EFFECT OF THE INTERACTION BETWEEN NEUTRONS AND NUCLEI ON THE WIDTH OF PARAMAGNETIC RESONANCE IN A NEUTRON BEAM

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Paramagnetic resonance in a neutron beam is analyzed by taking into account the interaction between the neutrons and the target nuclei. The effect of polarization of the target on the width of the resonance line is investigated. It is shown that the width of the resonance line is determined by incoherent processes resulting in the escape of neutrons from the beam. The results are analyzed under the assumption that paramagnetic resonance may be regarded as a photon absorption and emission process during transitions between levels of finite width.

In our preceding papers[1,2] we considered certain resonance phenomena that occur in beams of slow neutrons. The purpose of the present paper is to elucidate the effect of the width of the levels on these phenomena.

Let us consider the paramagnetic resonance of neutrons passing through a substance. The resonance transitions take place between levels that correspond to different potential energy of the neutrons in the external dc magnetic field, regardless of the magnitude of the kinetic energy. The total energy of the neutron does not change when it enters a target; there is only a redistribution of the energy between the kinetic and potential energies of the particle. Since the neutron wave function in a crystal decays with penetration, the neutron has an uncertainty in its momentum and therefore in its kinetic energy as well. Considering the constancy of the total energy of the penetrating neutron, this leads to an uncertainty in the potential energy of the interaction between the neutron and the dc magnetic field, and to the appearance of some width in the levels between which transitions take place. Hence, in the case of paramagnetic resonance of neutrons we have to do with a two-level system with a finite width of the levels that interact with the external ac field.

The equations that determine the behavior of this system with time have the form

$$idC_p(t)\,/\,dt = W_pC_p(t) + be^{i\omega t}C_q(t),$$
$$idC_q(t)\,/\,dt = be^{-i\omega t}C_p(t) + W_qC_q(t), \qquad (1)$$

where C_p, C_q are the components of the spin function of the neutrons; $W_{p,q} = E_{p,q} - i\gamma_{p,q}$, where $E_{p,q}$ and $\gamma_{p,q}$ are respectively the energy and damping of the levels p,q; b is the energy of interaction of the neutron with the transverse magnetic field, and ω is the frequency of this field.

The magnitude of the damping $\gamma_{p,q}$ is determined by processes that cause neutrons with spin parallel to the field (level p) and with spin antiparallel to the field (level q) to escape from the beam. In particular, for cold neutrons $\gamma_{p,q} = \rho v\sigma_{p,q}$, where ρ is the density of nuclei in the substance, v is the velocity of the neutrons, and $\sigma_{p,q}$ is the total cross-section for incoherent scattering in levels p and q, respectively.

If all the neutrons are initially polarized along the field, then $C_p(0) = 1$, $C_q(0) = 0$, and from the known solutions of these equations[3] it follows that the probability of observing at time t a neutron in level q is

$$|C_q(t)|^2 = \frac{1}{4}\frac{(2b)^2}{(\mathrm{Re}\,a)^2 + (\mathrm{Im}\,a)^2}(e^{-t\,\mathrm{Im}\,a} + e^{t\,\mathrm{Im}\,a} - 2\cos(t\,\mathrm{Re}\,a))\,e^{-(\gamma_p+\gamma_q)t};$$
$$a = [(\omega_0 - \omega)^2 + (2b)^2]^{1/2}, \quad \omega_0 = E_q - E_p - i(\gamma_q - \gamma_p). \qquad (2)$$

It can be shown that Eqs. (1) and (2) are valid for packets with dimensions $l' \gg \lambda\!\!\!^{-}$, but much less than the dimensions of the region in which the magnetic field acts. Therefore in transforming to spatial coordinates it is sufficient to substitute into Eqs. (1) and (2) the quantity $t = z/v$, where z is reckoned in the direction of motion of the neutrons. In particular, the probability of reorientation of the spin of the neutron as it leaves the interaction region is determined by Eq. (2) with $t = l/v$, where l is the length of this region. In the latter case the result of (2) is true also for plane waves; this is because we can use scattering theory to treat the problem of the flipping of a neutron spin after its passage through the interaction region. On the other hand, it is known that in the theory of scattering the packet and wave approaches are equivalent.[4]

