SCATTERING AND CONVERSION OF ELECTROMAGNETIC WAVES IN A TURBULENT PLASMA

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We consider the interaction between electromagnetic waves and a turbulent plasma consisting of cold ions and hot electrons moving at supersonic velocities with respect to the ions. Combination scattering of transverse electromagnetic waves by acoustic oscillations is treated as is the conversion of longitudinal waves into transverse waves as a result of the interaction with the acoustic waves. It is shown that in the presence of turbulence the strength of these processes is much greater than in the case of an equilibrium or quasiequilibrium plasma.

The dependence of the energy flux of scattered radiation on the frequency and direction of propagation of the incident and scattered waves is established. It is shown that this dependence is determined to a large extent by the spectral and angular distribution of the turbulent oscillations. In principle, this effect should make wave scattering and conversion useful for the experimental determination of turbulence spectra. Finally, we consider the spontaneous "emission" of a plasma due to the conversion of longitudinal waves into transverse waves as a result of the interaction with turbulent oscillations.

1. INTRODUCTION

IT is well known that the scattering and conversion of electromagnetic waves in a plasma are determined by the level of the fluctuations in the plasma.[1)] In cases in which the plasma is characterized by a high fluctuation level the intensity of the scattered radiation can be appreciable. In particular, scattering and wave conversion are very intense in a plasma approaching instability.[5-7]

In the present work we treat the interaction of electromagnetic waves with a plasma consisting of cold ions and hot electrons moving with respect to the ions at velocities greater than the velocity of ion-acoustic waves. If nonlinear effects are neglected, the acoustic oscillations in such a plasma are growing oscillations. Nonlinear effects do limit the growth of the random acoustic waves, however, and as a result there is established a stationary distribution of fluctuations characterized by high-amplitude random acoustic waves—this is the so-called state of stationary turbulence.[2)] The high fluctuation level in a turbulent plasma leads to intense scattering of electromagnetic waves and to the strong conversion of longitudinal waves into transverse waves. The present work is devoted to an investigation of these phenomena.

In Secs. 2 and 3 we investigate the scattering of transverse electromagnetic waves in a turbulent plasma. In particular, by measuring the frequencies of the acoustic satellites it should be possible to find the temperature of the electrons in the plasma. We show that the satelite intensity given as a function of the incident-wave frequency is very sensitive to the details of the turbulence spectrum. Hence, experiments on scattering of electro-magnetic waves in a turbulent plasma should be useful for verifying various theoretical conclusions concerning the nature of plasma turbulence.

In Sec. 4 we study the conversion of longitudinal waves into transverse waves as a result of the interaction with turbulent acoustic waves. As in the case of transverse-wave scattering, it appears

1)Scattering and wave conversion in a plasma have been treated extensively in the literature (cf.[1-4]). In all of this work, however, it has been assumed that the plasma is not highly unstable (although the plasma is not necessarily in equilibrium).

2)The mechanism responsible for the establishment of the stationary distribution of fluctuations in the case of the ion-acoustic instability has been treated by Kadomtsev and Petviashvili[8] and the spectral distribution of steady-state fluctuations has been studied by a number of authors.[8-10, 14] that this effect can also be used for plasma diagnostics and for the experimental establishment of the turbulence spectrum. We also consider the spontaneous "emission" from a turbulent plasma due to the conversion of random Langmuir waves into transverse waves; the energy "loss" from the plasma associated with this mechanism is also estimated.

2. SCATTERING OF TRANSVERSE ELECTROMAGNETIC WAVES

We first consider the scattering of transverse electromagnetic waves on a turbulent plasma consisting of cold ions and hot electrons. The scattering coefficient $d\sigma$ is the ratio of the scattered-wave intensity to the Poynting vector of the incident wave; this coefficient is related to the correlation function for the plasma electron density $\langle \delta n^2 \rangle$ by the familiar expression

$$d\sigma = \frac{1}{4\pi}\left(\frac{e^2}{mc^2}\right)^2 (1+\cos^2\vartheta)\langle \delta n^2\rangle_{q\Delta\omega}\, do'\, d\omega', \quad (1)$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$; $\Delta\omega = \omega' - \omega$; $\mathbf{k}$, ω ($\mathbf{k}'$, ω') are the wave vector and frequency of the incident (scattered) wave and ϑ is the scattering angle (the angle between $\mathbf{k}'$ and $\mathbf{k}$).

