SOVIET PHYSICS JETP VOLUME 22, NUMBER 3 MARCH, 1966

MACROCAUSALITY CONDITION IN NONLOCAL FIELD THEORY

A. N. LEZNOV

P. N. Lebedev Physical Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 16, 1965; resubmitted May 18, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 49, 784-786 (September, 1965)

It is shown that the well-known result of Stückelberg and Wanders [1] on the incompatibility of macrocausality and unitarity in nonlocal field theory is due to an unfavorable choice of the anti-hermitian part of the vertex graph. The validity of the macrocausality principle is discussed for a certain class of nonlocal theories.

1. According to Stückelberg and Wanders,[1] the matrix element for the scattering of m initial particles into a state of n final particles consists of terms of the form

$$\int d^4x_1\dots d^4x_m d^4y_1\dots d^4y_n \tilde{D}^c(x_1\dots y_n) \times \exp[i(p_1y_1+\dots+p_ny_n - k_1x_1-\dots-k_mx_m)]V_1(x_1)\dots V_n(y_n), \quad (1)$$

where $V_i(x_i)$ are four-dimensional regions with sufficiently smooth boundaries. The macrocausality condition of [1] consists in the requirement that the interaction over macroscopic time intervals be mediated by quanta with positive energy (cf. [2]). Mathematically, this is equivalent to separating $\tilde{D}^c$ in the region

$$x_1^0,\dots,x_t^0,\ y_1^0,\dots,y_s^0 \gg x^0_{t+1},\dots,x_m^0,\ y^0_{s+1},\dots,y_n^0$$

into two parts: the "short-range part" Δ with the property

$$\lim T^p\int d^4x_1\dots d^4y_n V_1(x_1)\dots V_n(y_n) \times\exp[i(p_1y_1+\dots-k_mx_m)]\Delta\to 0, \quad (2)$$

$$T\sim x_1^0-x_{t+1}^0\sim\dots\sim x_1^0-y_n^0\to\infty,$$

where p is arbitrary, and a part which has the "correct" spectral properties,

$$D^c(x_1\dots x_m;\ y_1\dots y_n) = \int d^4k_1\dots d^4p_n\exp[i(p_1y_1+\dots-k_mx_m)] \times\theta(k_{t+1}^0+\dots-p_n^0)D^c(k_1\dots p_n), \quad (3)$$

$$x_1^0,\dots,x_t^0,\ y_1^0,\dots,y_n^0\gg x^0_{t+1},\dots,x_m^0,\ y_1^0,\dots,y_n^0.$$

As was shown in [3], a sufficient condition for "short range" is the absence of close-lying singularities in the Fourier transform of the form factor in the complex plane. It follows from this that it suffices, for the fulfillment of (2) and (3), to require that there be no close-lying non-Feynman singularities in the matrix elements of the S matrix.

2. The S matrix was constructed in [1] by the method of Stückelberg-Bogolyubov: the imaginary parts of the coefficient functions of the expansion of the S matrix in terms of in operators are determined by the unitarity condition; the real parts are found by the macrocausality principle. The first order matrix element has the form

$$ie\int\Gamma(x_1,\ x_2;\ x_3)u_\varkappa^*(x_1)u_\varkappa(x_2)\varphi_\mu(x_3)d^4x_1\,d^4x_2\,d^4x_3,$$

where u_κ is a charged scalar field with mass κ, and φ_μ is a neutral field with mass μ. The requirements of unitarity, Lorentz and translation invariance, and macrocausality restrict the class of allowable Fourier transforms of the form factors to

$$\Gamma(p_1,p_2;\ k)=\Gamma(p_1^2,p_2^2;\ k^2)=\Gamma^*(p_1^2,p_2^2;\ k^2) =\Gamma(p_2^2,p_1^2;\ k^2);$$

$\Gamma(p_1^2,\ p_2;\ k^2)$ has no close-lying singularities.

The unitarity condition leads to the following expression for the imaginary parts of the second order matrix elements for the scattering process, M^{sc}, and vacuum polarization, M^v:

$$\operatorname{Im}M^{sc}\sim\delta(t-\mu^2),$$

$$\operatorname{Im}M^v\sim\theta(k^2-4\varkappa^2)(1-4\varkappa^2/k^2)^{1/2}|\Gamma(\varkappa^2,\varkappa^2;k^2)|^2.$$

By the macrocausality principle, we can write the real parts in the following form:

$$M^{sc}\sim\frac{F(t)}{t-\mu^2+i\varepsilon},$$

$$M^v\sim\int_{4\varkappa^2}^{\infty}\frac{dM^2|\Gamma(\varkappa^2;\varkappa^2;M^2)|^2(1-4\varkappa^2/M^2)^{1/2}}{k^2-M^2+i\varepsilon}+\varphi(k^2).$$

Here $F(t)$ and $\varphi(k^2)$ are real functions without close-lying singularities.

