VIOLATION OF BARRIER PENETRATION SYMMETRY FOR COMPOSITE PARTICLES

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For elementary particles the penetrability of any barrier in either direction is strictly the same [5]. However, the penetration coefficients for asymmetric barriers in opposite directions may be very different for composite particles if their energy is sufficient for actual excitation of the higher states of internal motion. This effect should manifest itself in various atomic and nuclear phenomena.

1. INTRODUCTION

THIS paper is a continuation of investigations of specific properties of composite particles moving in an external field [1,2]. Many properties of such systems can be studied on the simplest example of a one-dimensional three-body problem. For specific calculations we shall utilize a method whose correctness was investigated in [3].

Taking the internal structure of particles into account leads to essential changes in their properties compared to simple particles. The tunnel effect for composite particles turns out to be more pronounced [1], and at energies sufficient for the excitation of higher internal states penetration symmetry is violated for barriers of nonsymmetric shape. The latter phenomenon may be given the following qualitative explanation.

We consider a potential barrier which falls off sharply on the left and varies smoothly on the right (for example, the barrier shown in Fig. 1). Let a beam of composite particles in their ground state be incident on this barrier from the left. The kinetic energy of these particles will be $E - \varepsilon_g$ (E is the total energy, ε_g is the energy of the ground state of the internal motion of the composite particles). At the left edge of the barrier the incident particles will be strongly excited; in the excited state the kinetic energy of the center of mass turns out to be appreciably lower: $E - \varepsilon_e = E - \varepsilon_g - \Delta E$ (ε_e is the energy of the excited state for internal motion, $\Delta E = \varepsilon_e - \varepsilon_g$). This

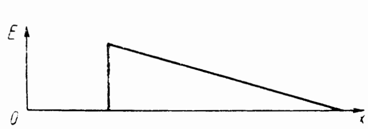

FIG. 1

will lead to a decrease in the penetrability, since the excited particles will be mainly reflected. But if the particles in their ground state are incident on the barrier from the right, then they will begin to be excited significantly only when they will have traversed practically the whole barrier. In this case it is easier for the excited particles to proceed to the left than to return, and, therefore, the possibility of excitation in practice does not make the penetration of the barrier more difficult. Thus, if the particles are incident on the barrier in their ground state the coefficient of transmission from the right D_g^- will be greater than the coefficient of transmission from the left D_g^+.

But if the particles incident on the barrier are in an excited state then we have the opposite picture. For particles incident from the left the intensity of transition to the ground state is large already at the beginning of the barrier and the added increment ΔE to the kinetic energy facilitates their overcoming the barrier. But particles incident from the right are forced to penetrate through the whole barrier with a low value of kinetic energy, and this strongly reduces the penetrability.

The arguments given above do not take into account the interference of waves moving in opposite directions. However, they do enable us to predict the result qualitatively with a high degree of probability in the majority of practically interesting cases.

Unfortunately, the complicated nature of the three-body problem makes a rigorous analysis of this phenomenon very difficult. In the general case in order to obtain precise estimates it is necessary to carry out the calculations with the aid of an electronic computer.

In Sec. 2 the basic equations are given describing the motion of a composite particle in an

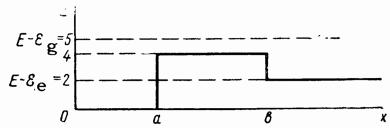

FIG. 2

external field, and it is shown that the ratio between the coefficients of barrier penetrability in opposite directions must depend on the shape of the barrier. The energy dependence of the effect under discussion at the threshold of excitation of the composite particle is given. In Sec. 3 a discussion of this effect is given in the limiting case of small asymmetry of the barrier. In Sec. 4 we give results of calculations by means of an electronic computer for a specific potential barrier (cf., Fig. 2).

2. BASIC EQUATIONS

The motion of two bound particles with the coordinates x_1 and x_2 and of masses m_1 and m_2 in an external field is described by the Schrödinger equation

$$\left[-\frac{1}{2M}\frac{\partial^2}{\partial R^2}-\frac{1}{2\mu}\frac{\partial^2}{\partial\rho^2}+V_{12}(\rho)+V_1\left(R+\frac{m_2\rho}{m_1+m_2}\right)+V_2\left(R-\frac{m_1\rho}{m_1+m_2}\right)\right]\Psi(R,\rho)=E\Psi(R,\rho), \tag{1}$$

where

$$M=m_1+m_2;\ \mu=m_1m_2/(m_1+m_2);$$
$$R=(m_1x_1+m_2x_2)/(m_1+m_2);$$

$\rho = x_1 - x_2$; V_{12} is the potential for the interaction between the two particles; V_i is the potential of the external field acting on the i-th particle.