The correlation function for the electron density at "intermediate" frequencies and long wavelengths $[q(T_i/M)^{1/2} \ll \omega \ll q(T_e/m)^{1/2},\ aq \ll 1]$ can be written in the form

$$e^2\langle \delta n^2\rangle_{\mathbf{q}\omega} = \pi^3 a^{-4}\{I(\mathbf{q})\delta(\omega - qs) + I(-\mathbf{q})\delta(\omega + qs)\}, \quad (2)$$

where T_e and T_i are the temperatures and m and M are the masses of electron and ion respectively, $s = (T_e/M)^{1/2}$ is the acoustic velocity, $a = T_e^{1/2}(4\pi e^2 n)^{-1/2}$ is the Debye radius and the function $I(\mathbf{q})$ characterizes the level of the fluctuations of the scalar potential.

In a plasma in which the electrons have a directed motion the function $I(\mathbf{q})$ is a sensitive function of the angle χ between the wave vector $\mathbf{q}$ and the electron velocity $\mathbf{u}$. In the stability region ($\cos\chi < s/u$) this function is given by a familiar relation from the linear theory [5,11,12]

$$I(\mathbf{q}) = a^2 T_e (2\pi)^{-2}\left(1 - \frac{u}{s}\cos\chi\right)^{-1}. \quad (3)$$

Near the boundaries of the stability region, where $\cos\chi \to s/u$, the quantity I increases sharply.

In the turbulence region ($\cos\chi > s/u$) the function $I(\mathbf{q})$ is of the form [10]

$$I(\mathbf{q}) = \frac{a^2 T_e}{(2\pi)^2} B(aq)^{-3}\rho(x)\left\{1 - \cos\chi + \left(1 - \frac{s}{u}\right)\lambda(x)\right\}^{-1} \quad (4)$$

(we assume for simplicity that $1 - s/u \ll 1$).

Here, $x = aq$

$$B = aT_e^2 u\varepsilon(4e^2 T_i s)^{-1}(\pi m/2M)^{1/2}, \quad (5)$$

while ρ and λ are functions defined by the equations

$$x\rho\frac{d\lambda}{dx} = \Psi_0(\lambda), \quad x\frac{d\rho}{dx} = \Psi_1(\lambda);$$

$$\Psi_0 = D^{-1}\left\{(1-\lambda)\ln\left(1+\frac{1}{\lambda}\right) + 1\right\},$$

$$\Psi_1 = D^{-1}\left\{\ln\left(1+\frac{1}{\lambda}\right) + \frac{1-\lambda}{\lambda(1+\lambda)}\right\},$$

$$D = \ln^2(1+1/\lambda) - 1/\lambda(1+\lambda), \quad (6)$$

and ϵ is a small parameter which characterizes the slope of the plateau of the electron distribution function [13] (in order-of-magnitude terms we have $\epsilon \sim (e^2 T_i \Lambda)^{1/2}(aT_e^2)^{-1/2}$; Λ is the Coulomb logarithm). Equation (4) holds when

$$q \gg (s\tau_i)^{-1}(1 - s/u)^{-1}(M/m)^{1/2},$$

where τ_i^{-1} is the ion collision frequency.

It has been shown earlier [10] that the functions ρ and λ are periodic functions of ln x with period

$$p = \rho_0 |c_+ + c_-|, \quad (7)$$

where ρ_0 is a constant determined by the initial distribution of fluctuations

$$c_+ = \int_0^\infty \Psi_0^{-1}\exp\{f^+(\lambda)\}d\lambda, \quad c_- = \int_{-1}^{-\infty}\Psi_0^{-1}\exp\{f^-(\lambda)\}d\lambda;$$

$$f^\pm = \int_{\lambda_\pm}^{\lambda}\Psi_1\Psi_0^{-1}d\lambda, \quad \lambda_+ = 1, \quad \lambda_- = -2.$$

Hence, the function $q^3 I(q)$, in accordance with (4), is a periodic function of the quantity ln (aq). At certain definite values of q, [$\ln(q/q_1) = np$ where q_1 is a quantity given by the initial distribution of fluctuations and $n = 0, \pm1, \ldots$] the amplitude of the turbulent wave is independent of the angle χ

$$I(\mathbf{q}) = a^2 T_e (2\pi)^{-2} B(aq)^{-3}\rho_0(1 - s/u)^{-1} \quad (\cos\chi > s/u). \quad (8)$$

