THE EFFECT OF ELECTRON SPIN ON THE QUANTUM OSCILLATIONS OF THE GALVANOMAGNETIC COEFFICIENTS OF n-TYPE InSb

S. T. PAVLOV, R. V. PARFEN'EV, Yu. A. FIRSOV, and S. S. SHALYT

Institute for Semiconductors, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 6, 1965

J. Exptl. Theoret. Phys. (U.S.S.R.) 48, 1565-1571 (June, 1965)

An experimental investigation showed the effect of electron spin on the quantum oscillations of the magnetoresistance and the Hall coefficient of n-type InSb. In connection with these experimental results, some theoretical problems are discussed regarding the effect of a strong quantizing magnetic field on the energy spectrum and the conditions of electron scattering in an n-type InSb crystal.

L. GUREVICH and A. Éfros[1] investigated theoretically the problem of the influence of electron spin on the Shubnikov–de Haas effect and, in particular, they predicted that an additional zeroth maximum, not dealt with previously in the theory, should be observed in the electrical conductivity curve of a degenerate semiconductor placed in a strong transverse quantizing magnetic field. The origin of this maximum is associated with the spin splitting of the $N = 0$ Landau level into two sublevels: $N = 0^+$ and $N = 0^-$ (the latter sublevel does not appear in the Shubnikov–de Haas effect). An experimental confirmation of this theoretical conclusion was reported in the papers of Amirkhanov, Bashirov, and Zakiev,[2] which described an investigation of the magnetoresistance in n-type InAs and n-type InSb in pulsed magnetic fields at $T = 20.4°K$.

An experimental investigation, reported in the present work, of the galvanomagnetic properties of single crystals of n-type InSb in an electromagnet providing a constant field at $T = 1.4°K$ showed (cf. Figs. 1 and 2) that in the region of the magnetic field where the transverse magnetoresistance curve exhibited the zeroth maximum, an oscillation of the Hall coefficient was also observed and this oscillation had a much greater amplitude than the oscillation of the same coefficient near the Landau levels with higher quantum numbers $(N \geq 1)$.[1)] The relatively weak oscillations of the Hall coefficient near the Landau levels with $N \geq 1$ may be explained simply by the influence of the

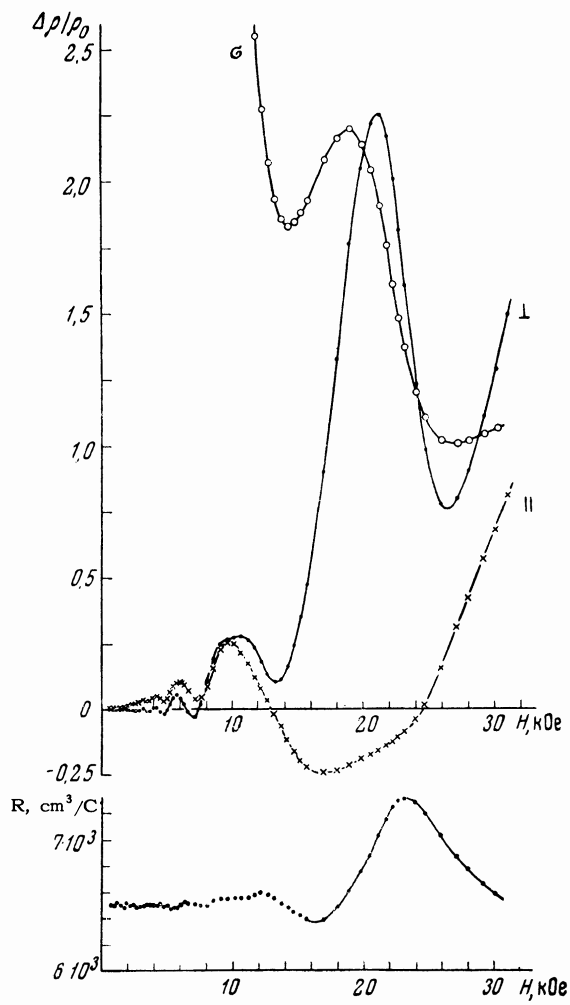

FIG. 1. Experimental curves of the dependences of the transverse (⊥) and longitudinal (||) magnetoresistance ($\Delta\rho/\rho_0$) and of the Hall coefficient (R) on the magnetic field intensity for sample No. 1. In $H < 10$ kOe, the electron density was $n = 9.6 \times 10^{15}$ cm^{-3} and the mobility was $u = 1 \times 10^5$ cm$^2\cdot$V$^{-1}\cdot$sec^{-1}. The temperature was $T = 1.4°K$. Curve σ, which is shown on an arbitrary scale results from a calculation of the magnetoconductivity.

