A LASER OPERATED WITH A SATURABLE FILTER

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A rate-equation analysis is performed of the processes in a laser in which a saturable filter is employed for Q switching. It is shown that two modes of excitation of the laser exist (a soft and hard regime). In the hard regime the excitation threshold of the laser is determined by the parameters of the system. It is also demonstrated that when the amplitude of the triggering signal exceeds threshold, a pulse of standard amplitude and width is produced in the system. If spontaneous decay can be neglected, the problem can be solved by quadrature. The condition for generation of a giant pulse and the maximum values of its properties are derived in this case. The limiting values of the decay time and the width of the giant pulse are found. The results of the calculations are compared with experiment.

It has been shown in a number of papers [1-4] that certain substances, in particular, solutions of certain metallic phthalocyanines, exhibit a decrease of absorption coefficient under the influence of light from a ruby laser. This property of the phthalocyanine solutions has found application in Q-switching lasers.

The kinetics of the processes which occur in a laser with such a filter may be described by rate equations. We assume that in a single-mode cavity there are two types of particles, each of which has two energy levels; the separation between these energy levels is the same for the two types of particles. It is assumed that initially the particles of the first (or "n") type (ruby) are in a negative temperature state, whereas the second (or "m") type of particles are in their ground state.

The system of rate equations for the level populations and the number of photons in the cavity is the following:

$$\frac{dn}{dt} = -2w_n\rho n - \frac{N_0+n}{\tau_n} + w_p(N_0-n),$$

$$N_0 = n_1 + n_2, \qquad n = n_2 - n_1,$$

$$\frac{dm}{dt} = -2w_m\rho m + \frac{M_0-m}{\tau_m},$$

$$M_0 = m_1 + m_2, \qquad m = m_1 - m_2,$$

$$\frac{d\rho}{dt} = w_n n\rho - w_m m\rho - \frac{\rho}{\tau_p} + \frac{N_0+n}{2k_n\tau_n} + \frac{M_0-m}{2k_m\tau_m},$$

$$k_{n,m} = \frac{8\pi\nu^2}{c^3}\Delta\nu_{n,m} V/\eta_{n,m}. \qquad (1)$$

The following notation has been introduced: n_1, n_2, m_1, m_2 are the total number of particles in the cavity in the "n" and "m" systems in the lower (subscript 1) and in the upper (subscript 2) levels; w_n and w_m are the probabilities for stimulated transitions per unit time; $w_{n,m} = \sigma_{n,m}c/V$, where $\sigma_{n,m}$ are the cross-sections for absorption in the respective systems, c is the velocity of light in the medium, V is the resonator volume; τ_n and τ_m are the spontaneous decay times for the upper state in system n and m respectively; ρ is the total number of photons in the cavity; τ_p is the decay time for the field in the cavity; the coefficients k_n and k_m describe the fraction of the spontaneous emission in systems "n" and "m" respectively, which occurs in the resonator mode under discussion; $k_{n,m}$ are the number of modes in the volume V within the line width $\Delta\nu_{n,m}$ divided by the quantum yield of the fluorescence $\eta_{n,m}$; w_p is the probability for exciting the "n" type particles via the pumping radiation.

The system (1) was solved with the M-20 electronic computer. In making the calculation the following values were assumed for the parameters of the saturable filter: $N_0 = 10^{20}$, $m_0 = M_0 = 2.5 \times 10^{15}$; $w_n = 1.2 \times 10^{-10}$ sec^{-1}, which for a resonator volume V = 10 cm^3 corresponds to an absorption cross section $\sigma_n = 4 \times 10^{-20}$ cm^2, $w_m = 4.8 \times 10^{-6}$ sec^{-1}, and correspondingly $\sigma_m = 1.6 \times 10^{-15}$ cm^2; $\tau_p = 10^{-9}$ sec. The spontaneous decay time τ_m and the initial conditions of the system (1) were varied, with the initial difference in populations n_0 and $m_0 = M_0$ chosen so that

$$w_n n_0 - w_m m_0 = 0. \qquad (2)$$

This means that there is no gain in the medium filling the resonator at the time t = 0.

