

strongly in the region of this temperature (and even passed through zero). However, the reason for the sharp drop of the magnetostriction of the Er ferrite below 50°K is not yet clear.

The magnetostriction of the gadolinium ferrite garnet changed its sign below the compensation point.<sup>[2,3]</sup> This was due to the different signs of the magnetostriction constants of the rare-earth and the "effective" iron sublattices: the magnetostriction constants of the former were negative while those of the latter were positive.

- <sup>1</sup>S. Iida, Phys. Lett. 6, 165 (1963).  
<sup>2</sup>K. Belov and A. V. Pedko, J. Appl. Phys. 31, Suppl. No. 5, 55S (1960).  
<sup>3</sup>Clark, DeSavage, and Callen, J. Appl. Phys. 35, No. 3, Part 2, 1028 (1964).

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### BEHAVIOR OF THE SPECIFIC HEAT $C_V$ OF PURE SUBSTANCES NEAR THE CRITICAL POINT

A. V. VORONEL', V. G. SNIGIREV, and Yu. R. CHASHKIN

Institute for Physico-technical and Radio-engineering Measurements

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EXPERIMENTAL studies<sup>[1,2]</sup> have shown that the specific heat  $C_V$  of argon and oxygen, plotted as a function of temperature, has a logarithmic singularity at the critical point. The slope of the curves for argon was found to be considerably less than that of the curves for oxygen. It was natural to expect the occurrence of a singularity at the critical point to be common among pure substances.

In the present study, we measured very care-

fully the specific heat of argon at the critical density (the density of argon in<sup>[4]</sup> differed by about 2% from the critical value) at temperature intervals as small as  $\approx 0.02$  deg K. The method of measurement was the same as in<sup>[2]</sup>. Since there are quite considerable discrepancies between the published values of the critical density of argon, we carried out measurements at several densities close to the critical value. The amount of the gas in the calorimeter was determined by weighing, the error in this measurement not having exceeded 0.1%.

Tables 1-3 list the values of the specific heat obtained experimentally, together with the corresponding temperature intervals  $\Delta T$ .

The curve corresponding to the density  $\rho_c = 0.533$  g/cm<sup>3</sup> should be regarded as closest to the critical density. The ordinate in Fig. 1 gives the so-called "configurational specific heat"  $C_V - (\frac{1}{2})iNk$  ( $i$  is the number of the degrees of freedom of the gas molecule), as used by Fisher.<sup>[3]</sup> As in earlier work, the semilogarithmic scale is used in Fig. 1, but temperature is

Table I. Specific heat of argon at  $\rho_c = 0.533$  g/cm<sup>3</sup>

$T, ^\circ K$	$\Delta T, ^\circ K$	$C_V, \text{J.mole}^{-1} \text{deg}^{-1}$	$T, ^\circ K$	$\Delta T, ^\circ K$	$C_V, \text{J.mole}^{-1} \text{deg}^{-1}$	$T, ^\circ K$	$\Delta T, ^\circ K$	$C_V, \text{J.mole}^{-1} \text{deg}^{-1}$
132.74	0.228	73.3	147.89	0.140	101.4	150.64	0.089	62.0
132.97	0.168	72.3	148.83	0.130	110.3	150.65	0.039	67.4
133.25	0.191	75.7	149.03	0.124	116.8	150.68	0.041	65.0
133.69	0.176	74.6	149.44	0.069	117.0	150.71	0.046	50.6
137.31	0.168	79.1	149.92	0.077	138.3	150.81	0.105	45.6
137.51	0.163	83.3	149.93	0.066	138.0	150.89	0.146	41.9
139.79	0.162	83.7	149.98	0.068	137.0	150.96	0.107	43.9
140.57	0.155	86.6	150.21	0.041	155.8	151.00	0.120	40.6
140.70	0.158	85.3	150.30	0.060	155.9	151.08	0.147	40.8
140.85	0.166	80.7	150.41	0.018	172.4	151.18	0.111	45.3
142.49	0.172	80.6	150.44	0.017	177.9	151.30	0.248	37.0
145.12	0.150	92.8	150.44	0.035	194.5	151.37	0.119	41.2
145.28	0.148	94.6	150.44	0.048	184.8	151.53	0.132	34.8
145.44	0.150	93.3	150.45	0.025	199.2	151.59	0.257	34.8
145.60	0.152	91.8	150.50	0.048	93.6	151.70	0.134	33.9
146.90	0.084	103.9	150.51	0.064	96.2	151.85	0.114	37.2
147.01	0.084	106.6	150.56	0.056	76.4	152.43	0.657	31.5
147.13	0.091	97.1	150.59	0.042	60.9	153.13	0.683	29.3
147.17	0.139	98.1	150.62	0.078	68.5	152.01	0.130	35.6
147.72	0.140	101.1	150.62	0.043	59.0			

