BEATS BETWEEN THE MODES OF A RUBY LASER

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High frequency modulation of radiation spikes from a ruby laser is investigated with an ultra-rapid photorecorder. A semi-confocal resonator with a 50 cm separation between the mirrors was employed in the laser. Selection of the angular modes was carried out by means of a special diaphragm inside the resonator. It is shown that higher order transverse modes are excited with increasing diaphragm diameter. It is concluded that the high frequency modulation of the intensity of the spikes is due to beats between the various modes. Inhomogeneous distortion of the ruby crystal appreciably affects the beat frequencies, the inhomogeneity being due to heating of the crystal by the pumping light.

INTRODUCTION

It has been shown in a series of papers [1-4] that in ruby lasers several cavity modes are simultaneously excited in each spike. Since in general the frequencies of these modes differ, the emission intensity is modulated by beats between the various modes. Thus in [5-9] it was shown that the ruby laser emission spikes were intensity modulated at a very high frequency. The modulation frequency observed by various methods is in the range from 5 to 500 Mc. A similar phenomenon occurs in other optical masers. For example in [10] beats were reported between the modes of a $CaWO_4:Nd^{3+}$ laser.

The previously published papers have not given a careful comparison of the observed beat frequencies calculated theoretically from the known dimensions and shape of the cavity. It has been shown only that some of the frequencies correspond to beats between axial modes, separated by the frequency interval $f_0 = c/2L'$. Therefore in the literature there is not complete agreement on the reasons for the high frequency modulation of the laser output. For example in [9] it was proposed that modulation at frequencies different from $c/2L'$ might be due to relaxation processes between two of the working levels or to non-uniformities in the crystal, etc.

In the present work we have studied the low frequency modulation of the laser output at frequencies lower than $c/2L'$, i.e., in the modulation frequency range whose interpretation is subject to the greatest doubts. To do this we used a semi-confocal cavity consisting of a spherical mirror and a plane mirror separated by the focal length of the sphere.

The resonant frequencies of a cavity with spherical mirrors are given by the expression[11,12]

$$\frac{2[L+l(\mu-1)]}{\lambda}=\frac{2L'}{\lambda}$$
$$=q+\frac{1}{\pi}(m+n+1)\arccos\sqrt{\left(1-\frac{L^*}{R_1}\right)\left(1-\frac{L^*}{R_2}\right)}, \quad (1)$$

where L is the separation between the cavity mirrors, and $L' = L + l(\mu - 1)$ is the optical path length, l is the crystal length, μ is the index of refraction, and $L^* = L - l(1 - 1/\mu)$ is the reduced length of the resonator with account of the effect of the crystal.

For the semi-confocal resonator $R_1 = 2L^*$, $R_2 = \infty$ and from Eq. (1) we derive the frequencies

$$\frac{2L'}{\lambda}=q+\frac{1}{4}(m+n+1). \quad (2)$$

It is clear from Eq. (2) that the frequencies of the semi-confocal resonator are degenerate, i.e., modes whose axial index q differs by unity correspond in frequency with modes for which the angular indices m or n differ by four. Hence in the semi-confocal cavity one can observe only discrete beat frequencies which are multiples of the frequency $f_T = f_0/4 = c/8L'$. In actuality the ruby crystal is optically non-uniform. This non-uniformity may be approximated by a lens of a focal length F[13], which lifts the mode degeneracy and alters the beat spectrum.

If a ruby crystal with an effective focal length F is placed in series with a plane mirror the system is equivalent to a plane parallel crystal with a spherical mirror having a radius of curvature $R_2 = F$.

Putting $R_1 = 2L^*$ in Eq. (1) and recalling that $L^*/F \ll 1$, we can expand Eq. (1) in powers of the small parameter L^*/F. As a result we obtain the following expression for the mode frequencies

$$\nu = \frac{c}{2L'}\left[q + (m+n+1)\left(\frac{1}{4} + \frac{1}{2\pi}\frac{L^*}{F}\right)\right]. \quad (3)$$

The frequencies of the beats between two modes having different axial indices are given from Eq. (3) by the expression

$$f = \nu_1 - \nu_2 = \frac{c}{2L'}\left[\Delta q + \Delta(m+n)\left(\frac{1}{4} + \frac{1}{2\pi}\frac{L^*}{F}\right)\right]. \quad (4)$$