It is assumed that a certain initial number of photons ρ_0 is "injected" into the cavity at the instant $t_0 = 0$. For an initial injection of photons ρ_0 above a threshold value $\rho_{thresh.}$ the laser processes under consideration evolve in an extremely short time ($\Delta t \ll \tau_n$, $1/w_p$, $k_m \tau_m$); hence during the calculation several aspects of the system of equations (1) were neglected, viz., the pumping and the spontaneous decay from the upper level of n-type particles were neglected in the first equation; in the second equation we neglected the spontaneous decay from the upper level of the m-type particles (this corresponds to taking $\tau_n = \infty$, $w_p = 0$, and $k_m \tau_m = \infty$).

For a sufficiently large trigger signal a very strong emission pulse occurs from n-type particles and the system produces an intense (so-called giant) pulse; somewhat before these processes occur the medium is completely bleached (that is, half of the m-type particles are transferred from the lower energy level to the upper level). Decreasing the initial injection of phonons ρ_0 delays the formation of the giant pulse. This is because of the increased time required to bleach the filter. Further decrease in the quantity ρ_0 results in inhibition of giant pulse formation, the initial number of photons being insufficient to bleach the medium.

At the instant during the formation of the giant pulse when ρ takes on its minimum value ρ_{min} the condition for self-excited oscillations is satisfied:

$$w_n n = w_m m + \tau_p^{-1} \qquad (3)$$

(initially $w_n n_0 < w_m m_0 + \tau_p^{-1}$ and without external triggering the system cannot begin to oscillate). Condition (3) means that the total gain in the system equals unity and that the time constant for the decay of the oscillations is infinite.

An interesting feature of this system is that it exhibits threshold properties; in particular, so long as the initial triggering signal ρ_0 does not exceed a certain threshold value ρ_{thresh} the system does not oscillate, but as soon as ρ_0 exceeds ρ_{thresh} by ever so little a pulse builds up (with standard, repeatable amplitude and duration between the half-power points). This pulse contains almost all of the energy previously stored by the n-type particles. Moreover once the filter has been bleached the growth of the laser pulse occurs at a rate which is independent of ρ_0 (but which is determined by the parameters of the system); also independent of ρ_0 is the rate at which the active medium is depleted. These phenomena are illustrated in Figs. 1 and 2. In Fig. 1 we give the time dependence of the number of photons (solid line) and the population inversion of the n-type particles (dashed line). The functions are normalized to their initial values. (The curves for the number of photons are plotted with a logarithmic scale.) The lifetime for spontaneous decay of the m-type particles is taken to be constant and equal to 10^{-9} sec; ρ_0 is a parameter that varies from curve to curve. For this choice of system constants a value of $\rho_0 = 1.5 \times 10^{13}$ is insufficient to cause laser action (this is shown by the dot-dashed curve on a non-logarithmic scale). Threshold for ρ_0 in the present case lies between 1.75×10^{13} and 1.5×10^{13}, the maximum pulse strength is $\rho_{max} = 0.35 n_0 = 0.35 N_0 = 3.5 \times 10^{19}$. In Fig. 2 we give the time dependence of the population inversion of the filter particles for the same values of the parameters; bleaching does not occur for $\rho_0 = 1.5 \times 10^{13}$; the curve n(t) has a minimum not equal to zero and the inversion of the active particles remains practically constant.

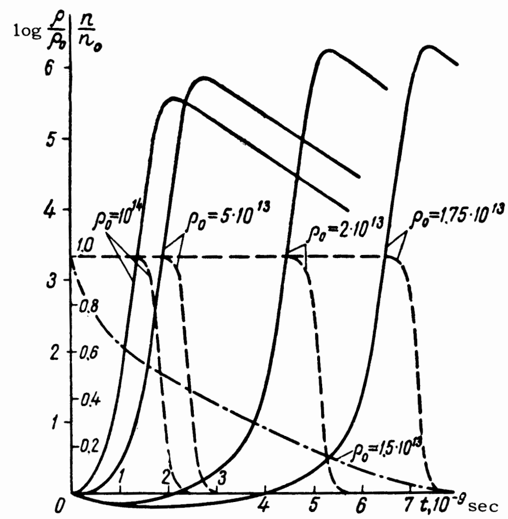

FIG. 1. Graphs of the time dependence of the number of photons at the resonance frequency and the population inversion of the active centers in the laser for various strengths of the triggering pulse ($n_0 = 10^{20}$, $m_0 = 2.5 \times 10^{15}$, $\tau_m = 10^{-9}$ sec.)

