BREMSSTRAHLUNG IN A STRONG RADIATION FIELD

F. V. BUNKIN and M. V. FEDOROV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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A method is developed for determining the cross sections for multiquantum bremsstrahlung and absorption in the presence of a strong electromagnetic field. Bremsstrahlung due to the scattering by a Coulomb potential is considered in detail. In the extreme case of small fields, the results are identical with the familiar perturbation-theory formulas. Asymptotic expressions are found for the bremsstrahlung cross sections in strong fields.

1. The classical problem of finding the cross section from bremsstrahlung or absorption is usually solved in the first Born approximation in the scatterer potential and in the first perturbation-theory approximation in the electromagnetic field.[1,2] For scattering in a Coulomb field there is a known solution which is not connected with the use of the Born approximation [3]. However, a more exact account of the Coulomb field introduces no appreciable changes in the result (see, for example, [4]).

Perturbation theory yields in practice only the probability of radiation (absorption) of a small number of quanta, and presupposes that the electromagnetic field is weak. The probabilities of the multiquantum processes are then small, so that perturbation theory gives satisfactory results.

At the present time, the development of methods for generating high-power laser radiation has made it possible to obtain exact solutions of many classical problems involving the interaction between an electromagnetic field and matter. In particular, the problem of the bremsstrahlung effect in a strong radiation field has become quite timely. Its practical interest is connected primarily with the fact that experiments were recently realized in which breakdown was produced in a gas by a focused laser beam.[5-8] In the theoretical analysis of the process of formation of the electron cascade [9-11] it is assumed that the acceleration of the electron is connected essentially with the single-quantum bremsstrahlung absorption. It is essential, however, to estimate the relative values of the cross sections of multiquantum processes at threshold fields.

The first attempt at finding the probabilities of multiquantum bremsstrahlung absorption or emission was made in [12]. However, the analysis in that paper is valid essentially only for a weak field, and the probabilities of the n-quantum processes are exaggerated by a factor $(c/v)^{2n}$.

The scattering of electrons by ions in the presence of a strong field was considered by Rand.[13] However, his solution method is not at all suitable for the problem and does not make it possible to calculate the cross sections for multiquantum bremsstrahlung emission or absorption. He carries through to conclusion only the calculations of the total cross section for the absorption of electromagnetic energy by an electron scattered by a Coulomb potential, and furthermore for only one particular case.

We develop in this paper a method which makes it possible to determine directly the cross sections of multiquantum induced emission and absorption when electrons are scattered by arbitrary objects in the presence of a strong electromagnetic field.

2. The behavior of an electron in an alternating electromagnetic field which does not depend on the spatial coordinates is well known. If $\mathbf{A}(t)$ is the vector potential of the electromagnetic field, then the wave functions of the electron are

$$\Psi_{\mathbf{p}} = \frac{1}{(2\pi\hbar)^{3/2}} \exp\left\{\frac{i}{\hbar}\left[\mathbf{pr} - \int_0^t \frac{(\mathbf{p} - e\mathbf{A}(\tau)/c)^2}{2m} d\tau\right]\right\}. \tag{1}$$

We shall assume henceforth that the field is monochromatic and therefore $\mathbf{A}(t) = -\omega^{-1}c\mathbf{E}\cdot\sin\omega t$. The energy of the electron in such a state oscillates about some mean value

$$\langle \overline{H} \rangle = \mathbf{p}^2/2m + (e^2/2mc^2)\overline{\mathbf{A}^2}$$

(the bar denotes time averaging). Therefore the transition from the state Ψ_p into the state $\Psi_{p'}$ is a transition in which the average energy changes by $(p'^2 - p^2)/2m$. The spontaneous transitions can always be neglected compared with the induced transitions, and consequently the probability of the transition $\Psi \rightarrow \Psi'$ with $(p'^2 - p^2)/2m = \pm n\hbar\omega$ determines the probability of absorption (emission) of n quanta of the field.

