q_3^2 - q_3^2 f) / q^2, \quad \pi_{44} = (q_3^2 - q_3^2 f) / q^2,$$
$$\pi_{34} = \pi_{43} = -q_4(q_3 - q_3 f) / q^2. \quad (24)$$

Here

$$q_4 = i\omega, \quad q_3 = \mathbf{kn}v_F, \quad f = \arcsin \beta / \beta\sqrt{1 - \beta^2},$$
$$\beta^2 = -q^2 / 4\Delta^2 = [\omega^2 - (\mathbf{kn}v_F^2)] / 4\Delta^2. \quad (25)$$

We expand $K_i(\mathbf{n}, k)$ and $\mathscr{V}(\mathbf{nn}')$ in spherical harmonics:

$$K_i = \sum_{lm} K_{lm}{}^i Y_{lm}(\mathbf{n}), \quad \frac{\rho}{4\pi} \mathscr{V}(\mathbf{nn}') = \sum_{lm} g_l Y_{lm}(\mathbf{n}) Y_{lm}^*(\mathbf{n}'). \quad (26)$$

We put

$$\pi_{ll_1m}^{ir} = \int Y_{lm}^*(\mathbf{n}) \pi_{ir} Y_{l_1m}(\mathbf{n}) d\mathbf{n}. \quad (27)$$

Substituting in (22) the explicit expressions for the quantities in (27), we arrive at a system of algebraic equations for K^i_{lm}:

$$K_{lm}{}^1 = \sum_{l_1} g_{l_1} (L - f + \beta^2 f)_{ll_1m} K^1_{l_1m}, \quad (28)$$

$$K_{lm}{}^4 = \sum_{l_1} g_{l_1} \left\{ \left(\frac{q_3^2 - q_3^2 f}{q^2} \right)_{ll_1m} K^4_{l_1m} + q_4 \left(\frac{q_3 - q_3 f}{q^2} \right)_{ll_1m} K^3_{l_1m} - \left(\frac{q_3 f}{2\Delta} \right)_{ll_1m} K^5_{l_1m} \right\} - 4\delta_{m0}\rho V(k) \left(\frac{q_3^2 - q_3^2 f}{q^2} \right)_{l00} K_{00}{}^4,$$

$$K_{lm}{}^3 = \sum_{l_1} g_{l_1} \left\{ -q_4 \left(\frac{q_3 - q_3 f}{q^2} \right)_{ll_1m} K^4_{l_1m} + \left(f + \frac{q_3^2 - q_3^2 f}{q^2} \right)_{ll_1m} K^3_{l_1m} + \left(\frac{q_4 f}{2\Delta} \right)_{ll_1m} K^5_{l_1m} \right\} + 4\delta_{m0}\rho V(k) q_4 \left(\frac{q_3 - q_3 f}{q^2} \right)_{l00} K_{00}{}^4,$$

$$K_{lm}{}^5 = \sum_{l_1} g_{l_1} \{ -(q_3 f/2\Delta)_{ll_1m} K^4_{l_1m} - (q_4 f/2\Delta)_{ll_1m} K^3_{l_1m} + (L + \beta^2 f)_{ll_1m} K^5_{l_1m} \} + 4\delta_{m0} \rho V(k) (q_3 f/2\Delta)_{l00} K_{00}{}^4. \quad (29)$$

The equation for K^1_{lm}, as in [3], has no non-vanishing solutions when $\mathbf{k} \neq 0$, and can be disregarded. When $l = 0$ the equation for K^4_{00} can be separated, accurate to terms of order g_0:

$$K_{00}{}^4 = g_0 \left(\frac{q_3^2 - q_3^2 f}{q^2} \right)_{00} K_{00}{}^4 - 4\rho V(k) \left(\frac{q_3^2 - q_3^2 f}{q^2} \right)_{00} K_{00}{}^4. \quad (30)$$

This equation has a solution $K^4_{00} \neq 0$ when $\omega \gg \Delta$, and corresponds to plasma oscillations with dispersion

$$\omega^2(\mathbf{k}) = \omega_{\mathrm{pl}}^2(0) + (3/_5 v_F^2 + 1/_3 g_0 v_F^2) k^2, \tag{31}$$

$$\omega_{\mathrm{pl}}^2(0) = 8\pi n e^2 / m. \tag{32}$$

Here n—electron density in band 2 or hole density in band 1. The fact that the value of $\omega_{\mathrm{pl}}^2(0)$ is double the value obtained for a metal is due to the presence of two types of carrier.