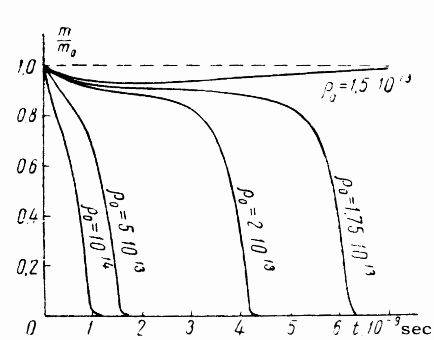

FIG. 2. The time dependence of population inversion in the filter for various initial triggering pulses ($n_0 = 10^{20}$, $m_0 = 2.5 \times 10^{15}$, $\tau_m = 10^{-9}$ sec).

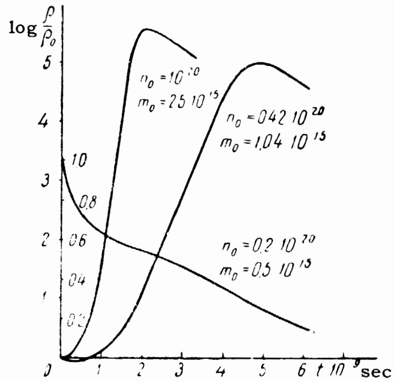

FIG. 3. The dependence of the number of photons in the laser for various degrees of initial inversion ($\rho_0 = 10^{14}$, $\tau_m = 10^{-9}$ sec).

If the n-type particles are not completely inverted at t = 0, then it is clear that the amplitude of the laser pulses and their growth rate will be smaller and that the delay will be greater than for the case of complete inversion. This situation is shown in Fig. 3. For $\rho_0 = 10^{14}$, $\tau_m = 10^{-9}$ sec, an initial inversion of the active particles $n_0 = 0.2 \times 10^{20}$ is insufficient for producing a laser pulse (the initial values of n_0 were chosen in accordance with condition (2)). To form a pulse in this case and to decrease the delay it is necessary to have a more intense triggering pulse.

The behavior of the system for different spontaneous decay times of the n-type particles is illustrated in Figs. 4–6. As in the previous discussion there is also a threshold in the parameter τ_m, assuming a constant initial triggering pulse ρ_0. For $\rho_0 = 10^{14}$ the threshold value of τ_m lies within the limits 10^{-10} sec < τ_m < 3×10^{-10} sec. An increase in the spontaneous lifetime τ_m, which is a measure of the "inertia" of the bleachable medium, leads

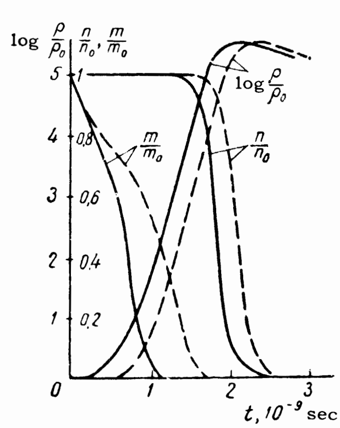

FIG. 4. Time dependence of the number of photons and the population inversion of the active centers in the filter for various spontaneous decay times of the filter [$\rho_0 = 10^{14}$, $\tau_m = 10^{-9}$ sec (solid curves), $\tau_m = 3 \times 10^{-10}$ sec (dashed curve)].

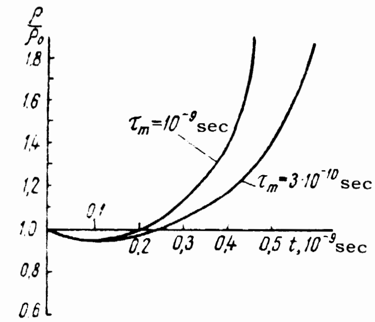

FIG. 5. The initial stage of the formation of a giant pulse for various spontaneous decay times of the filter ($\rho_0 = 10^{14}$).

to a more rapid formation of the standard giant pulse (Fig. 4). In Fig. 5 we show on a magnified scale the initial portions of the curves of the function $\rho(t)$. For $\rho_0 = 10^{14}$ and for $\tau_m = 10^{-10}$ sec the system does not oscillate; $\rho(t)$ goes to zero exponentially with a time constant $\tau \approx \tau_p$ and n remains essentially constant and equal to $n_0 = 10^{20}$, whereas m undergoes the variation shown in Fig. 6.

