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COUPLED MAGNETOELASTIC WAVES IN ANTIFERROMAGNETS IN STRONG MAGNETIC FIELDS

V. G. BAR'YAKHTAR, M. A. SAVCHENKO, and V. V. TARASENKO

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Coupled magnetoelastic waves in antiferromagnets in strong magnetic fields are considered for the case when the angle between the magnetic moments of the sublattices differs appreciably from π. Owing to exchange interaction, the most strongly coupled are the nonactivated spin and longitudinal sound waves. The relative corrections to the frequencies (phase velocities) of the sound and magnetic waves during magnetoacoustic resonance are of the order $(\delta M_0^2/\rho s^2)^{1/2}$. The amplitude of longitudinal sound oscillations induced by an alternating magnetic field is calculated. It is shown that excitation of longitudinal sound waves occurs only if the alternating magnetic field is parallel to the dc magnetic field.

1. Coupled magneto-elastic waves arise in an elastically deformed ferromagnet because of magnetostriction and ponderomotive forces.[1-3] The coupling between the spin and elastic waves, generally speaking, is small and is characterized by the dimensionless parameter $\zeta' = (M_0^2/\rho s^2)^{1/2}$, where M_0 is the magnetic moment per unit volume, ρ is the density of the substance, and s is the speed of sound ($\zeta' \ll 1$). Under conditions of magnetoacoustic resonance, when the frequencies and wave vectors of the spin and sound waves coincide, mixing of the branches of the energy spectrum of the ferromagnet occurs, and the corrections to the frequencies are found to be proportional to ζ'. Therefore the coupling between the elastic and spin waves in ferromagnets is most strongly manifest under conditions of magnetoacoustic resonance; far from resonance the relative corrections to the frequencies of the spin and sound waves are proportional to ζ'^2.

An analogous situation occurs in antiferromagnets in weak magnetic fields.[4,5] However, in sufficiently strong magnetic fields, the situation is entirely different. That is to say, if the external magnetic field is such that the angle 2θ between the magnetic moments of the sublattices is substantially different from zero and π ($\sin 2\theta \sim 1$), then the coupling between the spin and sound waves is due to the exchange and not the relativistic interaction between the magnetic moments. The role of the coupling parameter in this case is played by the quantity $\zeta = \gamma\zeta'\delta^{1/2} \cos\theta$, where $\gamma \sim 1$, $\delta \sim T_N/\mu M_0$ is the exchange interaction constant, T_N is the Néel temperature, and μ is the Bohr magneton. Since $\delta \gg 1$, the magnitude of ζ is substantially greater than ζ' and consequently a relatively strong interaction between the sound and spin waves takes place over the entire frequency range. Because of this, sound waves can be excited by means of an applied alternating field not only at the frequency of magnetoacoustic resonance but also over a wide range of frequencies.

The interaction between the spin and sound waves in this case is increased when there is magnetoacoustic resonance and is characterized by the quantity $(\delta M_0^2/\rho s^2)^{1/2}$.

By virtue of the isotropic character of the exchange energy, the spin waves are coupled to the longitudinal sound waves and only the nonactivated spin waves from the two spin branches interact with the longitudinal sound waves. The interaction between the spin waves and the transverse sound waves, on the other hand, is due to the relativistic part of the magnetostriction, and we shall not take it into consideration.

We note that after the magnetic sublattices are "flipped," the magnitude of the static deformation in sufficiently large external magnetic fields depends strongly on the magnetic field. For such values of the external magnetic field when the angle between the magnetic moments of the sublattices is $\pi/2$, the sign of the volume static deformations changes. Depending on the sign of the magnetostriction constant for these values of the magnetic field, there occurs a transition from uniform compression to expansion or vice versa.

Because of the interaction of the spin and longitudinal sound waves, the tensor at the high-frequency magnetic susceptibility $\chi_{ik}(\omega, \mathbf{k})$ contains not only the characteristic frequencies of the spin system, but also the frequencies of the longitudinal sound waves. The sound frequencies have only the component $\chi_{zz}(\omega, \mathbf{k})$, which means that the sound waves appear when the alternating magnetic field is polarized along the constant external magnetic field (parallel setting).