The expression for the transition probability (2) has a rather complex dependence on the width of the levels between which transitions occur.[1] We consider a few special cases. Let $\gamma_p = \gamma_q = \gamma$, which corresponds to paramagnetic resonance in an unpolarized target. Then

$$\operatorname{Re} a = [(E_q - E_p - \omega)^2 + (2b)^2]^{1/2}, \qquad \operatorname{Im} a = 0.$$

From the general expression (2), we obtain

$$|C_q(t)|^2 = \frac{(2b)^2}{(E_q - E_p - \omega)^2 + (2b)^2} e^{-2\gamma t} \times \sin^2 \frac{t}{2} [(E_q - E_p - \omega)^2 + (2b)^2]^{1/2}. \quad (3)$$

The dependence of the probability $|C_q(t)|^2$ on the frequency of the applied field ω at a given time is the same as in the absence of damping. It follows that a measurement of the intensity of the penetrating beam at some point of the target as a function of the frequency does not give any information about the attenuation of the beam.

Now let the following condition be satisfied:

$$|\gamma_q - \gamma_p| \gg 2b,$$

which is realized in a partially polarized target when the amplitude of the ac field is sufficiently small. The solution of (2) can then be written in the form

$$|C_q(t)|^2 \approx \frac{1}{4} \frac{(2b)^2}{(E_q - E_p - \omega)^2 + (\gamma_q - \gamma_p)^2} \times \{e^{-2\gamma_p t} + e^{-2\gamma_q t} - 2e^{-(\gamma_p + \gamma_q)t} \cos(E_q - E_p - \omega)t\}. \quad (4)$$

If $\gamma_p \gg \gamma_q$ or $\gamma_q \gg \gamma_p$, then, as can be seen, there are values of the time t such that the expression contained within the curly brackets in Eq. (4) takes a value close to unity. The dependence of the probability $|C_q(t)|^2$ on ω then has a Lorentz shape with a width determined by the difference in the widths of levels p and q.

It is easy to verify that the difference of the widths is

$$\gamma_p - \gamma_q = \alpha \mathscr{P},$$

where $\mathscr{P}$ is the degree of polarization of the target. In the special case of the scattering of cold neutrons by a partially polarized target, $\alpha = \rho v \sigma_{\uparrow\downarrow}$, where $\sigma_{\uparrow\downarrow}$ is the cross-section of incoherent scattering of the neutron by a nucleus with a spin antiparallel to the neutron spin.

In steady state the total number of neutrons in level q for a sufficiently thick target equals

$$N = J_0 \int_0^\infty |C_q(t)|^2 dt \approx \frac{A}{(E_q - E_p - \omega)^2 + \Gamma^2};$$

$$A = \frac{J_0}{8} \frac{(2b)^2 (\gamma_p + \gamma_q)}{\gamma_p \gamma_q}, \quad \Gamma^2 = (\gamma_p + \gamma_q)^2 \left(1 + \frac{b^2}{\gamma_p \gamma_q}\right), \quad (5)$$

where J_0 is the intensity of the incident beam. Obviously, the dependence of the total number of neutrons N in level q on the frequency of the applied field ω in the general case has the Lorentzian shape with the width determined by Γ. For a sufficiently small field, when $b^2/\gamma_p\gamma_q \ll 1$, the width Γ is determined purely by nuclear damping effects and equals the sum of widths of levels p and q.