We determine the probability of transition between the states Ψ_p during scattering by a potential $V(\mathbf{r})$. If we confine ourselves to the Born approximation, then the calculation is carried out in accordance with the standard perturbation-theory method, where it is necessary to choose the functions Ψ_p as the initial functions. As a result we find that the transition probability is of the form

$$w = \sum_{n=-\infty}^{+\infty} w^{(n)}, \qquad w^{(n)} \sim \delta\left(\frac{p'^2 - p^2}{2m} - n\hbar\omega\right). \tag{2}$$

The quantity $w^{(n)}$ determines the probability of absorption, of $|n|$ quanta when $n > 0$ and the emission probability when $n < 0$. Going over from probability to effective cross sections, we find that the cross section for the absorption of n quanta (emission of $|n|$ quanta when n is negative) is determined by the following expression:

$$\sigma^{(n)} = \frac{2m^2 v\hbar\omega\rho}{\pi\hbar^4 E^2 c} \int do \left| \int d\mathbf{r} V(\mathbf{r}) \exp\left\{\frac{imv}{\hbar}(\mathbf{n} - \lambda\mathbf{n}')\mathbf{r}\right\}\right|^2$$

$$\times \lambda J_n{}^2[\gamma(\cos\theta_0(1 - \lambda\cos\theta) - \lambda\sin\theta_0\sin\theta\cos\varphi)]. \tag{3}$$

Here E and ω are the intensity and frequency of the electromagnetic field, m and v are the mass and velocity of the electron, ρ the density of the scattering centers, J_n Bessel functions of order n, and $do = \sin\theta\, d\theta\, d\varphi$ the solid-angle element. The vectors $\mathbf{n}$ and $\mathbf{n}'$ are unit vectors in the directions of the momenta of the incident and scattered electrons. It is assumed further that

$$\mathbf{n} = \{0, 0, 1\}, \quad \mathbf{n}' = \{\sin\theta\cos\varphi,\ \sin\theta\sin\varphi,\ \cos\theta\}.$$

θ_0 is the angle between the direction of polarization of the electric field $\mathbf{E}$ and the vector $\mathbf{n}$. It is assumed that the vector $\mathbf{E}$ lies in the (xz) plane;

$$\lambda = \sqrt{1 + 2n\xi}, \quad \xi = \hbar\omega / mv^2, \quad \gamma = evE / \hbar\omega^2.$$

The main parameter which determines the magnitude of the field E is γ. It can be reduced to the form $(I/I_0)^{1/2}$, where $I = cE^2/8\pi$ is the energy flux density of the laser radiation, $I_0 = c\hbar^2\omega^4/8\pi e^2 v^2$ is the characteristic flux density. Comparison with the results of [12] shows that the characteristic current density found in that paper differs from the foregoing expression by a factor v^2/c^2. This leads to an overestimate of the probability of the n-quantum process by a factor $(c/v)^{2n}$.

If the velocity of the electron is $v \sim 10^8$ cm/sec and the frequency of the electromagnetic field is $\sim 3 \times 10^{15}$ sec^{-1} (the frequency of a ruby laser), then the parameter γ becomes of the order of unity at fields $E \sim 6 \times 10^7$ V/cm, that is, at radiation fluxes $I \sim 5 \times 10^{12}$ W/cm^2. Such fluxes are now already attainable in the focus of a laser. However, in experiments on the production of breakdown in gases, the threshold values of the field correspond to a flux density $I \sim 10^{10}$ W/cm^2, and consequently $\gamma \ll 1$. This indicates that the perturbation-theory results are perfectly satisfactory for this problem, since the cross sections of the many-quantum processes are negligibly small (unlike the predictions of [12]). This conclusion is confirmed also by direct calculations (see formula (12)).