Table II. Specific heat of argon at  $\rho = 0.530 \text{ g/cm}^3$ 

$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$	$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$	$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$
146.95	0.137	95.7	150.38	0.042	144.5	150.52	0.052	89.2
150.11	0.074	121.0	150.42	0.042	152.8	150.56	0.073	66.6
150.24	0.046	129.5	150.45	0.036	159.9	150.58	0.066	63.1
150.29	0.053	140.0	150.45	0.039	162.5	150.66	0.093	53.9
150.32	0.045	132.2	150.48	0.036	170.6	150.74	0.100	48.8
150.33	0.068	133.5	150.48	0.038	166.2			

Table III. Specific heat of argon at  $\rho = 0.538 \text{ g/cm}^3$ 

$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$	$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$	$T, ^\circ\text{K}$	$\Delta T, ^\circ\text{K}$	$C_v, \text{J.mole}^{-1} \cdot \text{deg}^{-1}$
147.00	0.144	89.5	150.21	0.084	118.6	150.58	0.064	58.4
149.14	0.411	109.1	150.27	0.058	121.2	150.67	0.066	51.2
149.56	0.392	114.0	150.40	0.046	139.0	150.74	0.087	49.6
149.93	0.280	116.0	150.42	0.054	132.1	151.35	0.550	39.3
						152.06	0.837	33.0

given in nondimensional units  $t = (T - T_C)/T_C$  for the sake of convenience. Figure 1 includes also the points representing the specific heats of oxygen [2] and of nitrogen, containing about 2% of impurities.

It is evident that the curves for both substances coincide within the limits of the experimental error. Moreover, the specific heat of nitrogen fits the universal curve at high values of  $t$ , and deviates from it as  $T_C$  is approached and as the density departs from its critical value (Fig. 2). Figure 2 shows the same curve (dashed) together with the points corresponding to the densities 0.530, 0.538, and 0.521  $\text{g/cm}^3$ . The latter curve was taken from [1] and it should be remembered that the accuracy of the determination of the density in [1] did not exceed 1–2%. It is clear that the slope of the curves decreases rapidly on departure from the critical density.

This result explains the difference between the slopes of the curves in [1] (0.521  $\text{g/cm}^3$ ) and [2], and agrees with the theoretical suggestions. [4] We

shall use it to estimate the numerical values of the coefficients in the formula

$$C_v = C_{v \text{ reg}} + 2\alpha_1 \ln [(t + \beta v^2)^2 + \gamma^2 v^4] + h(t/v^2). \quad (1)$$

From Fig. 1, it is easy to determine  $\alpha_1 = (0.58 \pm 0.03) Nk$  at  $v = 0$  (common logarithms are used in this figure).

At sufficiently high temperatures ( $t \approx 10^{-3}$ ), we can neglect the term  $\gamma^2 v^4$  and obtain the approximately equal estimates  $\beta \approx 5$  for curves 2 and 3 in Fig. 2; this value of  $\beta$  is more reliable than the estimate  $\beta \approx 0.1$  given in [4]. At sufficiently low values of  $t$ , such as  $t \approx 10^{-4}$ , we can neglect the terms  $t$  and  $\beta v^2$  and estimate the slope of the curve  $C_v(\ln v)$  as  $t \rightarrow 0$ . This slope is found to be approximately twice as great (1.93) as that of the curve  $C_v(\ln t)$  for  $v \rightarrow 0$ , which confirms the correctness of the expansion under the logarithm sign in expression (1).

Additional data are essential for an estimate of the value of  $\gamma$ .

In connection with the interpretation of our re-

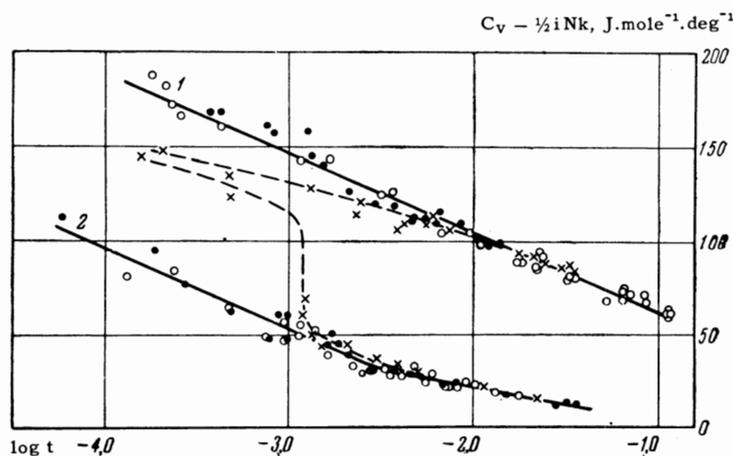


FIG. 1. Dependence of the "configurational specific heat" on  $\log t$  for various substances:  $\circ - \text{O}_2$ ,  $\circ - \text{Ar}$ ,  $\times - \text{N}_2 (+2\%)$  at  $v = 0$ .