The wave function Ψ for the system can be expanded in terms of the complete set of functions Φ_n describing the internal motion of the composite particle [4]:

$$\Psi=\sum_n \varphi_n(R)\Phi_n(\rho), \tag{2}$$

where Φ_n satisfy the equation (the dot denotes differentiation with respect to ρ)

$$-\frac{1}{2\mu}\ddot{\Phi}_n+V_{12}\Phi_n=\mathscr{E}_n\Phi_n. \tag{3}$$

For the functions φ_n describing the motion of the center of mass of the composite particle which is in the state n with respect to the internal motion we obtain the system of coupled equations:

$$\varphi_n''-2M(\mathscr{E}_n-E)\varphi_n=\sum_m w_{mn}\varphi_m, \tag{4}$$

where the mixing coefficients w_{mn}, characterizing the intensity of transitions between the internal states of the composite particle under the action of an external field have the form

$$w_{mn}(R)=2M\int_{-\infty}^{\infty}(V_1+V_2)\Phi_m\Phi_n\,d\rho. \tag{5}$$

We first consider the case when the energy of the particle is insufficient for a real excitation of higher internal states:

$$E-\mathscr{E}_g<\mathscr{E}_e-\mathscr{E}_g\ (\mathscr{E}_g=\mathscr{E}_1;\ \mathscr{E}_e=\mathscr{E}_n;\ n\geqslant 2), \tag{6}$$

i.e., the particle can recede to infinity only in the ground state, while φ_n for $n \geq 2$ fall off exponentially far from the barrier. The system (4) with such boundary conditions has two linearly independent solutions.[1)] We shall show that in this case the coefficients of penetrability through the barrier in opposite directions are strictly the same.[2)] For the two linearly independent solutions we choose the solution φ_-, describing the case when a unit flux of particles in the ground state is incident on the potential barrier from the right, and φ_-^*. The functions φ_{1-}, φ_{1-}^* have the asymptotic form

$$\varphi_{1-}=\begin{cases}e^{-ik_1'R}+B_{1-}e^{ik_1'R}, & R\to\infty\\ A_{1-}e^{-ik_1R} & R\to-\infty\end{cases};$$
$$\varphi_{1-}^*=\begin{cases}e^{ik_1'R}+B_{1-}^*e^{-ik_1'R}, & R\to\infty\\ A_{1-}^*e^{ik_1R} & R\to-\infty\end{cases}.$$

$$k=\sqrt{2M\left[E-\mathscr{E}_g-\sum_i V_i(-\infty)\right]};$$
$$k'=\sqrt{2M\left[E-\mathscr{E}_g-\sum V_i(\infty)\right]}. \tag{7}$$

The penetrability coefficient is $D^- = |A_{1-}|^2(k/k')$. With the aid of the solutions φ_- and φ_-^* we can construct any arbitrary solution of (4) which satisfies the same boundary conditions which hold for φ_n $(n \geq 2)$, for example, when a unit flux of particles in the ground state φ_+ is incident on the barrier from the left. The asymptotic form of φ_{1+} is the following:

$$\varphi_{1+}=\begin{cases}A_{1+}e^{ik_1'R}, & R\to\infty\\ e^{ik_1R}+B_{1+}e^{-ik_1R}, & R\to-\infty\end{cases}. \tag{8}$$

The penetrability coefficient in this case is $D^+ = |A_{1+}|^2(k'/k)$.

In order to construct φ_+ with the aid of φ_-, φ_-^* we multiply φ_- by $-B_{1-}^*/A_{1-}^*$, we divide φ_-^* by A_{1-}^* and add. We obtain:

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1)At energy sufficient for the excitation of m states of the composite particle the number of linearly independent solutions with φ_n for $n > m$ falling off at infinity will be 2m.

2)This is a generalization of the proof for simple particles to the case of a large number of coupled equations[5].

$$A_{1+} = (1 - |B_{1-}|^2) / A_{1-}^* = (k/k') A_{1-}, \qquad (9)$$

from which it follows that $D^+ = D^-$.

We now consider the case when the energy of the composite particle is such that the latter can recede to infinity in the ground and in the first excited states. In this case the system (4) has four linearly independent solutions satisfying the condition that φ_n for $n > 2$ fall off at infinity. Consequently, with the aid of a certain solution and its complex conjugate (two solutions) we can no longer construct any arbitrary solution for the system. Therefore, in this case we do not obtain a unique relation between two different processes and the ratio between the coefficients for the penetration of the barrier by particles moving in opposite directions will depend on the specific form of the barrier.