2. Consider a uniaxial antiferromagnet, the ground state of which in the absence of an external magnetic field is determined by two compensated magnetic sublattices. The Hamiltonian of such an antiferromagnet in an external magnetic field has the form

$$\mathscr{H} = \int dV \Big\{ \delta \mathbf{M}_1\mathbf{M}_2 - \frac{1}{2}\beta[(\mathbf{n}\mathbf{M}_1)^2 + (\mathbf{n}\mathbf{M}_2)^2] - \beta'(\mathbf{n}\mathbf{M}_1)(\mathbf{n}\mathbf{M}_2) - (\mathbf{M}_1 + \mathbf{M}_2, \mathbf{H}_0) + \delta\gamma(\mathbf{M}_1\mathbf{M}_2)u_{ii} + \frac{1}{2}\lambda_1 u_{ii}^2 + \frac{1}{2}\lambda_2\Big(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}\Big)^2 + \frac{1}{2}\rho\dot{u}_i^2 + \frac{1}{2}\alpha\Big[\Big(\frac{\partial \mathbf{M}_1}{\partial x_i}\Big)^2 + \Big(\frac{\partial \mathbf{M}_2}{\partial x_i}\Big)^2\Big] + \alpha'\frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i}\Big\}, \tag{1}$$

where $\mathbf{M}_j(\mathbf{r})$ is the density of the magnetic moment of the j-th sublattice ($j = 1, 2$), α, α', δ are exchange integrals, β and β' are magnetic anisotropy constants, H_0 is the constant uniform magnetic field, ρ is the density of the substance, $\mathbf{u}$ is the elastic displacement vector, $u_{ik} = \frac{1}{2}(\partial u_i/\partial x_k + \partial u_k/\partial x_i)$, λ_1, λ_2 are the elastic constants, and γ is the magnetostriction constant.

We shall assume that the antiferromagnet has magnetic anisotropy of the "easy-axis" type, i.e., $\beta - \beta' > 0$. This means that if the magnetic field directed along the preferred axis is less than the "flipping" field H_k, then the magnetic vectors of the sublattices will be oriented oppositely to one another in the ground state. On the other hand, in fields stronger than H_k the magnetic moments are rotated by some angle θ ($0 \le \theta < \pi/2$) relative to the preferred axis.

If the magnetic vectors of the sublattices and the deformation tensor are represented respectively in the form

$$\mathbf{M}_j = \mathbf{M}_{j0} + \mathbf{m}_j, \qquad u_{ik} = u_{ik}^0 + u_{ik}', \tag{2}$$

then the angle θ and the magnitude of the volume deformation are easily found from the condition that the energy (1) be a minimum with respect to small deviations of $\mathbf{m}_j$ and u_{ik}'. We then obtain the following equations for θ and u_{ii}^0:

$$[2\delta(1 + \gamma u_{ii}^0) - (\beta + \beta')]\cos\vartheta = H_0/M_0,$$
$$u_{ii}^0 = -\delta\gamma M_0^2 \cos 2\vartheta / \lambda_1. \tag{3}$$

It is seen from this that the magnitude of the volume static deformation depends on the external magnetic field even when $H_0 = 2^{-1/2}H_{\delta 0}$, where $H_{\delta 0} = (2\delta - \beta - \beta')M_0$, changes sign. In fields $H_0 > H_{\delta 1}$, where $H_{\delta 1} = (2\delta - \beta - \beta' - 2\delta^2\gamma^2 M_0^2/\lambda_1)M_0$, the magnetic moments line up along the $\mathbf{n}$ axis.

The quantities $\mathbf{m}_j$, which characterize the deviations of the magnetic moments of the sublattices from their equilibrium values, are conveniently written

$$\mathbf{m}_j = \mathbf{e}_{\zeta j}m_{j\zeta} + \mathbf{e}_{\eta j}m_{j\eta} + \mathbf{e}_{\xi j}m_{j\xi}, \tag{4}$$

where j is the label of the sublattice, and the unit vectors $\mathbf{e}_{\zeta j}$, $\mathbf{e}_{\eta j}$, $\mathbf{e}_{\xi j}$ are chosen in the following way: $\mathbf{e}_{\zeta j}$ is oriented in the direction of the magnetic moment of the j-th sublattice in the ground state, $\mathbf{e}_{\zeta j} = \mathbf{M}_{j0}/M_2$, $\mathbf{e}_{\xi j}$ is perpendicular to the plane of the magnetic moments $\mathbf{M}_{10}$, $\mathbf{M}_{20}$, and $\mathbf{e}_{\eta j} = \mathbf{e}_{\zeta j} \times \mathbf{e}_{\xi j}$.