In the general case formula (3) does not admit of further simplification. We shall consider below several particular cases, when the expression for the effective cross section $\sigma^{(n)}$ assumes a simpler form. First, however, we must stop to discuss the limits of applicability of the results. The approach itself implies the assumption that the electron motion is nonrelativistic. Consequently, the condition $v/c \ll 1$ must be satisfied. It has also been assumed that the electromagnetic field does not depend on the spatial coordinates. In other words, we are considering only the dipole approximation with respect to the field. The possibility of so simplifying the problem is connected with the condition $\omega x_0/c \ll 1$, where x_0 is a characteristic length parameter. We must choose x_0 to be the larger of the quantities v/ω (distance covered by the electron within one period of field oscillations) and $eE/m\omega^2$ (the amplitude of the electron oscillations in the alternating electromagnetic field). Consequently, the following conditions must be satisfied simultaneously:

$$v/c \ll 1, \qquad \gamma\xi(v/c) \ll 1. \tag{4}$$

We shall henceforth consider in detail the case of scattering in a Coulomb field. The condition for the applicability of the Born approximation has in this case, as is well known, the form

$$Ze^2/\hbar v \ll 1, \tag{5}$$

where Ze is the charge of the ion.

3. Thus, let us consider the bremsstrahlung effect in scattering by a Coulomb potential. In this case

$$\int V(\mathbf{r}) \exp\left\{\frac{imv}{\hbar}(\mathbf{n} - \lambda\mathbf{n}')\mathbf{r}\right\} d\mathbf{r}$$

$$= \frac{2\pi\hbar^2 \cdot Ze^2}{m^2 v^2 \lambda |\cos\theta - (1 + \lambda^2)/2\lambda|}. \tag{6}$$

We introduce the following notation: when $n > 0$

$$\sigma^{(n)} = \sigma_a^{(n)}, \ \sigma^{(-n)} = \sigma_e^{(n)}, \qquad \sigma_T^{(n)} = \sigma_e^{(n)} - \sigma_a^{(n)};$$

$\sigma_a^{(n)}$ and $\sigma_e^{(n)}$ are respectively the cross sections for bremsstrahlung absorption and emission of n quanta, and $\sigma_T^{(n)}$ the total cross section for the emission of n quanta. We consider the limiting cases of large and small values of the parameters γ and ξ.

a) $\gamma \ll 1$. In this case we can confine ourselves to the first terms of the expansion in a Bessel-function series, so that

$$\sigma^{(n)} = \frac{8\pi Z^2 e^4 \rho \hbar\omega}{m^2 c v^3 E^2 (n!)^2} \left(\frac{\gamma}{2}\right)^{2|n|} \frac{1}{\lambda} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta$$
$$\times \frac{[\cos\theta_0 (1 - \lambda\cos\theta) - \lambda \sin\theta_0 \sin\theta \cos\varphi]^{2|n|}}{(\cos\theta - (1+\lambda^2)/2\lambda)^2}. \quad (7)$$

When n = 1 this expression coincides with the results of Marcuse [2], where the bremsstrahlung's effect was considered in first order of perturbation theory.

Let us consider first the case of slow electrons, that is, $\xi \gg 1$. In this case there is no radiation, and the cross section for the absorption of n quanta is given by the expression

$$\sigma_a^{(n)} = \frac{8\pi Z^2 e^4 \rho}{m^{1/2} c (\hbar\omega)^{1/2} E^2 (n!)^2 2 n^{3/2}} \left(n \frac{e^2 E^2}{m\hbar\omega^3} \right)^n$$
$$\times \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta [\cos\theta_0 \cos\theta - \sin\theta_0 \sin\theta \cos\varphi]^{2n}. \quad (8)$$

Thus, the actual expansion parameter is the quantity $\gamma^2 \xi = e^2 E^2 / m\hbar\omega^3$, in accord with the result of Rand [6]. Expression (7) contains a dependence on the angle θ_0, but this dependence is quite weak. The total cross section for the absorption of one and two quanta (with account of second-order terms in the expansion of $\sigma_a^{(1)}$) for the angles $\theta_0 = 0$ and $\theta_0 = \pi/2$ is of form

$$\sigma_a^{(1)} + \sigma_a^{(2)} = \frac{2^7 \pi^2}{3} \frac{Z^2 e^6 \rho}{c m^{3/2} \hbar^{3/2} \omega^{7/2}}$$
$$\times \left[1 - \frac{3}{5} \left(1 - \frac{1}{2\sqrt{2}} \right) \frac{e^2 E^2}{m\hbar\omega^3} \right]. \quad (9)$$