There exists a more complicated relation between the processes. For example, with the aid of two different solutions of the system and their complex conjugates (altogether four solutions) we can construct any arbitrary solution satisfying the requirement that it be bounded at infinity ($E - \varepsilon_g < \varepsilon_2 - \varepsilon_g$). Thus, if we know the solutions for two processes, when a beam of particles in only the ground or only the first excited state is incident from $-\infty$, then with the aid of these solutions (and their conjugates) we can describe any other process, for example, when the particles are incident on the barrier from the right.

Near the threshold of excitation (above the threshold) the penetrability coefficients $D_g^\pm$ and $D_e^\pm$ have the following form:

$$D_g^\pm = D_g^0 + k_2 S^\pm; \quad D_e^\pm = k_2 N^\pm, \qquad (10)$$

where S and N are constants, D_g^0 is the coefficient of penetrability at the threshold $k_2 = \sqrt{2M(E - \varepsilon_2)}$. The theory of multichannel processes at the threshold[5,6] cannot predict the values of the constants S and N, but having measured them experimentally or having calculated them on an electronic computer for one value of the energy, we can by means of formula (10) predict the magnitude of the effect of penetrability asymmetry at any other arbitrary energy near the threshold:

$$D_g^+ - D_g^- = k_2(S^+ - S^-); \quad D_e^+ - D_e^- = k_2(N^+ - N^-).$$

3. THE CASE OF WEAK ASYMMETRY

The mechanism for the violation of the symmetry of the penetrability was qualitatively discussed in the introduction. A more rigorous argument can be given in the case of small asymmetry of the barriers.

We assume that V_i in (1) depends so smoothly on x that we can neglect transitions between the internal states when a composite particle passes through it. Then the motion of the composite particle in the i-th state will be described by the equation

$$-\varphi_i'' + w_{ii}\varphi_i = (E - \mathscr{E}_i)\varphi_i. \qquad (11)$$

We consider for each i-th state two solutions φ_{i+} and φ_{i-}, corresponding to particles incident on the barrier from the right and from the left:

$$\varphi_{i+}(R \to -\infty) = e^{ik_iR} + B_i e^{-ik_iR}; \quad \varphi_{i+}(R \to \infty) = A_i e^{ik_iR};$$

$$\varphi_{i-}(R \to -\infty) = A_i e^{-ik_iR};$$

$$\varphi_{i-}(R \to \infty) = e^{-ik_iR} + B_i e^{ik_iR}. \qquad (12)$$

We now add to the potential barrier a small nonsymmetric perturbation V′, which leads to a mixing of the states:

$$-\psi_i'' + (w_{ii} + w_{ii}')\psi_i - (E - \mathscr{E}_i)\psi_i = \sum_j w_{ij}'\psi_j. \qquad (13)$$

In the case when composite particles are incident on the barrier in the i-th state, the system (1) can be represented in the integral form

$$\psi_n = \varphi_{n+} \int_{-\infty}^{R} \frac{\varphi_{n-}}{\Delta_n} \sum_m w_{nm}' \psi_m \, dR + \varphi_{n-} \int_{R}^{\infty} \frac{\varphi_{n+}}{\Delta_n} \sum_m w_{nm}' \psi_m \, dR + C_n \varphi_{n\alpha}, \qquad (14)$$

where $C_i = 1$, $C_n = 0$ for $n \neq i$, while $\alpha = +$, if the particles are incident from the left and $\alpha = -$, if the particles are incident from the right, $\Delta_n = -iA_ik_i$.

We solve the system (14) by iterations. In the first approximation we obtain

$$\psi_n = \varphi_{n+} \int_{-\infty}^{R} \frac{\varphi_{n-}}{\Delta_n} \sum_m w_{nm}' C_m \varphi_{m\alpha} dR + \varphi_{n-} \int_{R}^{\infty} \frac{\varphi_{n+}}{\Delta_n} \sum_m w_{nm}' C_m \varphi_{m\alpha} dR + C_n \varphi_{n\alpha}. \qquad (15)$$

For the sake of simplicity we consider the case of only two equations in (1) corresponding to the ground and the first excited states. In the case when particles in their ground state are incident on the barrier from the left the coefficient D_g^+ has in the first approximation of perturbation theory the form

$$D_g^+ = |A_1|^2 \left| 1 + \int_{-\infty}^{\infty} \frac{\varphi_{1-}}{\Delta_1} w_{11}' \varphi_{1+} dR \right|^2 + \frac{k_2}{k_1} \left| \int_{-\infty}^{\infty} \frac{\varphi_{2-}}{\Delta_2} w_{21}' \varphi_{1+} dR \right|^2 |A_2|^2. \qquad (16)$$

When particles in the ground state are incident from the right we have

$$D_g^- = |A_1|^2\left|1+\int_{-\infty}^{\infty}\frac{\varphi_{1+}}{\Delta_1}w_{11}'\varphi_{1-}dR\right|^2 + \frac{k_2}{k_1}\left|\int_{-\infty}^{\infty}\frac{\varphi_{2+}}{\Delta_2}w_{21}'\varphi_{1-}dR\right|^2|A_2|^2, \quad (17)$$