The operators $m_{j\zeta}$, $m_{j\eta}$, $m_{j\xi}$ are connected with the Holstein-Primakoff operators[6] by the relations

$$m_{j\zeta} \cong -\mu a_j^+ a_j, \qquad m_{j\eta} \cong i(\mu M_0/2)^{1/2}(a_j - a_j^+),$$
$$m_{j\xi} \cong (\mu M_0/2)^{1/2}(a_j + a_j^+). \tag{5}$$

We limit the expressions for $m_{j\eta}$ and $m_{j\xi}$ to linear terms in the operators a_j, since we are not interested in relaxation processes.

The displacement vector $\mathbf{u}$ is related to the operators for the emission and absorption of phonons $b_{\mathbf{k}s}^+$ and $b_{\mathbf{k}s}$ in the following way:

$$\mathbf{u}(\mathbf{r}) = \frac{1}{(2\rho V)^{1/2}} \sum_{\mathbf{k},s} \frac{\mathbf{e}_{s\mathbf{k}}}{\omega_{s\mathbf{k}}^{1/2}}(b_{s\mathbf{k}}e^{i\mathbf{k}\mathbf{r}} + b_{s\mathbf{k}}^+ e^{-i\mathbf{k}\mathbf{r}}). \tag{6}$$

Using (4)—(6) and transforming from the operators a_j and a_j^+ to their Fourier components according to the formula

$$a_j = V^{-1/2}\sum_k a_{j\mathbf{k}}e^{i\mathbf{k}\mathbf{r}},$$

we obtain the following expression for the Hamiltonian of the antiferromagnet (1) in terms of the operators $a_{j\mathbf{k}}$, $a_{j\mathbf{k}}^+$ and $b_{s\mathbf{k}}$, $b_{s\mathbf{k}}^+$:

$$\mathscr{H} = \sum_{\mathbf{k}}\{[A_{\mathbf{k}}(a_{1\mathbf{k}}^+a_{1\mathbf{k}} + a_{2\mathbf{k}}^+a_{2\mathbf{k}}) + B_{\mathbf{k}}(a_{1\mathbf{k}}a_{2\text{-}\mathbf{k}} + a_{1\mathbf{k}}^+a_{2\text{-}\mathbf{k}}^+) + C_{\mathbf{k}}(a_{1\mathbf{k}}a_{2\mathbf{k}}^+ + a_{1\mathbf{k}}^+a_{2\mathbf{k}}) + \tfrac{1}{2}D_{\mathbf{k}}(a_{1\mathbf{k}}a_{1\text{-}\mathbf{k}} + a_{1\mathbf{k}}^+a_{1\text{-}\mathbf{k}}^+ + a_{2\mathbf{k}}a_{2\text{-}\mathbf{k}} + a_{2\mathbf{k}}^+a_{2\text{-}\mathbf{k}}^+)] + [\omega_{l\mathbf{k}}b_{l\mathbf{k}}^+b_{l\mathbf{k}} + \omega_{t\mathbf{k}}(b_{t_1\mathbf{k}}^+b_{t_1\mathbf{k}} + b_{t_2\mathbf{k}}^+b_{t_2\mathbf{k}})] + \tfrac{1}{2}\delta\gamma(\mu M_0/\rho\omega_{l\mathbf{k}})^{1/2}kM_0\sin 2\vartheta\,(b_{l\text{-}\mathbf{k}} - b_{l\mathbf{k}}^+) \times [a_{1\mathbf{k}} - a_{2\mathbf{k}} - a_{1\text{-}\mathbf{k}}^+ + a_{2\text{-}\mathbf{k}}^+]\}; \tag{7}$$