If $\ln(q/q_1) = p(n+\nu)$, where $\nu = c_+(c_+ + c_-)^{-1}$, the angular distribution of the turbulent acoustic waves is highly singular: the turbulent waves propagate either along the electron stream or along the surface of the Cerenkov cone

$$I(\mathbf{q}) = \frac{a^2 T_e}{(2\pi)^2}B \times (aq)^{-3}\rho_0 \begin{cases} \delta(1-\cos\chi) & \ln(q/q_1) \to p(n+\nu) - 0 \\ \delta(\cos\chi - s/u) & \ln(q/q_1) \to p(n+\nu) + 0. \end{cases} \quad (9)$$

We note that in contrast with the angular distribution of fluctuations averaged over angle the intensity of the fluctuations varies monotonically as the wave vector changes:

$$\bar{I}(q) \equiv \frac{1}{2}\int I(\mathbf{q})\,d\cos\chi \approx a^2 T_e (2\pi)^{-2} B (aq)^{-3}\rho_0. \qquad (10)$$

Substituting (3)—(9) for I in (1) and (2) we obtain the coefficient for scattering of transverse electromagnetic waves on random acoustic waves. In this case the quantity $d\sigma$ will depend differently on the wave vectors of the incident and scattered waves and assumes completely different orders of magnitude, depending on whether the condition $|\mathbf{k}\cdot\mathbf{u} - \mathbf{k}'\mathbf{u}| \geq |\mathbf{k} - \mathbf{k}'|s$ is satisfied or not. Introducing the angle $\theta(\theta')$ between the vectors $\mathbf{k}(\mathbf{k}')$ and $\mathbf{u}$ and the angle φ between the planes defined by $(\mathbf{k}, \mathbf{u})$ and $(\mathbf{k}', \mathbf{u})$ we can write this condition conveniently in the form

$$(\theta + \theta' - \pi)^2 + \varphi^2 \operatorname{tg}^2\theta \leqslant 8(1 - s/u). \qquad (11)$$

The inequality in (11) is evidently the condition that the acoustic wave participating in the scattering event must be turbulent.

If (11) is satisfied then the relation in (4) can be used for the function I. Substituting (4) in (1) and (2) we can find the scattering coefficient per unit solid angle and unit frequency interval for the scattered wave:

$$\frac{d\sigma}{do'\,d\omega'} = \frac{1}{4}\left(\frac{e^2}{nc^2}\right)^2 nB\frac{1+\cos^2 2\theta}{|ak\cos\theta|^3}\rho(x)$$
$$\times\left\{(\theta+\theta'-\pi)^2 + \varphi^2\operatorname{tg}^2\theta + 8\left(1-\frac{s}{u}\right)\lambda(x)\right\}^{-1}$$
$$\times\left\{\delta\left(\Delta\omega - 2ks\sin\frac{\vartheta}{2}\right) + \delta\left(\Delta\omega + 2ks\sin\frac{\vartheta}{2}\right)\right\} \qquad (12)^*$$

where $x = 2ak|\cos\theta|$.

If (11) is not satisfied, using (3) for I we obtain the scattering coefficient which follows from the linear theory:[5]

$$\frac{d\sigma}{do'\,d\omega'} = \frac{1}{4}\left(\frac{e^2}{mc^2}\right)^2 n(1+\cos^2\vartheta)$$
$$\times\left\{\left(1 - \frac{u(\cos\theta' - \cos\theta)}{2s\sin(\vartheta/2)}\right)^{-1}\delta\left(\Delta\omega - 2ks\sin\frac{\vartheta}{2}\right)\right.$$
$$\left. + \left(1 + \frac{u(\cos\theta' - \cos\theta)}{2s\sin(\vartheta/2)}\right)^{-1}\delta\left(\Delta\omega + 2ks\sin\frac{\vartheta}{2}\right)\right\}. \qquad (13)$$

In closing this section we note that the frequency shift in scattering $|\Delta\omega|$ is uniquely determined by the frequency of the incident wave and the scattering angle. Hence, if $\Delta\omega$ is measured it should be an easy matter to compute the acoustic velocity s and thus to obtain the electron temperature T_e.