D_g^+ and D_g^- differ by the amount

$$D_g^+ - D_g^- = \frac{k_2}{k_1}\left\{\left|\int_{-\infty}^{\infty}\frac{\varphi_{2-}}{\Delta_2}w_{21}'\varphi_{1+}dR\right|^2 - \left|\int_{-\infty}^{\infty}\frac{\varphi_{2+}}{\Delta_2}w_{21}'\varphi_{1-}dR\right|^2\right\}|A_2|^2. \quad (18)$$

Similarly we obtain for the excited state

$$D_e^+ - D_e^- = \frac{k_1}{k_2}\left\{\left|\int_{-\infty}^{\infty}\frac{\varphi_{1-}}{\Delta_1}w_{21}'\varphi_{2+}dR\right|^2 - \left|\int_{-\infty}\frac{\varphi_{1+}}{\Delta_1}w_{21}'\varphi_{2-}\right|^2\right\}|A_1|^2. \quad (19)$$

Let w′ be different from zero in a region small compared to k_1^{-1} near R_0, while R_0 is situated to the right of the barrier. We can then replace in the integrands the functions φ by their asymptotic values (12). Moreover, we shall take the functions φ outside the integrals as varying slowly in the region where w′ is different from zero:

$$D_g^+ - D_g^- = \frac{1}{k_1k_2}\left|\int_{-\infty}^{\infty}w_{21}dR\right|^2\Big[(1+|B_2|^2 + 2\operatorname{Re}B_2e^{-i(k_1+k_2)R_0})\left|\frac{A_1}{A_2}\right|^2 - (1+|B_1|^2+2\operatorname{Re}B_1e^{-i(k_1+k_2)R_0})\Big], \quad (20)$$

$$D_e^+ - D_e^- = -|A_2/A_1|^2(D_g^+ - D_g^-). \quad (20a)$$

In order to eliminate the effect of interference phenomena mentioned in the introduction, we shall average (20) also over an energy interval of such width that we can neglect the average of the oscillating quantities $2\operatorname{Re}\{B_i \exp(-i(k_1+k_2)R_0)\}$. After this we obtain

$$\overline{D_g^+ - D_g^-} > 0;\quad \overline{D_e^+ - D_e^-} < 0;\quad \overline{\left(\frac{D_e^- - D_e^+}{D_g^+ - D_g^-}\right)} < 1, \quad (21)$$

since

$$|B_i|^2 = 1 - |A_i|^2;\qquad |A_i|^2 > |A_2|^2.$$

4. SPECIFIC MODEL

For numerical calculations we have chosen the potential shown in Fig. 2. From considerations given in the Introduction one can predict from the outset with a high degree of confidence that particles incident on this barrier in the ground state, just as in the case of the potential in Fig. 1, will be able to penetrate the barrier easier when moving to the right, while particles incident in an excited state will be able to penetrate it easier when moving to the left. The potential V_{12} was chosen in the form of an infinite rectangular well of width π. The calculations were carried out for different values of the barrier width. The dependence of the coefficients of penetration through the potential barrier (at an energy E = 6.01) on the quantity $\Delta = b - a - \pi$ is shown in Fig. 3. The ratios are: $D_g^-/D_g^+ = 3.5$, and $D_e^+/D_e^- = 100 - 600$.

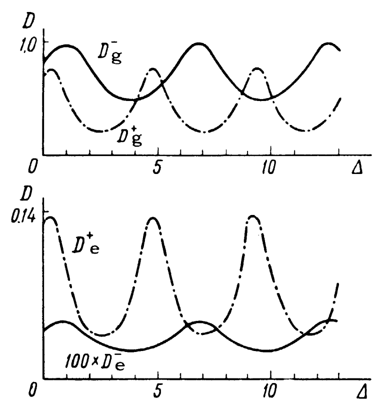

FIG. 3

As the total energy E increases the difference in the penetrability coefficients in opposite directions diminishes, which can be easily understood from the arguments given in the introduction. Figure 3 graphically demonstrates the effect of interference on the magnitude of the effect. As the barrier width is varied the conditions for the interference of waves moving in opposite directions are altered. This leads to periodic variations of the coefficient D. In some cases interference leads to a change in the sign of the effect under consideration, but on the average the effect is predicted correctly.

It should also be noted that a violation of the symmetry of barrier penetrability is also possible for simple particles in the case when the external field is set up by a system capable of being excited.

In conclusion we express our gratitude to S. N. Sokolov in discussions with whom the idea for the present paper originated.

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