The beat frequency will be a minimum when $\Delta(m+n) = 4$, $q = -1$. In this case

$$f_{min} = \frac{cL^*}{\pi L'F} \approx \frac{c}{\pi F}. \quad (5)$$

It follows from Eq. (4) that when the degeneracy has been lifted there should be additional beat frequencies as well as the frequencies of the type $kc/8L'$ (k an integer). In Fig. 1 the frequencies of the modes of the degenerate cavity are shown by the solid lines and the frequencies in the case where the degeneracy is lifted are shown by dashed lines. It is clear from Fig. 1 that when there is simultaneous excitation of modes with $n + m = 0$ and $m + n = 4$ one should observe the beat frequency $f_1 = f_{min} = c/\pi F$. If one were to simultaneously excite modes with $m + n = 0$ and $m + n = 3$ one would observe the beat frequency $f_2 = f_T - (3/4)f_{min}$. For simultaneous excitation of modes having a larger difference $\Delta(m+n)$ the beat spectrum would contain the frequencies

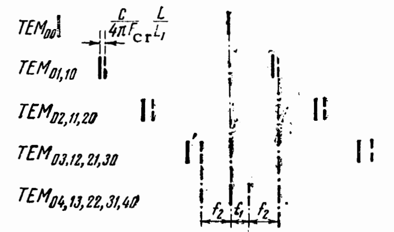

FIG. 1. The frequency spectrum of the semi-confocal cavity. Each row gives the frequencies of modes with different axial indices.

kf_{min} and the frequencies $f_T - (3/4 + k)f_{min}$. The presence of these frequencies complicates the beat spectrum and may make interpretation of the experimental data more difficult.

EXPERIMENTAL DETAILS

In the experimental study of the beats between modes we used external mirrors with multilayer dielectric coatings. The spherical mirror had a radius of curvature $R = 100$ cm. A ruby crystal 5 cm long and 7 mm in diameter was placed adjacent to the plane mirror. The side surface of the crystal was polished and the chromium ion concentration was 0.04%. The pumping source consisted of three IFK-2000 lamps. The laser was operated a factor four above threshold. The pumping energy was 6 kjoules. The total duration of laser action was about 800 μsec.

The laser output was recorded with a high speed rotating camera (SFR). When the SFR was operated in the time magnification mode we were able to simultaneously record both the near field

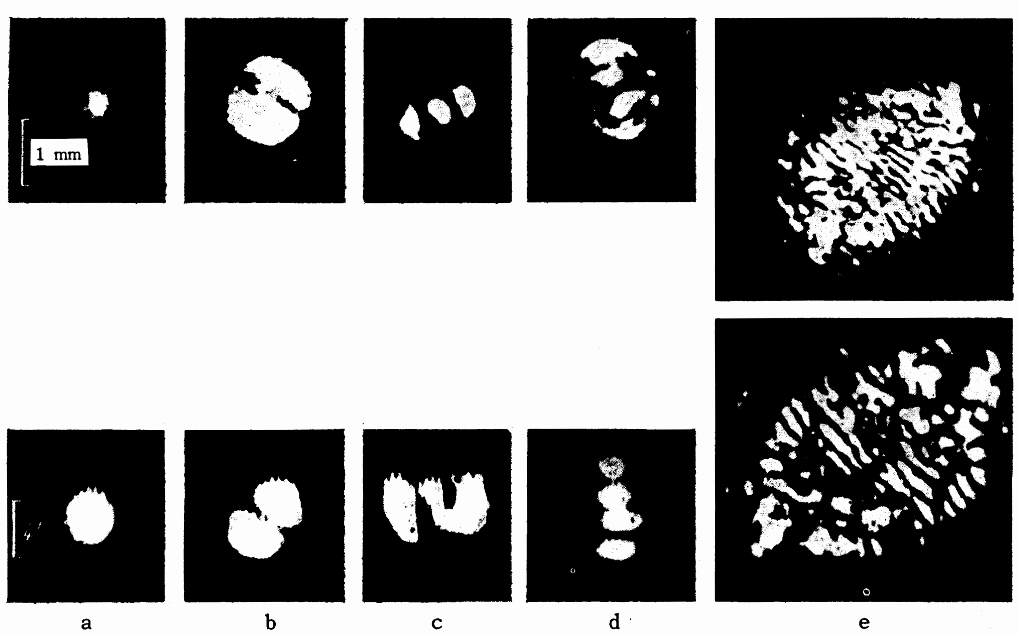

FIG. 2. Simultaneous radiation patterns in the near field (above) and in the far field (below) for individual spikes in the semi-confocal resonator: a-d, with a diaphragm of 1.35 mm diameter close to the plane mirror; e, without the diaphragm.

and far field patterns of the laser emission. The optics used to do this were similar to the arrangement used in [13].

By placing a circular diaphragm in front of the plane mirror of the semi-confocal resonator we were able to select particular angular modes, as was done in [14]. The diameter of the excitation volume of a mode at the mirror is given by the expression (cf. [13])

$$D = \varkappa_m \sqrt{L\lambda / \pi}.$$

Thus for a TEM_{00} mode D = 0.88 mm and for a TEM_{10}, D = 1.9 mm (measured between the 3% of maximum height points). The magnitude of κ_m increases with m approximately as $\sqrt{2m + 1}$ [3], and hence the diaphragm limits the maximum possible angular index of the mode.