*tg = tan.

3. ANGULAR DISTRIBUTION OF THE SCATTERED RADIATION

Integrating (12) and (13) with respect to ω' we obtain the scattering coefficient per unit solid angle. If (11) is satisfied then

$$\frac{d\sigma}{do'} = \frac{1}{4}\left(\frac{e^2}{mc^2}\right)^2 nB\frac{1+\cos^2 2\theta}{|ak\cos\theta|^3}\rho(x)$$
$$\times\left\{(\theta+\theta'-\pi)^2 + \varphi^2\operatorname{tg}^2\theta + 8\left(1-\frac{s}{u}\right)\lambda(x)\right\}^{-1}. \qquad (14)$$

If (11) is not satisfied then

$$\frac{d\sigma}{do'} = \frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2 n(1+\cos^2\vartheta)$$
$$\times\left\{1 - u^2(\cos\theta - \cos\theta')^2\left(2s\sin\frac{\vartheta}{2}\right)^{-2}\right\}^{-1}. \qquad (15)$$

The relations in (14) and (15) determine the angular distribution of the scattered radiation. Let us consider some of the characteristic features of this distribution.

1. If the angle $\pi - \theta'$ is not too close to θ or if the angle φ is not too small, the quantity $d\sigma/do'$ is independent of the frequency of the incident wave. In this case the scattering coefficient is of the same order of magnitude as in the absence of turbulence.

2. The scattering coefficient increases sharply as the vector $\mathbf{k}'$ approaches the surface of the critical cone defined by the equation

$$(\theta + \theta' - \pi)^2 + \varphi^2\operatorname{tg}^2\theta = 8(1 - s/u). \qquad (16)$$

The intensity of the scattered waves propagating inside the critical cone is, in order-of-magnitude terms, $B(ak)^{-3}$ times greater than the intensity of the waves propagating outside this cone.

3. The energy flux associated with the scattered waves is almost completely concentrated inside the critical cone. Introducing the integrated scattering coefficient σ, which characterizes the total energy flux of the scattered radiation, and using (10), we have

$$\sigma = \frac{\pi}{16}\left(\frac{e^2}{mc^2}\right)^2 nB\frac{1+\cos^2 2\theta}{|ak\cos\theta|^3}\rho_0. \qquad (17)$$

The contribution to the quantity σ from inside the critical cone exceeds the contribution outside the critical cone by a factor of approximately $B(ak)^{-3}$. We note that the integrated scattering coefficient is proportional to $(\omega\cos\theta)^{-3}$ and that it reaches a peak at

$$(k\cos\theta)^{-1} \sim s\tau_i(m/M)^{1/2}(1 - s/u).$$

4. If the vector $\mathbf{k}'$ lies inside the critical cone the scattering coefficient per unit solid angle is proportional to $(\omega \cos\theta)^{-3}$ and the coefficient of proportionality (for fixed values of θ, θ', and φ) is a periodic function of $\ln \omega$. At certain values of ω it follows from (9) that almost all of the scattered waves are propagated along the surface of the critical cone or along its axis:

$$\frac{d\sigma}{do'} = \frac{1}{4}\left(\frac{e^2}{mc^2}\right)^2 nB \frac{1+\cos^2 2\theta}{|ak\cos\theta|^3} \times \rho_0 \begin{cases} \delta([\theta+\theta'-\pi]^2+\varphi^2 \operatorname{tg}^2\theta) \\ \delta([\theta+\theta'-\pi]^2+\varphi^2 \operatorname{tg}^2\theta - 8[1-s/u]). \end{cases} \quad (18)$$

On the other hand, at other values of ω the flux of energy of the scattered radiation, as follows from (8), is uniform for all directions inside the critical cone

$$\frac{d\sigma}{do'} = \frac{1}{32}\left(\frac{e^2}{mc^2}\right)^2 nB \frac{1+\cos^2 2\theta}{|ak\cos\theta|^3} \rho_0 \left(1-\frac{s}{u}\right)^{-1}. \quad (19)$$

5. The period p of the oscillating scattering coefficient as a function of $\ln \omega$ is, in accordance with (7), determined by the initial distribution of fluctuations in the plasma. Regardless of the initial distribution of fluctuations, however, the following relation must be satisfied:

$$p = \frac{16}{\pi}\sigma\left(\frac{e^2}{mc^2}\right)^{-2}(nB)^{-1}\frac{|ak\cos\theta|^3}{1+\cos^2 2\theta}|c_+ + c_-|; \quad (20)$$

this expression relates the period of oscillation of the angular distribution of the scattered radiation to the integrated scattering coefficient.