here

$$A_k = \mu M_0[\alpha k^2 - {}^1\!/_2\beta \sin^2\vartheta + \delta'],$$

$$B_k = {}^1\!/_2\mu M_0 \sin^2\vartheta[2(\alpha' k^2 + \delta') - \beta'],$$

$$C_k = {}^1\!/_2\mu M_0[(\alpha' k^2 + \delta')(1 + \cos 2\vartheta) + \beta' \sin^2\vartheta],$$

$$D_k = {}^1\!/_2\beta\mu M_0 \sin^2\vartheta, \qquad \delta' = \delta - \delta^2\gamma^2 M_0^2 \cos 2\vartheta / \lambda_1,$$

while ω_{lk} and ω_{tk} are the frequencies of the longitudinal and transverse sound waves, respectively.

In order to find the spectrum of coupled magnetoelastic waves it is necessary to diagonalize the Hamiltonian (7). This process is conveniently carried out in two stages. First we introduce in place of the operators a_{jk} and a^+_{jk} the operators $\tilde{a}_{jk}$ and $\tilde{a}^+_{jk}$ according to the formulas

$$\tilde{a}_{1k} = 2^{-1/2}(a_{1k} + a_{2k}), \qquad \tilde{a}_{2k} = 2^{-1/2}(a_{1k} - a_{2k}). \quad (8)$$

The Hamiltonian (7) in terms of the new operators has the following form:

$$\mathscr{H} = \sum_k \{(A_k + C_k)\tilde{a}_{1k}^+\tilde{a}_{1k} + {}^1\!/_2(B_k + D_k)(\tilde{a}_{1k}\tilde{a}_{1-k} + \tilde{a}_{1k}^+\tilde{a}_{1-k}^+)$$
$$+ (A_k - C_k)\tilde{a}_{2k}^+\tilde{a}_{2k} + {}^1\!/_2(D_k - B_k)(\tilde{a}_{2k}\tilde{a}_{2-k} + \tilde{a}_{2k}^+\tilde{a}_{2-k}^+)$$
$$+ F_k[b_{lk}(\tilde{a}_{2-k} - \tilde{a}_{2k}^+) + b_{lk}^+(\tilde{a}_{2-k}^+ - \tilde{a}_{2k})] + \omega_{lk}b_{lk}^+b_{lk}$$
$$+ \omega_{t_1k}b_{1k}^+b_{1k} + \omega_{t_2k}b_{2k}^+b_{2k}\};$$
$$F_k = 2^{-1/2}\delta\gamma(\mu M_0/\rho\omega_{lk})^{1/2}kM_0 \sin 2\vartheta. \quad (9)$$

It follows from Eq. (9) that only the spin waves of the second branch, corresponding to the operators $\tilde{a}_{2k}$, are coupled to the longitudinal sound waves. The transverse sound waves do not interact with the spin waves in this approximation, and we shall not consider them further. The diagonalization of that part of the Hamiltonian (9) containing the operators $\tilde{a}_{1k}$ is well known.[7] We give here only the final results:

$$\mathscr{H}_1 = \sum_k [(A_k + C_k)\tilde{a}_{1k}^+\tilde{a}_{1k}$$
$$+ {}^1\!/_2(B_k + D_k)(\tilde{a}_{1k}\tilde{a}_{1-k} + \tilde{a}_{1k}\tilde{a}_{1-k}^+)] = \sum_k \varepsilon_{1k}c_{1k}^+c_{1k},$$
$$\varepsilon_{1k} = [(A_k + C_k)^2 - (B_k + D_k)^2]^{1/2}$$
$$= \mu M_0\{2\delta'[(\alpha + \alpha' \cos 2\vartheta)k^2$$
$$+ 2\delta' \cos^2\vartheta - (\beta - \beta')\sin^2\vartheta]\}^{1/2}. \quad (10)$$

The operators $\tilde{a}_{1k}$ are connected with the spin wave operators c_{1k} by the relations

$$\tilde{a}_{1k} = u_{1k}c_{1k} + v_{1k}^*c_{1-k}; \quad u_{1k} = \left[\frac{A_k + C_k + \varepsilon_{1k}}{2\varepsilon_{1k}}\right]^{1/2},$$
$$v_{1k} = -\left[\frac{A_k + C_k - \varepsilon_{1k}}{2\varepsilon_{1k}}\right]^{1/2}. \quad (11)$$