We obtain the second branch of the excitations with $l = 0$ by discarding the terms with $g_{l \neq 0}$ in the third and fourth equations of system (28):

$$K_{00}^3 = g_0\left(f + \frac{q_3^2 - q_3^2 f}{q^2}\right)_{00} K_{00}^3 + g_0 \frac{i\omega}{2\Delta} f K_{00}^5, \tag{33}$$

$$K_{00}^5 = -g_0 \frac{i\omega}{2\Delta} f K_{00}^3 + g_0 (L + \beta^2 f)_{00} K_{00}^5.$$

Solving (33) for $\beta \ll 1$, we obtain

$$\omega_0^2(k) = 1/_3 k^2 v_F^2 (1 + g_0). \tag{34}$$

We see thus that the acoustic-frequency branch exists independently of the plasma branch, unlike in a superconductor[3]. This result is physically obvious. In a superconductor, the pairs are charged and their motion is connected with long-range Coulomb forces, so that the acoustic branch is suppressed by the plasma waves. In the model considered here, the pair are neutral and interact with the aid of forces with finite radius.

Let us find the limiting frequencies of the excitations at $\mathbf{k} = 0$ for large l. For these excitations $K_{lm}^4 = 0$ and we have

$$K_{lm}^3 = g_l f K_{lm}^3 + g_l \frac{q_4 f}{2\Delta} K_{lm}^5,$$

$$K_{lm}^5 = -g_l \frac{q_4 f}{2\Delta} K_{lm}^3 + g_l (L + \beta^2 f) K_{lm}^5. \tag{35}$$

Equating the determinant (35) to zero we obtain

$$(1 - g_l L - g_l \beta^2 f)(1 - g_l f) - g_l^2 \beta^2 f = 0. \tag{36}$$

When $g_l \ll g_0$, the value of ω is close to 2Δ and

$$f(\beta) = \frac{\pi}{2}(1 - \beta^2)^{-1/2}.$$

Hence

$$\omega_l^2(0) = 4\Delta^2(1 - \alpha_l^2), \qquad \alpha_l = \pi g_l. \tag{37}$$

Let us consider now the influence of secondary interactions (other than (3)) on the spectrum of the collective excitations. We write down the matrix element (2) in the form

$$\begin{pmatrix} \mathbf{p}_1 n_1 & \mathbf{p}_2' n_2' \\ \mathbf{p}_1' n_1' & \mathbf{p}_2 n_2 \end{pmatrix} = (\gamma_4\gamma_\alpha)_{n_1 n_1'} V^{\alpha\beta}(\mathbf{p}_1, \mathbf{p}_1'; \mathbf{p}_2', \mathbf{p}_2)(\gamma_4\gamma_\beta)_{n_2' n_2}. \tag{38}$$

Summation over α and β is implied. In the notation of (5), (6), and (38), the Hamiltonian (1) takes the form

$$H = \sum \bar{a}_{\mathbf{p}}(-i\gamma_3 E + \gamma_4 \xi E) a_{\mathbf{p}} + 1/_2 \sum \bar{a}_{\mathbf{p}_1}\gamma_\alpha a_{\mathbf{p}_1'} V^{\alpha\beta} \bar{a}_{\mathbf{p}_2'}\gamma_\beta a_{\mathbf{p}_2}. \tag{38'}$$

Formula (12) for the self-energy part goes over

$$\Sigma(p) = -i \int d^4p' \gamma_\alpha G(p') \gamma_\beta V^{\alpha\beta}(\mathbf{p} - \mathbf{p}'). \tag{12'}$$

We will rewrite expression (14) for Σ in the form

$$\Sigma = \gamma_1 \Delta_1 + i\gamma_5 \Delta_2. \tag{14'}$$

G(p) then takes the form

$$G(p) = \frac{-i\hat{p} + \gamma_1\Delta_1 - i\gamma_5\Delta_2}{p^2 + \Delta^2}, \qquad \Delta^2 = \Delta_1^2 + \Delta_2^2. \tag{15'}$$

Substituting (14′) and (15′) in (12′) we obtain for Δ_1 and Δ_2 the equation

$$\gamma_1\Delta_1 + i\gamma_5\Delta_2 = -i \int \frac{d^4p'}{p'^2 + \Delta^2} V^{\alpha\beta}(p - p')\gamma^\alpha$$
$$\times (\gamma_1\Delta_1 - i\gamma_5\Delta_2)\gamma^\beta. \tag{16'}$$

We write down (16′) as follows:

$$\Delta_1 = -i \int \frac{d^4p'}{p'^2 + \Delta^2}(A_{11}\Delta_1 - iA_{15}\Delta_2),$$

$$\Delta_2 = -i \int \frac{d^4p'}{p'^2 + \Delta^2}(-iA_{51}\Delta_1 - A_{55}\Delta_2),$$

$$0 = -i \int \frac{d^4p'}{p'^2 + \Delta^2}(A_{41}\Delta_1 - iA_{45}\Delta_2),$$

$$0 = -i \int \frac{d^4p'}{p'^2 + \Delta^2}(A_{31}\Delta_1 - iA_{35}\Delta_2), \tag{39}$$

where

$$A_{ir} = A_{ri} = 1/_4 \operatorname{Sp} \gamma_i \gamma^\alpha \gamma_r \gamma^\beta V^{\alpha\beta}. \tag{40}$$

A_{ir} can be readily expressed in terms of $V^{\alpha\beta}$:

$$A_{ir} = 2V^{ir} + (2V^{11} - \operatorname{Sp} V)(\delta_{ir} - 2\delta_{i1}\delta_{1r}). \tag{41}$$