Let us consider now the case of sufficiently fast electrons, when $n\xi \ll 1$. When n = 1, the cross sections can be calculated for arbitrary values of θ_0. As a result of the integrations, allowing for the smallness of the parameter ξ, we obtain

$$\sigma_{a,e}^{(1)} = \frac{8\pi^2 Z^2 e^6 \rho}{m^2 v c \hbar\omega^3} \Big[(3\cos^2\theta_0 - 1) + (1 - \cos^2\theta_0) \ln\frac{2}{\xi}$$
$$\mp (3\cos^2\theta_0 - 1)\xi \ln\frac{2}{\xi} \pm 2\xi \cdot \cos^2\theta_0 \Big];$$
$$\sigma_T^{(1)} = \frac{2^4 \pi^2 Z^2 e^6 \rho}{m^3 v^3 c \omega^2} \left[(3\cos^2\theta_0 - 1) \left(\ln\frac{2}{\xi} \right) - 2\cos^2\theta_0 \right]. \quad (10)$$

This confirms the deductions of Marcuse [2,4] that the induced bremsstrahlung prevails over absorption if $\theta_0 = 0$, that is, if the incident electrons move along the electric-field vector.

When n > 1, the calculation of the cross sections in the case of arbitrary values of θ_0 does not lead to simple expressions. We therefore present below only the formulas for small angles $|\theta_0| \ll 1$.

When n = 2 and $\theta_0 = 0$

$$\sigma_{a,e}^{(2)} = \frac{\pi^2 Z^2 e^8 \rho v E^2}{m^2 \hbar^3 \omega^7 c} \left[\frac{8}{3} + \frac{5}{2}(2\xi)^2 \pm \frac{26}{3}(2\xi)^3 \mp 8(2\xi)^3 \ln\frac{1}{\xi} \right],$$
$$\sigma_T^{(2)} = \frac{2^7 \pi^2 Z^2 e^8 \rho E^2}{m^5 v^5 c \omega^4} \left(\ln\frac{1}{\xi} - \frac{13}{12} \right). \quad (11)$$

When n > 2 and $\theta_0 = 0$ we have, accurate to terms of higher orders in the small parameter $n\xi$,

$$\sigma_{a,e}^{(n)} = \frac{8\pi^2 Z^2 e^4 \rho \hbar\omega}{m^2 v^3 c E^2 (n!)^2} \left(\frac{evE}{\hbar\omega^2} \right)^{2n} \left[\frac{1}{2n-1} \pm \frac{n-2}{n-1} \frac{n\xi}{2} \right],$$
$$\sigma_T^{(n)} = -\frac{8\pi^2 Z^2 e^4 \rho \hbar^2 \omega^2}{m^3 v^5 c E^2} \frac{n}{(n!)^2} \frac{n-2}{n-1} \left(\frac{evE}{\hbar\omega^2} \right)^{2n}. \quad (12)$$

When θ_0 is small the cross sections differ little from their values when $\theta_0 = 0$

$$\delta\sigma_T^{(2)} = \sigma_T^{(2)}(\theta_0) - \sigma_T^{(2)}(0) = \frac{8\pi^2 Z^2 e^8 \rho E^2}{m^3 v c \hbar^2 \omega^6} \cdot \ln\frac{1}{\xi} \cdot \theta_0^2. \quad (13)$$