The imaginary part of the third order vertex graph M^3 in the neutral particle channel has the form

$$\operatorname{Im} M^3 \sim \Lambda(p_1^2)\Lambda(p_2^2)\Lambda(k^2)\theta(k^2-4\varkappa^2) \times \frac{1}{\Phi^{1/2}}\left\{\int_{A_2}^{A_1}\frac{F(t)-1}{t}dt+\int_{A_2}^{A_1}\frac{dt}{t}\right\}; \quad (4)$$

$$\Phi \equiv (p_1^2+p_2^2-k^2)^2-4p_1^2p_2^2,$$

$$A_{1,2}=\varkappa^2-\mu^2+{}^1/{}_2(p_1^2+p_2^2-k^2)\pm[(1-4\varkappa^2/k^2)\Phi]^{1/2},$$

$$\Lambda(p_1^2)\equiv\Gamma(\varkappa^2,p_1^2;\mu^2), \qquad \Lambda(k^2)\equiv\Gamma(\varkappa^2,\varkappa^2;k^2).$$

The separation of the integral (4) into two parts has been carried out as in [1], i.e., in such a way that the second term coincides with the corresponding local expression whose real part, which satisfies microcausality, is known from the local theory. The first term in (4) contains a "close-lying" cut, which, in accordance with the sufficient macrocausality condition, must be circumvented by the Feynman prescription. Hence,

$$M^3 \sim \Lambda(p_1^2)\Lambda(p_2^2)\int_{4\varkappa^2}^{\infty}\frac{dM^2 f(p_1^2,p_2^2;M^2)\Lambda(M^2)}{k^2-M^2+i\varepsilon}$$

$$f(p_1^2,p_2^2;k^2)\equiv\frac{1}{\Phi^{1/2}}\int_{A_2}^{A_1}\frac{F(t)-1}{t}dt.$$

It is seen from the structure of this expression that it has no close-lying singularities in the invariants p_1^2, p_2^2 for a choice of $F(t)$ which has no singularities in the finite plane, e.g., $F(t) = e^{-t^2/\Lambda^2}$ $[F(0)=1]$. From this follows that the sufficient macrocausality condition is fulfilled.

3. Prokhorov [4] has shown the applicability of the reduction formalism of Lehmann, Symanzik, and Zimmermann to a whole class of nonlocal theories. The matrix element (1) is written in the form

$$\langle m|S|n\rangle \sim \int d^4x_1\ldots d^4y_n \exp[i(p_1y_1+\ldots-k_mx_m)] \times W_{x_1}\ldots K_{y_n}\langle 0|T(A(x_1)\ldots A(y_n))|0\rangle, \quad (5)$$

where $A(x)$ is an operator of the interpolating field, and $K \equiv \Box - \mu^2$. Let us consider the function under the integral in (5) in the region

$$x_1^0,\ldots,x_t^0,y_1^0,\ldots,y_s^0 \gg x_{t+1}^0,\ldots,x_m^0,y_{s+1}^0,\ldots y_n^0$$

and find its Fourier transform:

$$\int d^4x_1\ldots d^4y_n \exp[i(-k_1x_1-\ldots+p_1y_1)]K_{x_1}\ldots K_{y_n} \times \langle 0|T(A(x_1)\ldots A(y_s))T(A(x_{t+1})\ldots A(y_n))|0\rangle$$
$$=\sum_N\int d^4x_1\ldots d^4y_n\exp[i(p_1y_1+\ldots-k_mx_m)]K_{x_1}\ldots K_{y_n} \times\langle 0|T(A(x_1)\ldots A(y_s))|N\rangle\langle N|A(x_{t+1})\ldots A(y_n)|0\rangle.$$

Using the translation invariance and the positiveness of the energy in the intermediate states, we easily find that the last expression contains

$$\delta^4(p_N-(k_{t+1}+\ldots-p_n)),$$

which implies that the Fourier transform of D^C contains only frequencies such that

$$k_{t+1}^0+\ldots+k_m^0-p_{s+1}^0-\ldots-p_n^0>0.$$

Thus the macrocausality condition is formally satisfied in a whole class of nonlocal theories for which the reduction formula holds.

In conclusion the author expresses his deep gratitude to D. A. Kirzhnits for his constant interest and numerous discussions during the course of this work.

[1] E. C. G. Stückelberg and G. Wanders, Helv. Phys. Acta **27**, 667 (1954).

[2] M. Fierz, Helv. Phys. Acta **23**, 731 (1950).

[3] M. Chretien and R. E. Peierls, Nuovo cimento **10**, 668 (1953).

[4] L. V. Prokhorov, JETP **43**, 476 (1963), Soviet Phys. JETP **16**, 341 (1963); Dissertation, Leningrad State University, 1964.

Translated by R. Lipperheide
105