In view of the foregoing there is in principle another way of exciting the laser.

By a suitable choice of the excitation power one can produce an initial value n_0 such that condition (3) for self-excited oscillations will be fulfilled. In this case the oscillation builds up from a level ρ which is determined by spontaneous emission. This level is very small and the delay will be quite long; under certain conditions it will be longer than the duration of the exciting flash. By contrast, if one applies a triggering pulse to the system it reaches the state described by (3) with a significant excess of photons and hence delay time can be made small compared with the duration of the flash lamp pulse and small in comparison to the decay time of the population difference for n-type particles.

When one may neglect spontaneous decay in the m-system it is simple to obtain the analytical form

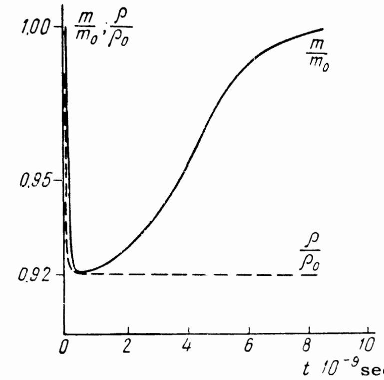

FIG. 6. Graphical time dependence of population inversion of the filter for a τ_m which does not exceed the critical value ($\rho_0 = 10^{14}$, $\tau = 10^{-10}$ sec).

of the dependence of ρ on n. One may also estimate the pulse width and the delay time. These expressions have the following form:

$$\rho = \frac{n_0}{2\beta}\ln\frac{n}{n_0} - \frac{n_0}{2\alpha}\left[1-\left(\frac{n}{n_0}\right)^{\alpha}\right] + \frac{n_0}{2}\left(1-\frac{n}{n_0}\right) + \rho_0, \quad (4)$$

$$m = \frac{n_0}{2\alpha}\left(\frac{n}{n_0}\right)^{\alpha}. \quad (5)$$

The instant t at which n takes on a given value n(t) is determined by the expression

$$t = \frac{1}{2w_n}\int_{n(t)}^{n_0} d(\ln n) \Big/ \left\{\frac{n_0}{2\beta}\ln\frac{n}{n_0} - \frac{n_0}{2\alpha}\left[1-\left(\frac{n}{n_0}\right)^{\alpha}\right] + \frac{n_0}{2}\left(1-\frac{n}{n_0}\right) + \rho_0\right\},$$

$$\beta = w_m \tau_p n_0, \qquad \alpha = w_m / w_n. \quad (6)$$

When t = 0, n_0 and m_0 are related by condition (2) and β is the factor which measures the extent to which the requirement for self-oscillation is exceeded in the cavity containing the saturated filter.

The condition for formation of the giant pulse is

$$\rho_{min} > 0 \text{ or } \rho_0 > n_0 / 2\alpha\beta^2. \quad (7)$$

For $\alpha \gg 1$,

$$\rho_{max} = \rho_0 + \frac{1}{2}n_0 - \frac{1}{2}\left(\frac{1}{\beta} + \frac{1}{\beta}\ln\beta\right), \quad (8)$$

and for $\alpha \gg 1$ and $\beta \gg 1$

$$\rho_{max} = {}^1\!/_2 n_0 + \rho_0.$$

The pulse width t_n between the half-power point and the peak when $\alpha \gg 1$ and $\beta \gg 1$ falls between the following limits

$$\tau_p > t_n > \tau_p / 2. \quad (9)$$

The delay time falls between the limits

$$[2w_n\beta(\rho_0 - \rho_{min})]^{-1} > t_{del} > (2w_m\beta\rho_0)^{-1}. \quad (10)$$

The maximum power P_{max} radiated by the laser is

$$P_{max} = \rho_{max} h\nu / \tau_p. \quad (11)$$

From the information contained in [1-3] it appears that the phthalocyanines have very short lifetimes in the excited state, less than 4×10^{-9} sec. Transition to the triplet state from the excited singlet level occurs in a time of the order $10^{-7} - 10^{-8}$ sec. Hence for a laser in which the saturable filter consists of vanadium phthalocyanine or cryptocyanine the first variant of the machine calculation is probably the most suitable. However it is clear that one might find substances in which population of the lower level by decay from the upper state occurs much more slowly. Ruby is such a material if one thinks of absorption bands and the fluorescence in the R lines.