When n > 2

$$\delta\sigma_{a,e}^{(n)} = \sigma_{a,e}^{(n)}(\theta_0) - \sigma_{a,e}^{(n)}(0) = \frac{4\pi^2 Z^2 e^4 \rho \hbar\omega}{m^2 v^3 c E^2 (n!)^2} \left(\frac{evE}{\hbar\omega^2} \right)^{2n}$$
$$\times \Big[-\frac{8n^2}{2n+1} \pm \frac{2n-1}{(2n-3)(n-1)}$$
$$\times (-4n^3 + 12n^2 - 6n - 7) \frac{n\xi}{2} \Big] \cdot \theta_0^2,$$
$$\delta\sigma_T^{(n)} = -\frac{4\pi^2 Z^2 e^4 \rho \hbar^2 \omega^2}{m^3 v^5 c E^2} \frac{n}{(n!)^2} \frac{2n-1}{(2n-3)(n-1)}$$
$$\times (-4n^3 + 12n^2 - 6n - 7) \left(\frac{evE}{\hbar\omega^2} \right)^{2n} \cdot \theta_0^2. \quad (14)$$

b) $\gamma \gg 1$. In this case we use everywhere, except in a narrow region of values of the argument, the asymptotic representation of the Bessel functions.

Let us consider separately the cases when the electrons move along the electric field ($\theta_0 = 0$) and perpendicular to it ($\theta_0 = \pi/2$).

When $\theta_0 = 0$ the use of the asymptotic values of the Bessel function for $\gamma \gg 1$ leads to the following expressions for the cross sections:

$$\sigma_a^{(n)} = \frac{2^6 \pi^3 Z^2 e^3 \rho\omega}{c E^3 n^2} \left\{ 2\ln\gamma + \ln[2n\xi(1 + 2n\xi)] + \frac{2}{n\xi} \right\}. \quad (15)$$

When $n\xi \le 1/2$

$$\sigma_e^{(n)} = \frac{2^6 \pi^3 Z^2 e^3 \rho\omega}{c E^3 n^2} \left[\ln\frac{1 - \sqrt{1 - 2n\xi}}{1 + \sqrt{1 - 2n\xi}} + \frac{2}{n\xi}\sqrt{1 - 2n\xi} \right],$$

$$\sigma_T{}^{(n)} = -\frac{2^6\pi^3 Z^2 e^3 \rho\omega}{cE^3 n^2}\Big\{2\ln\gamma + \ln[(1+2n\xi)(1+\sqrt{1-2n\xi})^2] + \frac{2}{n\xi}(1-\sqrt{1-2n\xi})\Big\} < 0. \tag{16}$$

When $n\xi \geq 1/2$ there is no radiation, so that $\sigma_T^{(n)} = -\sigma_a^{(n)}$, where $\sigma_a^{(n)}$ is given by (15).

We now consider the case when the initial velocity of the electron is perpendicular to the electric field, $\theta_0 = \pi/2$. Transformation of the general expression (3) yields for the cross section

$$\sigma^{(n)} = \frac{2^{11}\pi Z^2 e^3 \rho\hbar^2\omega^3}{cE^3 m^2 v^4}\left|\frac{1-\lambda}{1+\lambda}\right| \times \frac{1+\lambda^2}{(1+\lambda)^2(1-\lambda)^4}\cdot\ln(\lambda\gamma), \tag{17}$$

and when $|n|\xi \leq 1/2$ the number n can be either positive or negative that is, both absorption and emission take place. When $|n|\xi \geq 1/2$ there is no emission, that is, $\sigma^{(n)} = 0$.

For fast electrons and sufficiently small n, that is, if $n\xi \ll 1$, this yields

$$\sigma_{a,e}^{(n)} = \frac{2^5\pi Z^2 e^3\rho\hbar^3\omega^4}{cE^3 m^3 v^6 n^3}(1 \mp n\xi)\ln\gamma,$$

$$\sigma_T{}^{(n)} = \frac{2^6\pi Z^2 e^3\hbar^4\omega^5\rho}{cE^3 m^4 v^8 n^2}\ln\left(\frac{evE}{\hbar\omega^2}\right) > 0. \tag{18}$$

However, this result still does not give grounds for assuming that in the case when $\theta_0 = \pi/2$ and $\xi \ll 1$ the induced bremsstrahlung prevails over absorption at large fields. This is connected with the fact that at sufficiently large n we always have $n\xi > 1/2$ and consequently $\sigma_e^{(n)} = 0$ and $\sigma_T^{(n)} = -\sigma_a^{(n)} < 0$. The sign of the total emission cross section $\sigma_T = \Sigma n\sigma_T^{(n)}$ remains undetermined.