In a similar way we can diagonalize that part of the Hamiltonian (9) that contains only the operators $\tilde{a}_{2k}$. The canonical transformation in this case has the form

$$\tilde{a}_{2k} = u_{2k}\tilde{c}_{2k} + v_{2k}^*\tilde{c}_{2-k}^+; \quad u_{2k} = \left[\frac{A_k - C_k + \varepsilon_{2k}}{2\varepsilon_{2k}}\right]^{1/2},$$
$$v_{2k} = \left[\frac{A_k + C_k - \varepsilon_{2k}}{2\varepsilon_{2k}}\right]^{1/2} \quad (12)$$

and the energy

$$\varepsilon_{2k} = [(A_k - C_k)^2 - (B_k - D_k)^2]^{1/2}.$$

Note that ϵ_{2k} is the nonactivated branch of the spin waves of the antiferromagnetic after "flipping" of the sublattice magnetic moments. Substituting the magnitudes of the coefficients A_k, B_k, C_k, and D_k into the expression for ϵ_{2k} and assuming that $(ak) \ll 1$, we obtain

$$\varepsilon_{2k} = vk, \quad (13)$$

where v is the phase velocity of the nonactivated spin waves, equal to

$$v = \mu M_0 \sin\vartheta[(\alpha - \alpha')(2\delta' - \beta - \beta')]^{1/2}. \quad (14)$$

Using Eq. (12), we may now write the Hamiltonian (9) in the form

$$\mathscr{H} = \sum_k \{\varepsilon_{2k}\tilde{c}_{2k}^+\tilde{c}_{2k} + \omega_{lk}b_{lk}^+b_{lk} + \varepsilon_{1k}\tilde{c}_{1k}^+\tilde{c}_{1k}$$
$$+ F_k(u_{2k} - v_{2k})$$
$$\times(b_{lk}\tilde{c}_{2-k} + b_{lk}^+\tilde{c}_{2-k}^+ - b_{lk}\tilde{c}_{2k}^+ - b_{lk}^+\tilde{c}_{2k})\}. \quad (15)$$

Diagonalization of this Hamiltonian is also carried out by means of a canonical uv-transformation, but in this case the operators $\tilde{c}_{2k}$ and b_{lk} should be superpositions of both the new "spin" and the new "phonon" operators, i.e.,

$$\tilde{c}_{2k} = u_{22}c_{2k} + u_{23}d_{3k} + v_{22}^*c_{2-k}^+ + v_{23}d_{3-k}^+,$$
$$b_{lk} = u_{32}c_{2k} + u_{33}d_{3k} + v_{32}^*c_{2-k}^+ + v_{33}^*d_{3-k}^+. \quad (16)$$

The magnitudes of u and v in (16) are given in the Appendix.

Using the equations of motion and the commutation relations for the operators $\tilde{c}_{2k}$, b_{lk}, we obtain the following dispersion equation, which determines the frequencies of the coupled magnetoelastic waves:

$$(\omega^2 - \varepsilon_{2k}^2)(\omega^2 - \omega_{lk}^2) - 4\omega_{lk}^2\varepsilon_{2k}^2\zeta^2 = 0. \quad (17)$$

Since the nonactivated branch of the energy of the spin waves and the frequency of the longitudinal sound ω_{1k} depend linearly on the wave vector k, the frequencies of the coupled magnetoelastic waves will be linear functions of the wave vectors.

The phase velocities of the coupled magnetoelastic waves are determined from Eq. (17) and equal to

$$v_2^2 = {}^1/_2(v^2 + s^2) + [{}^1/_4(v^2 - s^2)^2 + 4\zeta^2 v^2 s^2]^{1/2},$$
$$v_3^2 = {}^1/_2(v^2 + s^2) - [{}^1/_4(v^2 - s^2)^2 + 4\zeta^2 v^2 s^2]^{1/2}. \quad (18)$$

If in the antiferromagnet the phase velocity of the sound waves is greater than the phase velocity of the spin waves,[1] then

$$v_2^2 \approx v^2\left[1 - \frac{4}{s^2 - v^2}\zeta^2 s^2\right],$$
$$v_3^2 \approx s^2\left[1 + \frac{4}{s^2 - v^2}\zeta^2 v^2\right]. \quad (19)$$