In Figs. 2a-e we show photographs of the intensity distribution of the laser emission both on the mirror (above) and in the far field (below), obtained with a diaphragm of 1.35 mm diameter. Under these conditions the TEM_{00} and TEM_{10} modes and other simple modes were excited. The exposure of each frame takes 1.6 μsec; the interval between frames is 3.2 μsec.

It should be noted that with the 1.35 mm diaphragm these simple modes were excited rather rarely. This is explained by the fact that although the TEM_{00} mode has the maximum Q, corresponding to the smallest diffraction losses, the excitation of this mode alone does not make effective use of the active medium over the whole diameter of the diaphragm. More efficient use occurs when there is simultaneous excitation of other modes with larger angular indices, which have a correspondingly larger region of excitation. This is evident in Fig. 3 in which we show a photograph of the near field distribution of the emission. The center of the diaphragm is occupied by the TEM_{00} mode and a mode of higher order is visible around the edges. In Fig. 2e we give photographs of the near and far field patterns of the laser emission for operation without the diaphragm. The diameter of the excited region is 4.5 mm. It

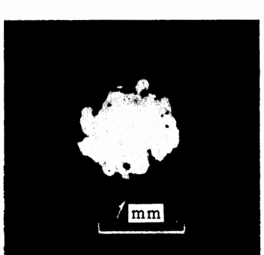

FIG. 3. Simultaneous excitation of several angular modes. The near field distribution (1.35 mm diaphragm).

is clear from the photograph that there is no emission from the upper region of the crystal. This is because internal modes are excited in this region of the crystal. This follows from the results of [15], in which interference methods were used to show (on the same crystal) that the uppermost part of the crystal is heated much more strongly than the lower part of the crystal (Fig. 4f in [15]). Internal modes were also reported in [16].

The beats between modes were observed with the SFR operated in the photorecording mode [2]. The laser output was projected on the entrance slit of the SFR. For a mirror rotation rate of 75000 rpm the resolving power of the apparatus was sufficient to record beats up to 75 Mc. Since the duration of a single revolution of the mirror is 800 μsec, i.e., equal to the duration of the laser action, there is no possibility of multiple exposure. The duration of the recording is 100 μsec, and its temporal position corresponded with maximum pumping. It follows from Eq. (2) that for R = 100 cm and f_0 = 268 Mc and f_T = 67 Mc.

When the limiting aperture had a diameter of 1 mm beats were not observed. This is because in this case only the TEM_{00} mode was excited and hence one would observe beats only between axial modes at a frequency of 268 Mc, which is beyond the resolution of the apparatus. When the diameter of the diaphragm was increased to 1.4 mm, beats appeared. One of the recordings is shown in Fig. 4a. The beat frequency is 52 Mc. It is clear from the recording that intensity modulation is not observed over the whole slit but rather in positions corresponding to the locations of the various modes. The frequency distribution of the observed beats is shown in Fig. 5a. The distribution exhibits two maxima, the first at frequency 12 Mc and the second at 54 Mc. The first maximum corresponds to the beat frequency $f_1 = f_{min}$ and the second to the frequency $f_2 = f_T - (3/4) f_{min}$. From this we find $f_T = (3/4) f_1 + f_2$ = 63 Mc, which is in rather good agreement with the theoretical value of 67 Mc. From Eq. (7) we find the focal length of the crystal F = 800 cm, which is in satisfactory agreement with the results of [15] in which the same crystal was investigated and in which a value of 600–800 cm was obtained for F.

From Fig. 5a we see that the width of each maximum is about 10 Mc. One should note, however, that the distribution of beat frequencies was constructed from fifteen recordings obtained for various laser pulses. Owing to non-uniform initial temperatures, there could have been different values of the parameter F for the various

