ON THE THEORY OF GAS LASERS

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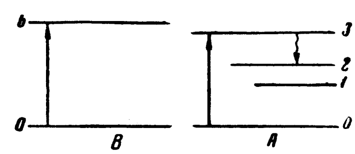
The problem of the emission of radiation by a gas laser is solved by semiclassical theory methods. The dependence is obtained of the density of the field of induced radiation on the parameters of the resonator, on the probabilities of excitation per unit time of atoms in the plasma of the gas discharge and on the relaxation characteristics of the system. In the calculation the motion of the emitting particles, the degeneracy of the levels and the random orientation of the dipole moments of the transition under consideration are taken into account, and also the existence of a mechanism for the excitation of the lower level.

AT the present time a large number of articles exists devoted to the study of lasers. In them the spectral characteristics and the general regularities in the generation of radiation are investigated. Nevertheless, there is still no satisfactory quantitative theory, and for the calculation of power and of the threshold of generation one is restricted only to rough estimates. In particular, for gas lasers this is explained by considerable difficulties associated with the necessity of taking into account a large number of physical phenomena both in the calculation of the interaction of the coherent field of the radiation with the randomly moving atoms of the medium, and also in calculating the rates of excitation in the plasma of the gas discharge.

In the present paper we propose a simple method of calculating the threshold and the power emitted by a gas laser. The calculation is carried out for the experimental conditions of Javan et al.[1] near the threshold of oscillation. The result is expressed in terms of the known parameters of the resonator and of the medium and of parameters which can be obtained either by a measurement, or by an independent subsidiary calculation under the specific operating conditions of the laser. In view of the complexity of the complete calculation this result can serve as a first step in the direction of overcoming these difficulties.

The diagram shows a typical set of energy levels utilized in the operation of gas lasers with electronic excitation. An extensive literature (cf., for example,[2]) has been devoted to the clarification of the kinetic processes in the plasma of a discharge which are the most essential ones for the operation of a laser. We only note that in the discharge both level 3 and level 2 become populated



and induced transitions occur between them. The latter circumstance can appreciably raise the oscillation threshold. Javan et al.[1] have observed linearly polarized radiation belonging to one type of oscillation near the threshold of oscillation. As a rule levels 3 and 2 are degenerate. In our calculation we shall be guided by the above remarks.

We shall describe the radiation field in the resonator $\mathbf{e}_{32}E_{32}(r)\exp(-i\omega_{32}t)$ with the aid of Maxwell's equations, and the processes occurring in the medium under the influence of the field by means of the kinetic equation for the density matrix in the interaction representation $R_{m\alpha,n\beta}$ ($m\alpha$ and $n\beta$ are states corresponding to the degenerate energy levels E_m and E_n). Taking into account the selection rules for linearly polarized radiation ($D_{i\alpha,k\beta} = D_{ik}^{(\alpha)}\delta_{\alpha\beta}$), we shall write the equation in the form ($\hbar = c = 1$)

$$\left(\Delta - \frac{\partial^2}{\partial t^2}\right)E_{32}(r)\,e^{-i\omega_{32}t} = 4\pi\frac{\partial}{\partial t}[\sigma E_{32}(r)\,e^{-i\omega_{32}t} + iN\omega_{32}e^{-i\omega_{32}t}\langle R_{32}{}^{(\alpha)}\mathbf{D}_{32}{}^{(\alpha)}\mathbf{e}_{32}\rangle] \quad (1)$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\nabla\right)R_{ik}{}^{(\alpha)} = -i\sum_m (E_{im}(r)\,\mathbf{e}_{im}\mathbf{D}_{im}{}^{(\alpha)}R_{mk}{}^{(\alpha)} - R_{im}{}^{(\alpha)}\mathbf{D}_{mk}{}^{(\alpha)}\mathbf{e}_{mk}{}^{(\alpha)}E_{mk}(r)) + \gamma_{ik}{}^{(\alpha)}R_{ik}{}^{(\alpha)}, \quad (2)$$

where σ is the conductivity corresponding to the radiation losses on reflection from the mirrors, which, as can be shown with the aid of the formulas of dispersion theory, are considerably greater than the dissipation of radiation in the medium; $D_{i\alpha,k\beta}$ is the matrix element of the dipole moment for the transition between the states $i\alpha$ and $k\beta$; $D \cdot e = D \cos\theta$; N is the concentration of the atoms of the working substance; v is the velocity of motion of an atom interacting with the radiation; $\langle\dots\rangle$ denotes averaging over all possible orientations of the atoms; γ are the relaxation parameters. It is necessary to average over v and α in the final results. In subsequent calculations we omit the indices α.

By comparing the laws describing the variation of the field in an empty resonator obtained on the one hand with the aid of reflection coefficients and on the other hand with the aid of σ, we can easily obtain the formula

$$2\pi\sigma = (T + L) / 2l, \tag{3}$$

where l is the length of the resonator, L and T are the coefficients of absorption and transmission of the mirrors $T + L \ll 1$.

Assuming that the field is homogeneous over the cross section of the resonator we represent E(r) and R(r) in the form

$$E_{32}(r) = E_{32}(x) = E_{32}\cos k_{32}x = E_{32}\cos\xi, \tag{4}$$

$$R_{32}(r) = R_{32}(x) = r_{32}^+\cos\xi + r_{32}^-\sin\xi,$$
$$R_{mm} = R_m = \text{const}, \tag{5}$$

where k_{32} is the modulus of the propagation vector, the x axis is chosen along the resonator axis, E_{32}, r^+, r^- do not depend on x and t. We can represent R in such a form only for weak radiation fields (for $ED/\gamma \ll 1$, cf.[3]).