[1)] A similar oscillation of the Hall coefficient was reported by Frederikse and Hosler.[3] However, their corresponding magnetoresistance curve did not exhibit the zeroth maximum sufficiently clearly and they did not discuss this maximum.

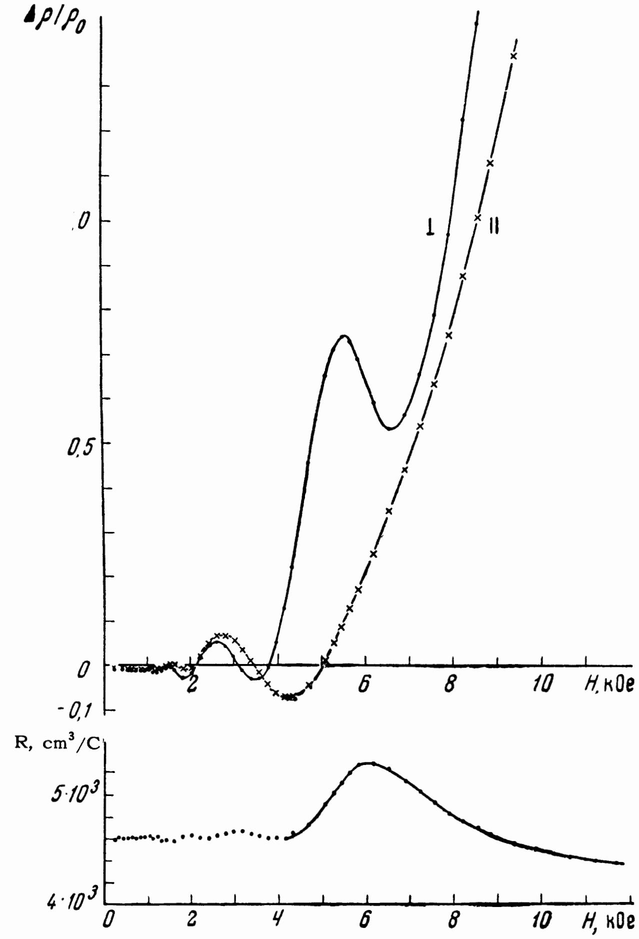

FIG. 2. Experimental curves of the dependences of the transverse and longitudinal magnetoresistance and of the Hall coefficient on the magnetic field intensity for sample No. 2. In H < 4 kOe, the electron density $n = 1.35 \times 10^{15}$ cm^{-3}, and the mobility $u = 1.1 \times 10^5$ cm^2·V^{-1}·sec^{-1}; the temperature was T = 1.4°K.

oscillation of the magnetoconductivity coefficient σ_{xx} on the coefficient $\rho_{xy} = RH$.[4] Allowance for this weak effect is insufficient to account for the amplitude of the oscillation of ρ_{xy} near N = 0$^+$ and gives an incorrect phase. The physical nature of the relatively strong oscillation of the Hall coefficient is not yet clear. It may possibly be associated with a change in the electron density in a magnetic field.

It is evident from Figs. 1 and 2 that the zeroth maxima are absent in the experimental curves of the longitudinal (||) magnetoresistance. This circumstance is, according to Éfros,[5] associated with the low probability of electron scattering accompanied by spin flip.

The theory of the Shubnikov–de Haas effect considers the influence of a magnetic field on the component of the electrical conductivity tensor, σ_{xx}, related to the experimentally determined components of the magnetoresistance tensor ρ_{xx} and ρ_{xy} by the expression[2)]

$$\sigma_{xx} = \rho_{xx} / (\rho_{xx}^2 + \rho_{xy}^2). \tag{1}$$

Therefore, when comparing the experimental results with the theoretical predictions, it is necessary to plot first the curve $\sigma_{xx}(H)$, allowing for the oscillations of both ρ_{xx} and ρ_{xy}, and to determine the position, H_0^+, of the zeroth maximum of this curve.

It is evident from the experimental curves in Fig. 1 for the transverse (⊥) magnetoresistance and the Hall coefficient R and from the dependence $\sigma_{xx}(H)$ calculated from these curves (the latter is shown in Fig. 1 on an arbitrary scale) that the positions of the zeroth maxima in the $\rho_{xx}(H)$ and $\sigma_{xx}(H)$ curves are widely separated (by 2 kOe).

Comparison of the experimental data for $\sigma_{xx}(H)$ with the theory is complicated by two factors, one of which is the possible dependence of the electron density n on the magnetic field. According to the theory, H_0^+ is the value of the magnetic field at which the zeroth spin Landau sublevel

$$\varepsilon_0{}^+ = \frac{1}{2}\hbar\frac{eH}{m^*c} + \frac{e\hbar}{2m_sc}H \tag{2}$$

intersects the chemical potential level. The chemical potential is a function of the electron density n and of the magnetic field H. Equating the chemical potential to the energy ϵ_0^+, we obtain an equation for the determination of the magnetic field H_0^+. Usually, in the theory of the Shubnikov–de Haas effect, it is assumed that n is independent of H. If n = n(H), then the form of the equation used to determine H_0^+ changes. However, the exact form of the dependence n(H) is not known since there is no certainty that the observed oscillation of the Hall coefficient is solely due to a density effect.