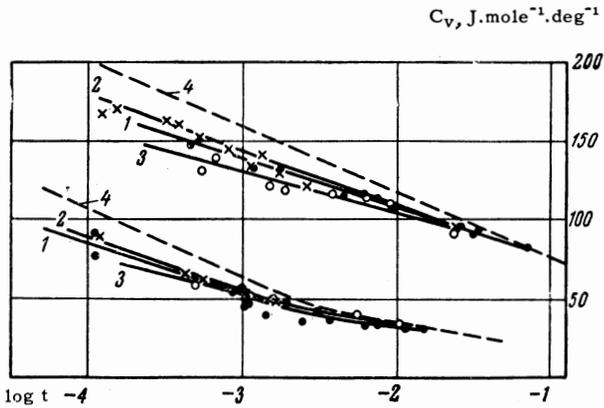


FIG. 2. Dependence of the specific heat of argon on  $\log t$  at the densities: 1) 0.521 g/cm<sup>3</sup> (O); 2) 0.530 g/cm<sup>3</sup> (×); 3) 0.538 g/cm<sup>3</sup> (O); 4) 0.533 g/cm<sup>3</sup>.

sults by Fisher,<sup>[3,5]</sup> we attempted to analyze the present results on the logarithmic scale ( $\log C_v$  as a function of  $\log t$ ). Such an analysis showed that (at  $T > T_C$ ) a power dependence (with an exponent  $1/4$ ), as well as a semilogarithmic dependence, were equally compatible with the data within the limits of the scatter of the points. How-

ever, we are still of the opinion that the logarithmic dependence is more likely both because it fits the most reliable points and because we can retain the concept of a discontinuity, which seems important to us.

At present, we see no reason to review the propositions advanced in<sup>[4]</sup>.

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<sup>1</sup> Bagatskiĭ, Voronel', and Gusak, JETP 43, 728 (1962), Soviet Phys. JETP 16, 517 (1963).

<sup>2</sup> Voronel', Chashkin, Simkin, and Popov, JETP 45, 828 (1963), Soviet Phys. JETP 18, 568 (1964).

<sup>3</sup> M. E. Fisher, Preprint, Rockefeller Institute, New York, 1964.

<sup>4</sup> Azbel', Voronel', and Giterman, JETP 46, 673 (1964), Soviet Phys. JETP 19, 457 (1964).

<sup>5</sup> M. E. Fisher, J. Math. Phys. (New York) 5, 944 (1964).

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## A POSSIBLE METHOD FOR STUDYING FERMİ SURFACES

Yu. V. SHARVIN

Institute for Physics Problems, Academy of Sciences, U.S.S.R.

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WITH measurements of the conductivity of metal samples in a magnetic field it is possible to observe various size effects if the electron free path is sufficiently large in comparison to the dimensions of the sample. We describe here an idea for an experiment which allows us to produce inside a metal single crystal something like a beta spectrograph with focusing of the electrons in a longitudinal magnetic field.

If there is an elliptic turning point on the Fermi surface for a given direction of the uniform magnetic field, then electrons located in the vicinity of this point of momentum space will be focused by the magnetic field, i.e., the electrons that have

emerged from some point inside the metal collect again at a point which lies on the same line of force at a distance  $L$  from the first point. The distance  $L$  is related to the field strength  $H$  by

$$2\pi\hbar/\sqrt{K} = eHL/c, \quad (1)$$

where  $K$  is the Gaussian curvature of the Fermi surface at the turning point and  $n$  is an integer (if additional conditions are satisfied, focusing can also be realized on other parts of the Fermi surface; we do not discuss this here).

To observe the focusing effect it is proposed to measure the resistance of the sample between two contacts of very small size, which could be made, for example, by means of thin wires contacting the surface of the sample at two points on opposite sides of it. If the conditions for focusing are fulfilled for these points, there must be periodic minima in the dependence of the sample resistance on  $H$ .

One could also suggest another more practical and convenient arrangement of the experiment, in which the current is passed between the first microcontact and an auxiliary contact. Then the potential difference between the second microcontact and the other auxiliary contact will be ex-