For slow electrons or for large values of $n(n\xi \gg 1)$, formula (17) leads to the expression

$$\sigma_a{}^{(n)} = \frac{2^6\pi^3 Z^2 e^3\rho\omega}{cE^3 n^2}\ln\left(n\frac{e^2E^2}{m\hbar\omega^3}\right). \tag{19}$$

The change in the cross sections for small deviations from the angles $\theta_0 = 0$ and $\theta_0 = \pi/2$ is determined by the second derivatives of the cross sections with respect to θ_0, since

$$(d\sigma_{a,e,T}^{(n)}/d\theta_0)_{\theta_0=0} = (d\sigma_{a,e,T}^{(n)}/d\theta_0)_{\theta_0=\pi/2} = 0.$$

We have left out from the formulas that follow the factor $2^5\pi^3 Z^2 e^3\rho\omega c^{-1}E^{-3}n^{-2}$:

$$\left(\frac{d^2\sigma_a{}^{(n)}}{d\theta_0^2}\right)_{\theta_0=0} \sim -\frac{4\gamma^2}{n^3\xi^3}[n^2\xi^2 + 4n\xi + 2(1+n\xi)\sqrt{1+2n\xi} + 2], \tag{20}$$

$$\left(\frac{d^2\sigma_e{}^{(n)}}{d\theta_0^2}\right)_{\theta_0=0} \sim \frac{1}{n^3\xi^3}\Big[-(2-n\xi)\ln\frac{(1-\sqrt{1-2n\xi})^2}{2n\xi} + \frac{\sqrt{1-2n\xi}}{2n^2\xi^2}(4n^2\xi^2 - 13n\xi + 12)\Big]; \quad n\xi \leqslant 1/2$$

$$\left(\frac{d^2\sigma_e{}^{(n)}}{d\theta_0^2}\right)_{\theta_0=0} = 0, \quad n\xi \geqslant 1/2, \tag{21}$$

$$\left(\frac{d^2\sigma_{a,e}^{(n)}}{d\theta_0^2}\right)_{\theta_0=\pi/2} \sim \mp\frac{4\gamma^2}{n^3\xi^3}[n^2\xi^2 \pm 4n\xi + 2(1\pm n\xi)\sqrt{1\pm 2n\xi} + 2]. \tag{22}$$

Thus, at angles θ_0 close to 0 or to $\pi/2$ we have

$$\sigma_{a,e}^{(n)}\left(\frac{\pi}{2}+\delta\theta_0\right) \approx \sigma_{a,e,T}^{(n)}\left(\frac{\pi}{2}\right) + \frac{(\delta\theta_0)^2}{2}\left(\frac{d^2\sigma_{a,e,T}^{(n)}}{d\theta_0^2}\right)_{\theta_0=\pi/2}.$$

$$\sigma_{a,e,T}^{(n)}(\delta\theta_0) \approx \sigma_{a,e,T}^{(n)}(0) + \frac{(\delta\theta_0)^2}{2}\left(\frac{d^2\sigma_{a,e,T}^{(n)}}{d\theta_0^2}\right)_{\theta_0=0}.$$

The condition for the second term of the expansions to be small compared with the zero-order term takes the form $|\delta\theta_0| \ll \gamma^{-2}\ln\gamma$ in all cases, except the expansion of $\sigma_e^{(n)}(\theta_0)$ when θ_0 is close to zero. In this case the less stringent condition $|\delta\theta_0| \ll 1$ should be satisfied.

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