The second factor is associated with the broadening of the Landau levels due to the electron scattering. Allowance for this smooths out the singularities (of the infinity type) in the density of states of electrons in a magnetic field.

Dingle[6] postulated that the broadening of the density of states may be allowed for formally by introducing an additional parameter $k_0T_D = \hbar/\tau$ (T_D is the Dingle temperature, τ is the relaxation time of electrons). When $T_D > T_{exper}$, the corrections due to the dependence of the density

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2)An isotropic semiconductor is considered. The magnetic field is directed along the z axis. The subscripts of the tensor components indicate the directions of the current and the electric field.[4]

of states on the scattering play the key role in the determination of the form of the magnetoresistance curve.

If we ignore the effect of the broadening of the Landau levels due to the electron scattering and do not allow for the possible dependence of n on H in the expression for the chemical potential, the theoretical treatment leads to the following results.

According to Gurevich and Éfros,[1] the formula for the identification of the zeroth maximum ($N = 0^+$) in the transverse electrical conductivity curve $\sigma_{xx}(H)$ has the form

$$H_{0^+} = n^{2/3}\frac{\hbar c}{e}(\sqrt{2}\pi^2)^{2/3}(m_s/m^*)^{1/3}. \quad (3)$$

Partial degeneracy (due to the non-zero test temperature) may be allowed for by multiplying the right-hand part of Eq. (3) by the factor

$$[1 + 0.53(k_0 T m_s c / \hbar e H)^{1/2}]^{-2/3}. \quad (4)$$

In the calculation of H_0^+, we assumed that the spectroscopic splitting factor of InSb was $|g| = 50$ and that the spin mass was $m_s = 2m_0/|g| = 0.04\,m_0$. The effective mass was assumed to be $m^* = 0.013\,m_0$. When the electron density was determined from the value of the Hall coefficient T in the magnetic field region preceding the zeroth maximum, we found for sample No. 1 (2.8 × 1.3 × 17 mm, Fig. 1): $n = 9.6 \times 10^{15}$ cm^{-3} and $H_0^+ = 24$ kOe. When the electron density was determined from the value of R near the zeroth maximum, it was found that $n = 8.5 \times 10^{15}$ cm^{-3} and $H_0^+ = 22.1$ kOe. The latter value was in better agreement with the experimental value $H_0^+ = 19$ kOe. The situation was the same for sample No. 2 (2.2 × 2.1 × 17 mm), for which we found experimentally that the zeroth maximum in the ρ_{xx} curve was at H = 5.5 kOe, while the Hall coefficient rose from 4.6×10^3 cm^3/C ($n = 1.35 \times 10^{15}$ cm^{-3}) to 5.3×10^3 cm^3/C ($n = 1.17 \times 10^{15}$ cm^{-3}).

Similar results were also obtained in an investigation of four other samples having different carrier densities and different geometrical factors.

The general form of the experimental curves for the transverse magnetoresistance ($\Delta\rho/\rho_{0\perp}$) indicated that the investigated effects were strongly affected by the scattering of electrons (in the present case, by ionized impurities). A finite relaxation time always leads to a broadening of the Landau levels. We shall assume that in our case this broadening can be represented by the Dingle temperature. Then a very rough allowance for this factor may be made by replacing the root singularity $(\epsilon - \epsilon_0^+)^{-1/2}$ in the density of electron states near the zeroth spin level[3)] with $\mathrm{Re}\,(\epsilon - \epsilon_0 + ik_0T_D)^{-1/2}$, which transforms the density-of-states curve from a curve tending to infinity at the point $\epsilon = \epsilon_0^+$ to a curve with a maximum lying in the region of energy values exceeding ϵ_0^+ by about k_0T_D. If we then assume that the condition for resonance is the coincidence of the region of maximum density of states with the chemical potential level, then in the determination of H_0^+ this leads to the multiplication of the right-hand part of Eq. (3) by a factor of the type

$$[1 + (k_0 T_D m_s c / \hbar e H)^{1/2}]^{-2/3}. \quad (5)$$

It is understood that the expression obtained in this way can give only the tendency of the oscillation maxima to shift toward weaker magnetic fields in the strong-scattering case ($T_D > T_{exper}$) but cannot be used to determine the g-factor quantitatively.