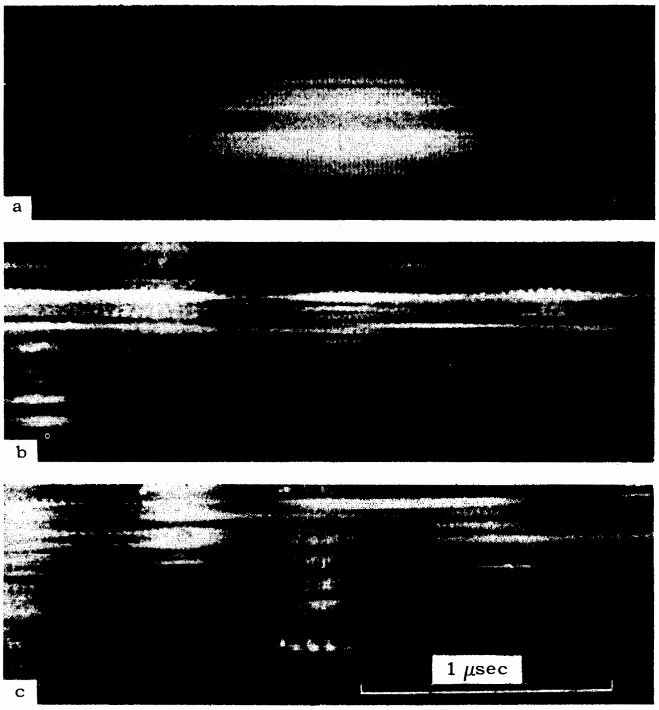

FIG. 4. Time resolution of the far field pattern: a- semi-confocal cavity with a 1.35 mm diaphragm and temperature +20°C; b and c- plane mirrors without a diaphragm and the ruby at −165°C.

laser pulses. Heating of the crystal during the exposure of a frame also plays a role. For comparison we list the beat frequencies obtained from a single frame: 19, 15, 14, 58, 53 and 52 Mc.

In Fig. 5b we give the beat frequency distribution for laser operation without a diaphragm. Since very complicated angular modes are excited in this case (cf. Fig. 2e) the beat frequency distribution is more complicated than for operation with a diaphragm.

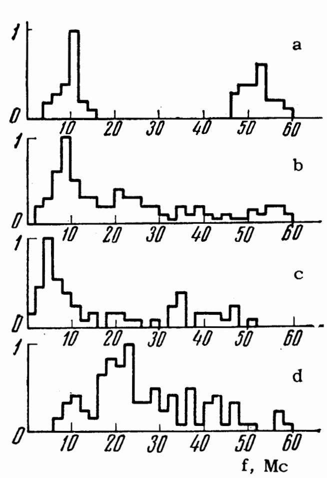

FIG. 5. The frequency distribution of beats: a – for the plane parallel resonator with 1.35 mm diaphragm and a ruby temperature +20°C; b – for the semi-confocal resonator without a diaphragm and a ruby temperature at +20°C; c – for parallel mirrors and ruby temperature +20°C; d – for plane mirrors and the ruby at temperature −165°C.

In addition to beats in the semi-confocal resonator we also investigated beats in a plane parallel resonator. In this case the reflecting coatings were deposited directly on the end faces of the crystal. The beat frequency distribution for the plane parallel resonator is shown in Fig. 5c at 300°K and in Fig. 5d for 108°K. All the curves are normalized to their maximum. At 108°K the beats occur considerably more often. Nearly every spike is observed to be modulated, whereas at 300°K only every twentieth is modulated. At low temperatures the emission is modulated simultaneously at a given frequency and its harmonics (cf. Fig. 4b, where we have the frequencies 12 and 36 Mc). Figure 4c shows a case in which different parts of the crystal are modulated at different frequencies, viz, 10 and 34 Mc. Moreover in this figure one can see that the phase of the modulation is not constant over the whole angular distribution which indicates that the phases of the modes giving rise to beats are different at different angles.

DISCUSSION OF RESULTS

Our study has shown that several different modes are excited during each spike of a ruby laser. One encounters two types of combinations of different modes.

1. As was shown previously [1-3] several spikes are excited in each mode. These modes have the same transverse indices m and n but different axial indices q. This may be explained as follows. Modes with the same angular indices have very nearly the same Q's. The gain coefficients in laser action in these modes are also very nearly the same since the separation between them $f_0 = c/2L'$ is small compared to spontaneous emission line width. As was shown experimentally in [4] and calculated theoretically in [17] in this case these modes have very high probability of being simultaneously excited in a single spike, since the conditions for exciting them are identical. This gives rise to beats at frequency f_0. However since the Q and the gain coefficient for the modes are slightly different, one finds that as a rule in different spikes one excites different combinations of axial modes [2,3].

2. If beats with the "axial" difference frequency f_0 are observed in almost every spike then simultaneous excitation of modes with different transverse indices is considerably less probable. This

is supported by our finding that at 300°K beats between angular modes are quite rare. The increase in the number of beats at lower temperature and for increasing pumping is explained by the increase in gain coefficients; in this case differences in Q have less effect on the kinetics of laser action and it is possible to excite a large number of modes (cf. also [4]).

The results obtained on the beat frequencies make it possible to estimate the line width of an individual mode. Since the experimentally observed beat frequencies are no lower than 2 Mc, the width of a single mode cannot be larger than this quantity. On the other hand the duration of the spikes in the irregular spiking regime is $\tau = 0.5-1\ \mu$sec and the width of a single mode cannot be less than the quantity $\Delta f \approx 1/\tau$, that is, 1—2 Mc. Thus one may assert that Δf is about 1—2 Mc, which is in agreement with the value obtained in [18].

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