Experimentally the magnitude of Γ can be determined, for example, by measuring the dependence on the frequency ω of the number of scattered cold neutrons with the original polarization. Actually, in a crystal the cold neutrons can scatter only with a flipped spin.[5] Therefore only neutrons that have first undergone a spin flip in the ac magnetic field can possess the original spin direction after scattering. Clearly, the number of such neutrons will depend on the frequency ω in resonant fashion with a line width equal to Γ.

Paramagnetic resonance is frequently regarded as a process in which emission or absorption of a radiofrequency photon takes place. It is then assumed that the incident radiation is not monochromatic and has a certain scatter of frequencies $\Delta\omega$ in the region $\omega = E_p - E_q$. It can be shown that this approach is possible when this inequality is fulfilled:

$$|E_q - E_p|^{-1} \ll t \ll b^{-1}.$$

In accordance with the second part of the inequality it is always true in the photon approach that $\Delta\omega \gg b$. Let us consider paramagnetic resonance in our case from this point of view.

If the frequency scatter $\Delta\omega$ is greater than the width of the levels, then the width of the resonance is determined solely by the magnitude of $\Delta\omega$ and is independent of γ_p and γ_q. Let us now consider the case when $\Delta\omega \ll \gamma_{p,q}$. Under these conditions the radiation can be considered to be monochromatic.

[1]We note that due to the relation $\Delta t \Delta E > \hbar$, the effective width of the resonance depends also on l. When $v/l > \gamma_p - \gamma_q$ and $v/l > b$, this dependence becomes the decisive factor. and if $v/l > E_q - E_p$, the width becomes so large that the resonance effects cannot in general be detected.

To be specific, let there be a transition from level p to level q. The rate of change of the population of the q level is

$$\frac{d|C_q(t)|^2}{dt} = \frac{dP}{dt} - \gamma_q |C_q(t)|^2, \tag{6}$$

where the term dP/dt represents the change in the population of the level q due to photon absorption, and the term $-\gamma_q |C_q(t)|^2$ represents the decrease in the population due to the presence of a width γ_q. From this,

$$dP/dt = d|C_q|^2/dt + \gamma_q |C_q|^2. \tag{7}$$

The transition probability per unit time from level p to level q, averaged over the lifetime of level p, takes the form

$$\left\langle \frac{dP}{dt} \right\rangle = \gamma_p \gamma_q \int_0^t |C_q(t)|^2 dt + \gamma_p |C_q(t)|^2, \quad t \gg \frac{1}{\gamma_p}, \ \frac{1}{\gamma_q}$$

or

$$\left\langle \frac{dP}{dt} \right\rangle \approx \gamma_p \gamma_q \int_0^\infty |C_q(t)|^2 dt = \frac{b^2(\gamma_p + \gamma_q)}{2[(E_q - E_p - \omega)^2 + (\gamma_p + \gamma_q)^2]}. \tag{8}$$

It is clear that the number of absorbed and emitted photons is proportional to (8). Obviously, the dependence of $\langle dP/dt \rangle$ on the frequency of the applied magnetic field has a Lorentz shape with a width equal to the sum of the widths of the levels. It is easily shown that Eq. (8) is valid also in the case when one of the levels has zero width.

[1] V. G. Baryshevskiĭ and M. I. Podgoretskiĭ, JETP **47**, 1050 (1964), Soviet Phys. JETP **20**, 704 (1965).

[2] Baryshevskiĭ, Lyuboshitz, and Podgoretskiĭ, Yadernaya Fizika **1**, 27 (1965), Soviet J. Nucl. Phys. **1**, 19 (1965).

[3] N. F. Ramsey, Molecular Beams, Oxford, 1956 (Russ. Transl., IIL, 1960).

[4] S. S. Schweber, An Introduction to Relativistic Quantum Field Theory, Row, Peterson, 1961 (Russ. Transl., IIL, 1963).

[5] A. Akhiezer and I. Pomeranchuk, Nekotorye voprosy teorii yadra (Some Problems in Nuclear Theory), Gostekhizdat, 1950.

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