Thus, the nature of the functions $d\sigma/do'$ and σ is intimately related to the details of the turbulence spectrum. It would then appear that an experimental investigation of the angular distribution of the scattered radiation should serve as a valuable means of verifying various aspects of the theory of plasma turbulence.

In closing this section we note, as pointed out earlier,[14] that in addition to the stationary distribution of fluctuations in a turbulent plasma it is also possible to have an "oscillating" distribution of fluctuations, in which case the correlation function I(q) is a periodic function of ln t. All of the conclusions stated in Secs. 2 and 3 also hold for the oscillating distributions of turbulent acoustic waves. In this case oscillations in the angular distribution of the scattered waves are observed as the frequency of the incident wave is changed and are also observed as periodic fluctuations in time for fixed values of ω.

4. CONVERSION OF LONGITUDINAL WAVES INTO TRANSVERSE WAVES

In addition to causing scattering of transverse waves, fluctuations in a plasma are responsible for other scattering and conversion processes in which longitudinal waves are involved: for example, conversion of Langmuir waves into transverse waves (and vice versa) or scattering of Langmuir waves on acoustic waves. It is evident that all of these processes will be much stronger in a turbulent plasma than in an equilibrium or quasiequilibrium plasma.

In the present work we limit ourselves to conversion of Langmuir waves into transverse waves as a result of interaction with acoustic waves. The power P produced as a result of the conversion radiation is related to the correlation function I(q) by the familiar expression (cf. for example [6]):

$$P = V(2\pi)^{-3}\int Q d\mathbf{k}',$$

$$Q = \pi^3 e^2 \Omega^2 (8T_e^2)^{-1} \sin^2\vartheta |E_0|^2 \Big\{ I(-\mathbf{k}) \times \delta\left(\frac{c^2k'^2}{2\Omega} - \frac{3}{2}\Omega a^2 k^2 - \mathbf{ku} - sk\right) + I(\mathbf{k}) \times \delta\left(\frac{c^2k'^2}{2\Omega} - \frac{3}{2}\Omega a^2k^2 - \mathbf{ku} + sk\right)\Big\}, \quad (21)$$

where E_0 is the amplitude and $\mathbf{k}$ is the wave vector of the incident longitudinal wave, ϑ is the angle between $\mathbf{k}'$ and $\mathbf{k}$; $\Omega = (4\pi e^2 n)^{1/2} m^{-1/2}$ is the plasma frequency (V is the volume of the plasma). The relation in (21) is obtained under the assumption that k is not too small, $a^2k^2 \gg s^2 T_e (mc^4)^{-1}$.

Greatest interest attaches to the conversion of Langmuir waves for which the angle θ between the vectors $\mathbf{k}$ and $\mathbf{u}$ satisfies the condition

$$\cos^2\theta \geqslant (1 - s/u)^2. \quad (22)$$

This inequality represents the condition that the acoustic wave participating in the conversion must be turbulent. Assuming that (22) is satisfied and substituting (4) in (21) we find:

$$Q = \frac{\pi e^2}{32mc^2}\frac{\Omega}{k_\pm'}\sin^2\theta' |E_0|^2 \frac{B}{(ak)^3} \times \frac{\rho(x)\,\delta(k'-k_\pm')}{(1\pm\cos\theta)+(1-s/u)\lambda(x)}, \quad (23)$$

where $x = ak$, θ' is the angle between $\mathbf{k}'$ and $\mathbf{u}$

$$k_\pm'^2 = 3k^2\frac{T_e}{mc^2}\left\{1+\frac{2}{3}\left(\frac{m}{M}\right)^{1/2}(ak)^{-1}\left(\pm 1+\frac{u}{s}\cos\theta\right)\right\}$$

and the upper (lower) sign corresponds to the case $\cos\theta < 0$ ($\cos\theta > 0$). The total radiated power is then

$$P = \frac{e^2\Omega|E_0|^2 V}{128\pi mc^2} B \frac{\rho(x)k_\pm'}{(ak)^3}\left\{1 \pm \cos\theta + \left(1-\frac{s}{u}\right)\lambda(x)\right\}^{-1}. \quad (24)$$

[If $ak < \frac{2}{3}(m/M)^{1/2}[(u/s)|\cos\theta| - 1]$ then $k_+'^2 < 0$; in this case the quantities Q, and P vanish when $\cos\theta < 0$.]