A more precise theoretical allowance for the influence of the finite relaxation time on the position of the zeroth maximum H_0^+ can be made on the basis of the analysis due to Bychkov,[7] who has shown that the qualitative conclusions of Dingle are satisfied if $\Omega\tau \gg 1$ (Ω is the cyclotron frequency). However, the initial assumption of this analysis—scattering on a short-range potential—is not, strictly speaking, valid in the case of scattering by ionized impurities, which is of interest to us. A theoretical treatment is difficult because of the absence of a small parameter. In the investigated samples of InSb, the electron density (and the impurity concentration) n, the effective Bohr radius $a_B^* = \hbar^2\kappa/m^*e^2$ (κ is the permittivity), and the exact electron-scattering amplitude f are related by the expressions $a_B^* n^{1/3} \approx 1$ and $fn^{1/3} \approx 1$, while in the model of impurities having a short-range potential $fn^{1/3} \ll 1$, and in the theory of heavily doped semiconductors a $n^{1/3} \gg 1$. These estimates show that, in the present case, we can use neither the theory of heavily doped semiconductors nor the short-range potential approximation.

By increasing the carrier density (to $n \approx 10^{18}$ cm^{-3}) and the magnetic field intensity, we can move to a region where the theory of heavily

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[3)] We recall that the density of states of electrons possessing energy ϵ, Landau quantum number N, and spin quantum number $s = \pm 1$ is, without allowance for the Dingle temperature, proportional to

$$\left[\varepsilon - \hbar\Omega\left(N + \frac{1}{2}\right) + s\frac{\hbar e}{2m_s c}H\right]^{-1/2}.$$

doped semiconductors is valid ($a_B^* n^{1/3} \gg 1$), which facilitates the theoretical interpretation of the results because of the appearance of a convenient theoretical parameter, although at present the question whether it is possible to use the Dingle temperature in this limiting case has not yet, to the authors' knowledge, been settled.

In the short-range potential model, which can be analyzed exactly,[7] the shift of the oscillation maxima into the region of weaker magnetic fields is due to a change in the resonance condition as a result of the allowance for the scattering, referred to above, and due to a change in the chemical potential. The latter occurs because to define the chemical potential in terms of the density it is necessary to use the density-of-states function and allow for its change due to the scattering. In the case considered in [7], this would lead to the multiplication of the right-hand part of Eq. (3) by a factor of the order of

$$[1 + (k_0 T_D m_s c \,/\, \hbar eH)^{2/3}]^{-2/3}. \qquad (6)$$

Thus, the approximate nature of the conclusions precludes our using the factor (5) or (6) to find the value of H_0^+ more exactly. However, the shift of the theoretical value H_0^+ toward weaker magnetic fields, which is indicated by these corrections, improves the agreement with the experimental data.

It is possible that the relatively strong oscillation of the Hall coefficient in the region of the zeroth maximum is associated with a special density effect, which appears in strong magnetic fields. Bychkov[7] has shown that, in the short-range potential approximation, bound states may be formed in a strong magnetic field. Then the localization of the electron wave function should be less than the volume per one electron. These bound states lie above the conduction band edge, but below the zeroth Landau level, and their binding energy is

$$\varepsilon_b = \frac{\hbar eH}{2m^* c} - \frac{f^2 e^2 H^2}{3\pi^2 m^* c^2}. \qquad (7)$$

At sufficiently low temperatures, when the condition $\gamma = n_{impur} l_H / f \ll 1$ is satisfied (the magnetic quantum length is $l_H = \sqrt{c\hbar/eH}$), these bound states are stable with respect to both thermal perturbations and to the interaction between impurities.

In the investigated samples of InSb $n_{impur} \approx n$, i.e., in relatively weak magnetic fields (in the region of the oscillation maxima with $N \geq 1$), we have $\gamma \gg 1$ and there cannot be any bound states. However, as the magnetic field intensity is increased and the zeroth maximum is approached, we have $\gamma \rightarrow 1$, i.e., the conditions for the appearance of bound states become more favorable. If we were able to show, within the framework of a rigorous theory, that bound states can appear in real crystals, then such local levels could act as a reservoir storing some of the free electrons. In that case, the density effect (the change in the free electron density in a magnetic field) would occur even if the electron density were temperature-independent in the absence of a magnetic field.

It is known that the effect is absent when $a_B^* n^{1/3} \gg 1$ and, therefore, a relative decrease in the zeroth maximum of the Hall coefficient compared with the zeroth maximum of the magnetoresistance in the region of high carrier densities ($n \approx 10^{18}$ cm^{-3}) would be an indirect confirmation of the proposed hypothesis of the physical nature of the zeroth maximum of the Hall coefficient.

Further experimental studies of the nature of the observed oscillations would require an investigation, in a quantizing magnetic field, of the thermoelectric power which, as shown by Obraztsov,[8] is governed completely by the electron entropy in the zeroth approximation with respect to $(\Omega\tau)^{-1}$.

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Translated by A. Tybulewicz
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