We have studied experimentally the various operating regimes of a laser using a solution of vanadium phthalocyanine in nitrobenzene as a saturable filter. The experimental arrangement is shown in Fig. 7. The numbers in the figure denote the following: 1—a 100% reflecting mirror, 2—a cell containing the phthalocyanine solution (the cell length is 5 mm and its initial absorption is 90%), 3—a ruby rod 120 mm long and 10 mm in diameter with uncoated ends, 4—a plane parallel glass plate whose purpose will be described below, 5—a 70% transmitting mirror. Two experiments were carried out, which differed in that the ends of the ruby were arranged parallel to the mirrors in the first experiment. Ordinary laser action occurred first using the 30% mirror and the reflection from the end face of the ruby. This provided the triggering pulse to the cavity, i.e. the pulse designated as ρ_0 in the previous calculation. In the second experiment the ruby was aligned to make a small angle (about 0.5°) with the mirrors. In this case the glass plate 4 was removed. Thus the glass plate served to ensure that the cavity losses were the same in both experiments regardless of the alignment of the ruby.

In the first experiment we observed a giant pulse when the pumping intensity was sufficient to produce ordinary laser action between the 30% reflecting mirror and the end face of the ruby. The emitted pulse had a duration of 10 nsec between the peak and the half-power point and had an energy of 1 joule. The pulse occurred 300 μsec after the firing of the flash lamp. In this experiment n_0 did not exceed 0.6×10^{20}; $\beta \le 3.3$; $\tau_p = 1.5 \times 10^{-9}$ sec. According to formula (8) ρ_{max} in this case could have attained the value 10^{19} and P_{max} the value 1.9×10^9 W.

In the second experiment the flash lamp intensity was considerably increased but no ordinary laser action due to a reflection from the internal surfaces of the resonator occurred. After approximately 400—600 μsec (the duration of the flash pulse between the 30% point and the peak was 500 μ sec) there occurred a pulse 20—25 nsec long

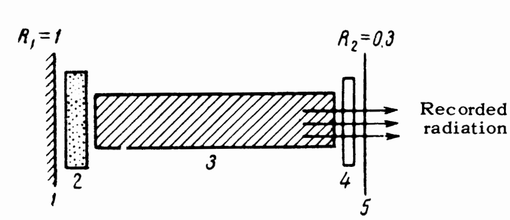

FIG. 7. Experimental setup.

(from the half-power point to the maximum) with an energy less than $1/_{15}$ joule.

These experiments demonstrate the existence of a considerable delay in the development of a giant pulse when excitation occurs without a triggering pulse. The small energy of the pulse in the second experiment can be explained by the decrease in the population inversion in the ruby towards the end of the flash lamp pulse. Comparison of the expected values of the pulse duration and its power with the experimental values shows poor agreement. The small power is easily explained by the nonuniform pumping of the crystal. It is more difficult to explain the increase in the pulse width. In our view there are two likely mechanisms. The first of these is the following. We assume that the field which is present initially in the cavity is nonuniform over the resonator cross-section. This is a completely natural assumption since the laser puts out spikes in ordinary operation[5]. In such a case the resonator is divided into parts (in the plane of the mirror) which differ in the amount of initial excitation. The coupling between these parts is small, since it occurs only by diffraction. As shown previously, the delay preceding the appearance of the pulse depends on the magnitude of the initial excitation. The radiation observed from the whole end face of the mirror will then be a series of pulses which together form the broad pulse which was observed experimentally.

The second mechanism has previously been discussed by Hellwarth[6], viz., cross relaxation between the R lines of ruby and in particular relaxation between the $\overline{E}$ and $2\overline{A}$ levels, from the first of which the depopulation occurs.

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