If (22) is not satisfied then, in accordance with (3) and (21), the quantities Q and P assume the form

$$Q = \frac{\pi e^2}{32mc^2}\frac{\Omega}{k'}\sin^2\vartheta|E_0|^2\left\{\frac{\delta(k'-k_+')}{1+(u/s)\cos\theta} + \frac{\delta(k'-k_-')}{1-(u/s)\cos\theta}\right.$$

$$P = \frac{e^2\Omega|E_0|^2 V}{128\pi mc^2}\left\{k_+'\left(1+\frac{u}{s}\cos\theta\right)^{-1} + k_-'\left(1-\frac{u}{s}\cos\theta\right)^{-1}\right\} \quad (25)$$

[if $ak < \frac{2}{3}(m/M)^{1/2}(1 - (u/s)\cos\theta)$ the second terms in these expressions are omitted]. It is evident from a comparison of (24) and (25) that when (22) is satisfied the radiated power is approximately $B(ak)^{-3}$ times greater than in the case in which (22) is not satisfied.

Let us now consider briefly the dependence of the radiated power on the wave vector of the incident wave, assuming that (22) is satisfied. It is evident that the radiated power is especially large when k is small and that a maximum is reached when

$$k \sim (s\tau_i)^{-1}(M/m)^{1/2}(1 - s/u)^{-1}.$$

Further, in accordance with (24) the quantity P is proportional to $k_\pm' k^{-3}$ and the coefficient of proportionality is an oscillating function of ln k. For certain values of k intense radiation of transverse waves occurs only when $\cos\theta \approx \pm 1$ or $\cos\theta \approx \pm s/u$. For certain other values the radiated power is uniform for all angles satisfying (22).

We note that an experimental investigation of the conversion of Langmuir waves into transverse waves would be of especial interest in connection with plasma turbulence since the quantity $P(\mathbf{k})$ (if the $k_\pm'$ factor is neglected) is proportional to $I(\mp\mathbf{k})$; hence, the measurement of this quantity should yield a direct representation of the turbulence spectrum.

The conversion of longitudinal waves into transverse waves can also be of interest as a possible mechanism for the loss of energy from a plasma, the point being that there are always random Langmuir waves in the plasma and the amplitude of these waves is determined by the temperature of the plasma electrons. By interacting with turbulent acoustic waves the random Langmuir waves can be converted into transverse waves which can then be radiated from the plasma. The intensity of this spontaneous emission is easily determined by substituting $|E_0^2|_{\mathbf{k}} \sim T_e$ in (21) and carrying out the integration over $\mathbf{k}$.

Without giving the details of the calculations here we wish to note the following characteristic features of this emission:

1. The power radiated in connection with this emission (for one particle species) is of order

$$\frac{P}{nV} \sim \Omega T_i R, \quad R \sim \left(\frac{e^2 T_e}{amc^2 T_i}\right)^{3/2}\left(\frac{m\Lambda}{M}\right)^{1/2}$$

This power is $(aT_e^2 m\Lambda)^{1/2}(e^2 T_i M)^{-1/2}$ times greater than the power carried away by transverse waves in the absence of turbulence.

2. The function $Q(\mathbf{k}')$, which gives the spectral and angular distributions of the radiated transverse waves, is proportional to $\sin^2\theta'$ where θ' is the angle between $\mathbf{k}'$ and $\mathbf{u}$.

3. Waves characterized by small values of k' are of special interest. The maximum radiated power is

$$Q \sim T_e\Omega^2\tau_i\left(\frac{e^2\Lambda}{aT_i}\right)^{1/2}\left(1-\frac{s}{u}\right)\frac{m}{M}\sin^2\theta',$$

which is reached when

$$k'^2 \sim \Omega(c^2\tau_i)^{-1}(M/m)^{1/2}(1 - s/u)^{-1}.$$

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