The quantities $V^{\alpha\beta}$ are in turn expressed in terms of the matrix elements $\begin{pmatrix} n_1, & n_2' \\ n_1', & n_2 \end{pmatrix}$ with the aid of relations that are the inverse of (38):

$$V^{\alpha\beta} = \sum_{n_1, n_1', n_2', n_2} \frac{1}{4}(\gamma^\alpha\gamma^4)_{n_1' n_1} \begin{pmatrix} n_1 & n_2' \\ n_1' & n_2 \end{pmatrix} (\gamma^\beta\gamma^4)_{n_2 n_2'}. \tag{42}$$

Introducing for the different interactions (2) the notation

$$\begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} = V,$$

$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}^* = W, \qquad \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} = S,$$

$$\begin{pmatrix} 2 & 2 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}^* = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}^* = R_2,$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}^* = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}^* = R_1,$$

$$R_2 \pm R_1 = 2R_\pm \tag{43}$$

and substituting (42) in (41), we obtain

$$A_{11} = V + \mathrm{Re}\, W, \quad A_{44} = V + S, \quad A_{33} = -V + S,$$
$$A_{55} = -V + \mathrm{Re}\, W, \quad A_{14} = 2\,\mathrm{Re}\, R_+, \quad A_{13} = 2i\,\mathrm{Re}\, R_-,$$
$$A_{15} = -i\,\mathrm{Im}\, W, \quad A_{43} = 0, \quad A_{45} = -2i\,\mathrm{Im}\, R_-,$$
$$A_{35} = 2\,\mathrm{Im}\, R_+. \tag{44}$$

We shall assume that the additional interactions W, S, and R are small compared with V. As noted above, these interactions can greatly influence only the acoustic branch (34). In view of this we confine ourselves to an analysis of their first harmonics. For the S-components of the matrix elements (43) (we shall denote them by lower-case letters) we can choose the phases of the Bloch functions ψ_{pn} in such a way that $v = g_0$ does not change, and w becomes real and positive. Then $A_{15} = 0$ and the first two equations of (39) take the form

$$\Delta_1 = -i(v+w)\int \frac{\Delta_1 d^2p}{p^2+\Delta^2}, \quad \Delta_2 = -i(v-w)\int \frac{\Delta_2 d^2p}{p^2+\Delta^2}. \tag{45}$$

The third and fourth equations in (39) yield $A_{41} = A_{31} = 0$.

Of the two possible solutions of the system (45), a large value of the gap corresponds to the solution $\Delta_1 = \Delta$ and $\Delta_2 = 0$, and we obtain for Δ the expression

$$1 = (v+w)\ln(2\omega_0/\Delta) \equiv (v+w)L. \tag{19'}$$

The Green's function (15′) takes the form (15).

The spectrum of the collective excitations is obtained from the equations

$$K_i(\mathbf{n}, k) = \tilde{\pi}_{is}(\omega, \mathbf{nk})\rho\int A_{sr}(\mathbf{nn}')K_r(\mathbf{n}', k)\,d\mathbf{n}'/4\pi$$
$$-\tilde{\pi}_{i4}4\rho V(k)\int K_4(\mathbf{n}', k)\,d\mathbf{n}'/4\pi, \tag{22'}$$

where

$$\tilde{\pi}_{ir} = \frac{i}{4}\,\mathrm{Sp}\int \gamma_i G(p_1)\gamma_r G(p_2)\,d^2p. \tag{46}$$

From (22′) we obtain for the low-lying branch of the excitations, corresponding to the solution $K^4_{00} = 0$, $K^3_{00} \neq 0$, and $K^5_{00} \neq 0$, using (46), (44), and (24), the system of equations

$$K_{00}{}^3 = \left[(v-s)\left(f + \frac{q_3{}^2 - q_3{}^2 f}{q^2}\right)_{00} + 2ir^+\left(\frac{q_4 f}{2\Delta}\right)_{00}\right]K_{00}{}^3$$
$$+\left[2ir^+\left(f + \frac{q_3{}^2 - q_3{}^2 f}{q^2}\right)_{00} + (v-w)\left(\frac{q_4 f}{2\Delta}\right)_{00}\right]K_{00}{}^5,$$
$$K_{00}{}^5 = \left[-(v-s)\left(\frac{q_4 f}{2\Delta}\right)_{00} + 2r^+(L+\beta^2 f)_{00}\right]K_{00}{}^3$$
$$+\left[-2ir^+\left(\frac{q_4 f}{2\Delta}\right)_{00} + (v-w)(L+\beta^2 f)_{00}\right]K_{00}{}^5. \tag{33'}$$

Neglecting the terms that are quadratic in the interaction, we obtain from (33′)

$$\omega^2(\mathbf{k}) = \omega_0{}^2(0) + {}^1/_3 k^2 v_F{}^2, \tag{47}$$

where

$$\omega_0{}^2(0) = 4\Delta^2\frac{2w}{v^2}. \tag{48}$$

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