Taking into account (4) and (5), neglecting in the equations for R the cross terms in virtue of the noncoherence of the pumping "fields" for levels 3 and 2 and denoting by q_3 and q_2 the terms describing the processes for populating the states corresponding to the degenerate energy levels 3 and 2 we can recast equations (1) and (2) in the following form:

$$E_{32} = -i(N\omega_{32}/\sigma)\langle r_{32}^+ D_{23}\cos\theta\rangle; \tag{6}$$

$$(kvd/d\xi + \gamma_{32})R_{32} = iU_{32}(R_3 - R_2)\cos\xi,$$
$$(kvd/d\xi + \gamma_3)R_3 = q_3 + 2\,\mathrm{Im}\,(U_{32}R_{32}^*)\cos\xi, \tag{7}$$
$$(kvd/d\xi + \gamma_2)R_2 = q_2 - 2\,\mathrm{Im}\,(U_{32}R_{32}^*)\cos\xi.$$

where $U_{32} = E_{32}D_{32}\cos\theta$.

Equations (6), (7) admit solutions of the form

$$U_{32} = U_{32}^*, \quad r_{32} = i\rho_{32}, \tag{8}$$

where $\rho_{32}^* = \rho_{32}$. Substituting (4), (5), (8) into (6), (7), we obtain

$$E_{32} = (N\omega_{32}/\sigma)\langle\rho_{32}^+ D_{23}\cos\theta\rangle, \tag{9}$$

$$\rho_{32}^+(v,\theta) = U_{32}(\tilde{q}_3 - \tilde{q}_2)$$
$$\times\left[\frac{(kv)^2}{\gamma_{32}} + \gamma_{32} + U_{32}^2\frac{\gamma_3+\gamma_2}{\gamma_3\gamma_2}\right]^{-1}; \tag{10}$$
$$\tilde{q}_3 = q_3/\gamma_3, \quad \tilde{q}_2 = q_2/\gamma_2.$$

Averaging over the Maxwellian velocity distribution and over the isotropic orientations of the atoms we obtain for $ED/\gamma \ll 1$, $\gamma/k\bar{v} \ll 1$

$$\langle\rho_{32}^+\cos\theta\rangle = \frac{\sqrt{\pi}}{3k\bar{v}}(\tilde{q}_3 - \tilde{q}_2)E_{32}D_{32}$$
$$\times\left[1 - 0.3|E_{32}|^2|D_{32}|^2\frac{\gamma_3+\gamma_2}{\gamma_3\gamma_2\gamma_{32}}\right],$$
$$\langle\rho_{32}^-\rangle_v = 0. \tag{11}$$

Substituting (11) into (9) and taking into account the fact that $\gamma_{32} = (\gamma_3 + \gamma_2)/2$ (cf.[4]), while the quantities γ, q and D depend on α, we obtain after averaging the result over α

$$|E_{32}|^2 = \frac{5\hbar^2\Gamma_3\Gamma_2}{3S_{32}}[(\tilde{Q}_3 - \tilde{Q}_2)Q^{-1} - 1]. \tag{12}$$

From this we determine the threshold rates of excitation $\tilde{Q}_{3,2} = Q_{3,2}/g_{3,2}\Gamma_{3,2}$:

$$(\tilde{Q}_3 - \tilde{Q}_2)_{\text{thresh}} = Q = \frac{3\hbar\bar{v}(L+T)}{4\pi^{3/2}S_{32}Nl},$$

$$S_{32} = \sum_{\alpha,\beta}|D_{3\alpha,2\beta}|^2, \tag{13}$$

where $\Gamma_{3,2} = (\tau_{3,2})^{-1}$ and $\tau_{3,2}$ are the lifetimes of the levels in the discharge, g are the statistical weights of the levels; $Q_{3,2}$ are the probabilities of excitation per unit time to appropriate levels for atoms in the discharge. The intensity of the transition S_{32} can be easily obtained knowing the oscillator strength for the transition $3 \leftrightarrow 2$[5]. The value of Q_3 is determined by the self absorption of spontaneous radiation and by collisions of the first and of the second kinds, Q_2 is determined by the electronic excitation from the metastable levels 1 and by the diffusion of metastable atoms towards the walls of the discharge tube. The dependence of Q and Γ on the kinetic processes in the plasma of the gas discharge determines the complex dependence of the radiated power on the partial pressure of the components of the gas mixture, on the radius of the discharge tube and on the discharge current. These parameters can be obtained either in preliminary experiments, or they can be calculated in a consistent theory with the aid of available empirical formulas[2,6].

Note (November 21, 1964). Recently a paper by Lamb [7] has appeared which, in particular, touches upon questions discussed in the present paper. From our point of view the methods described in the present paper, and, in particular, the method of taking the motion of particles into account in the equation itself, is more convenient and has enabled us to solve the problem outlined above more rapidly and more simply. Moreover, Lamb does not take into account the degeneracy of the levels and the random orientation of the dipole moments of the particles interacting with the radiation. The latter circumstance is essential for the problem proposed by us and leads to a quantitative (and for strong fields should also lead to a qualitative) difference in the results.

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