From these expressions it follows that the relative additions to the phase velocities of the sound and magnetic waves for the values $\delta \sim 10^3$, $M_0 \sim 10^3$ cgs emu, $\gamma \sim 1$, $\rho \sim 10$ g/cm^3, $v \sim 10^5$ cm/sec, $s_1 \sim 10^5$ cm/sec, are of the order $\zeta^2 \sim 10^{-2}$. Consequently, both the corrections to the velocities of the sound (elastic) and spin (magnetic) waves amount to several per cent even outside the region of magnetoacoustic resonance. In addition, these corrections depend on the external magnetic field, since a change in the latter leads to a change in the phase velocities of the sound waves.

When the phase velocity of the sound waves is less than the phase velocity of the spin waves, then, as is seen from (19), as the external magnetic field H_0 is increased, the phase velocity of the spin waves v decreases. This leads to a fulfillment of the resonance condition (equality of frequencies) between the sound and spin waves at certain values of the external magnetic field when $v(H_0) = s$. The interaction between the spin and sound waves is then particularly strong, and the relative corrections to the corresponding magnitudes of the phase velocities turns out to be of the order of

$$\Delta v / v = 2\zeta \sim 10^{-1}.$$

From this relation it follows that the relative change in the phase velocities can be as high as 10%.

By considering the resonant interaction of the spin and sound waves in antiferromagnets in strong magnetic fields, when $\omega_{1k} = \epsilon_{2k}$, we find

$$H_0 = H_{\delta 0}(1 - s_l^2 / v_0^2)^{1/2}, \qquad v_0 = v|_{\vartheta=\pi/2}. \quad (20)$$

Condition (20) can be fulfilled in antiferromagnets of the type $\alpha - Fe_2O_3$, Cr_2O_3, or NiO.

3. Through the use of the interaction of the spin and sound waves, sound waves may be excited in an antiferromagnet by means of an applied magnetic field, and spin waves by applied ultrasound. In order to determine the amplitude of sound waves excited by an applied alternating magnetic field, we shall calculate the average value of the operator u_{ll}.

The state of an antiferromagnet in an alternating magnetic field $\mathbf{h}(\mathbf{r}, t)$, to the accuracy of terms linear in $\mathbf{h}(\mathbf{r}, t)$, is described by the density matrix

$$\rho(t) = \rho_0 + i\int_{-\infty}^{t} dt' \int d\mathbf{r}' [M_i(\mathbf{r}', t')\rho_0] h_i(\mathbf{r}' t'), \quad (21)$$

where ρ_0 is the equilibrium density matrix and $M_i = M_{1i} + M_{2i}$ is the magnetic moment of the antiferromagnet. The average value of the volume deformations of the body is determined by means of the operator u_{ll} averaged with the density matrix $\rho(t)$. We then have:

$$u_{ll}(\mathbf{r}, t) = \int_{-\infty}^{\infty} dt' d\mathbf{r}' L_s^R(\mathbf{r} - \mathbf{r}', t - t') h_s(\mathbf{r}', t'), \quad (22)$$

where

$$L_s^R(\mathbf{r} - \mathbf{r}', t - t') = i\theta(t - t')\langle[u_{ll}(\mathbf{r}, t) M_s(r', t')]\rangle,$$
$$\theta(t - t') = \begin{cases} 1, & t - t' > 0 \\ 0, & t - t' < 0. \end{cases}$$

Here the angular brackets $\langle \ldots \rangle$ indicate that the averaging is carried out with the equilibrium density matrix ρ_0.

Transforming to Fourier components in Eq. (22), we find

$$u_{ll}(\mathbf{k}, \omega) = L_s^R(\mathbf{k}, \omega) h_s(\mathbf{k}, \omega),$$
$$L_s^R(\mathbf{k}, \omega) = \int d\mathbf{r} \int dt\, e^{-i\mathbf{kr} + i\omega t} L_s^R(\mathbf{r}, t). \quad (23)$$

Using Eqs. (16), (22), (23), and (A-4), we find an expression for the vector component $L_s^R(\mathbf{r}, t)$:

$$L_x = L_y = 0, \; L_z = -\frac{\zeta\zeta'}{\sqrt{\delta'}} \frac{(vs)^{3/2}}{v_3^2 - v_2^2}\left(\frac{k}{M_0}\right)$$
$$\times \left\{\left(\frac{s}{v_2}\right)^{1/2}\left(\frac{1}{\omega - \omega_2 + i\Gamma} - \frac{1}{\omega + \omega_2 + i\Gamma}\right) + \left(\frac{v}{v_3}\right)^{1/2}\left(\frac{1}{\omega - \omega_3 - i\Gamma} - \frac{1}{\omega + \omega_3 + i\Gamma}\right)\right\}, \quad (24)$$

where $\omega_2 = kv_2$, $\omega_3 = kv_3$, $\Gamma \to 0$. It follows from (24) that longitudinal sound waves are excited in an antiferromagnet only if the alternating magnetic field is polarized along the direction of the resultant magnetization, i.e., excitation of longitudinal

[1]This occurs in the transition-metal carbonates.

sound in antiferromagnets in strong magnetic fields, due to exchange magnetostrictive interaction, is effected when the external dc field H_0 and the alternating field $\mathbf{h}$ are parallel to one another (parallel setting).

In conclusion, we find the tensor of the high-frequency magnetic susceptibility of the antiferromagnet. If the body is situated in an external alternating magnetic field, then, as is well-known, an alternating component of the magnetization is induced in it:

$$\Delta m_i(\mathbf{r},t) = m_i(\mathbf{r},t) - m_i^0 = \int_{-\infty}^{\infty} dt' \int d\mathbf{r}' \chi_{il}(\mathbf{r}-\mathbf{r}', t-t') h_l(\mathbf{r}',t'). \tag{25}$$

Here the tensor of the magnetic susceptibility χ_{il} has the following form:

$$\chi_{il}(\mathbf{r}-\mathbf{r}', t-t') = i\theta(t-t')\langle[M_i(\mathbf{r},t)M_l(\mathbf{r}',t')]\rangle. \tag{26}$$

Transforming to Fourier components in Eq. (25), we obtain

$$\mathbf{m}(\mathbf{k},\omega) = \chi(\mathbf{k},\omega)\mathbf{h}(\mathbf{k},\omega),$$
$$\chi(\mathbf{k},\omega) = \int dt \int d\mathbf{r}\, e^{-i\mathbf{kr}+i\omega t}\chi(\mathbf{r},t). \tag{27}$$

Using Eqs. (26), (27), (16), and (A-4), we find expressions for the components of the susceptibility tensor:

$$\chi_{ik} = \begin{pmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & \chi_{zz} \end{pmatrix}; \quad \chi_{xx} = 4\delta'(\mu M_0)^2 \frac{\cos^2\vartheta}{\varepsilon_{1k}^2-\omega^2},$$

$$\chi_{yy} = \frac{1}{\delta'}\frac{\varepsilon_{1k}^2}{\varepsilon_{1k}^2-\omega^2}, \quad \chi_{xy} = -\chi_{yx} = 2i\mu M_0\cos\vartheta\frac{\omega}{\varepsilon_{1k}^2-\omega^2}, \quad \chi_{zz} = \frac{vk^2}{\delta'}$$
$$\times\left\{4\zeta^2\left(\frac{s}{v_3}\right)\left(\frac{vs}{v_3^2-v_2^2}\right)\left(\frac{v^2}{v_3^2-v_2^2}\right)\frac{v_3}{\omega_3^2-\omega^2} + \left(\frac{v}{v_2}\right)\left(\frac{s^2-v_2^2}{v_3^2-v_2^2}\right)\frac{v_2}{\omega_2^2-\omega^2}\right\}. \tag{28}$$

The values of the components of the susceptibility tensor $\chi_{ik}(\mathbf{k},\omega)$ for $\mathbf{k}=0$ agree with those calculated by Kaganov and Tsukernik.[8]

From (28) it follows that the χ_{zz} component of the high-frequency susceptibility tensor has not one, but two poles ($\omega=\omega_2$, $\omega=\omega_3$), corresponding to the frequencies of the coupled magnetoelastic waves.

APPENDIX

In order to find the characteristic frequencies of the coupled magnetoelastic waves and the values of the quantities u and v in Eq. (16), we make use of the equations of motion

$$\dot{\tilde{c}}_{2k} = i[\mathcal{H}, \tilde{c}_{2k}], \qquad \dot{b}_{lk} = i[\mathcal{H}, b_{lk}]. \tag{A-1}$$

From this we obtain the following system of homogeneous equations:

$$\begin{aligned} u_{22}(\omega-\varepsilon_{2k}) + F_k(u_{2k}-v_{2k})(u_{32}-v_{32}) &= 0,\\ v_{22}(\omega+\varepsilon_{2k}) + F_k(u_{2k}-v_{2k})(u_{32}-v_{32}) &= 0,\\ u_{32}(\omega-\omega_{lk}) + F_k(u_{2k}-v_{2k})(u_{22}-v_{22}) &= 0,\\ v_{32}(\omega+\omega_{lk}) + F_k(u_{2k}-v_{2k})(u_{22}-v_{22}) &= 0. \end{aligned} \tag{A-2}$$

By setting the determinant of this system equal to zero, we obtain the dispersion equation (17). To determine the quantities u and v, it is necessary to take into account that the operators $\tilde{c}$ and b satisfy the Bose commutation rules. Then u and v have the following form:

$$u_{32} = -\frac{2\varepsilon_{2k}(u_{2k}-v_{2k})F_k}{(\omega_2-\omega_{lk})(\omega_2+\varepsilon_{2k})}u_{22},$$

$$u_{23} = -\frac{2\omega_{lk}(u_{2k}-v_{2k})F_k}{(\omega_3-\varepsilon_{2k})(\omega_3+\omega_{lk})}u_{33}, \quad v_{22} = \frac{\omega_2-\varepsilon_{2k}}{\omega_2+\varepsilon_{2k}}u_{22},$$

$$v_{33} = \frac{\omega_3-\omega_{lk}}{\omega_3+\omega_{lk}}u_{33}, \quad v_{32} = -\frac{2\varepsilon_{2k}(u_{2k}-v_{2k})F_k}{(\omega_2+\varepsilon_{2k})(\omega_2+\omega_{lk})}u_{22},$$

$$v_{23} = -\frac{2\omega_{lk}(u_{2k}-v_{2k})F_k}{(\omega_3+\omega_{lk})(\omega_3+\varepsilon_{2k})}u_{33},$$

$$u_{22} = \frac{\omega_2+\varepsilon_{2k}}{2(\omega_2\varepsilon_{2k})^{1/2}}\left(\frac{\omega_2^2-\omega_{lk}^2}{\omega_2^2-\omega_3^2}\right)^{1/2},$$

$$u_{33} = \frac{\omega_3+\omega_{lk}}{2(\omega_3\omega_{lk})^{1/2}}\left(\frac{\omega_3^2-\varepsilon_{2k}^2}{\omega_3^2-\omega_2^2}\right)^{1/2}. \tag{A-3}$$

The relations (A-3) are greatly simplified when the spectrum of the spin waves follows a linear dispersion law. Then we have

$$u_{32} = -\frac{2\zeta v(vs)^{1/2}}{(v_2-s)(v_2+v)}u_{22}, \quad u_{23} = -\frac{2\zeta s(vs)^{1/2}}{(v_3+s)(v_3-v)}u_{33},$$

$$v_{22} = \frac{v_2-v}{v_2+v}u_{22}, \qquad v_{33} = \frac{v_3-s}{v_3+s}u_{33},$$

$$v_{32} = -\frac{2\zeta v(vs)^{1/2}}{(v_2+v)(v_2+s)}u_{22},$$

$$v_{23} = -\frac{2\zeta s(vs)^{1/2}}{(v_3+s)(v_3+v)}u_{33},$$

$$u_{22} = \frac{v+v_2}{2(vv_2)^{1/2}}\left(\frac{v_2^2-s^2}{v_2^2-v_3^2}\right)^{1/2},$$

$$u_{33} = \frac{v_3+s}{2(v_3s)^{1/2}}\left(\frac{v_3^2-v^2}{v_3^2-v_2^2}\right)^{1/2}. \tag{A-4}$$

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