ON THE MOTION OF THE MAGNETIC MOMENT

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An equation of motion is obtained for the magnetic moment; the effects of energy dissipation and the inertia of the magnetic moment are taken into account in the expression for the effective field. Expressions are given for the resonance frequency for a sphere and for the process of magnetization reversal in a thin plate.

THE change of the magnetic moment in time is completely determined by the equation of motion[1]

$$\mathbf{M} = -\gamma[\mathbf{M}\mathbf{H}_{\text{eff}}], \qquad (1)^*$$

where $\mathbf{H}_{\text{eff}}$ is the sum of all fields acting on the magnetic moment $\mathbf{M}$. The general expression for $\mathbf{H}_{\text{eff}}$ can be obtained by a consistent application of Landau and Lifshitz's concept of effective fields. We have

$$\mathbf{H}_{\text{eff}}\, d\mathbf{M} = -d\,(\Phi_p + T\Delta S + \Phi_k), \qquad (2)$$

where Φ_p is the potential energy of the system, ΔS is the nonequilibrium increase in entropy,[2] and Φ_k is the kinetic energy of the system.

The orienting field

$$\mathbf{H}_0 = -d\Phi_p / d\mathbf{M} \qquad (3)$$

is the sum of the external field $\mathbf{H}_e$ and various fields depending on the position of the magnetic moment $\mathbf{M}$ in the material—the demagnetizing field, the anisotropy field and the field due to deformation of the body.

The friction field $\mathbf{H}_f$ follows from the relation $\mathbf{H}_f d\mathbf{M} = -T d\Delta S$ or

$$\mathbf{H}_f\ \mathbf{M} = -T\frac{d\Delta S}{dt} = -P, \qquad (4)$$

where P is the power dissipation. Tne latter, being a scalar quantity, must be a quadratic function of time derivatives of $\mathbf{M}$ of various order:

$$P = \alpha\dot{\mathbf{M}}^2 + \beta\ddot{\mathbf{M}}^2 + \cdots \qquad (5)$$

Contributions to the terms in P proportional to $\dot{\mathbf{M}}^2$ arise from the Joule loss ($P \sim E^2/\rho \sim \dot{\mathbf{M}}^2$), the loss from internal friction, etc. Contributions proportional to $\ddot{\mathbf{M}}^2$ arise, in particular, from magnetic dipole radiation ($P = (2/3c^3)\,\ddot{\mathbf{M}}^2$). (We shall assume isotropic dissipation of power.) From (4) and (5), confining ourselves to the terms written out explicitly, we get

$$\mathbf{H}_f = -\alpha\dot{\mathbf{M}} + \beta\dddot{\mathbf{M}}. \qquad (6)$$

The kinetic energy of a moving magnetic moment is composed of the energy of the electric field produced by it, $\Phi_E \sim E^2 \sim \dot{\mathbf{M}}^2$, the kinetic energy of the rotating magnetic field $\Phi_H = \frac{1}{2}J_H\omega^2\sin^2\vartheta$, the energy of the induced magnetic field, the energy of lattice waves excited through the magnetostrictive coupling, and so on. (Here J_H is the equatorial moment of inertia due to the magnetic field produced by the magnetic moment $\mathbf{M}$, that is, the moment of inertia relative to an axis perpendicular to $\mathbf{M}$; ω is the angular velocity of precession of the moment in question, and ϑ is the angle between $\mathbf{M}$ and ω.) In general we can write the kinetic energy in the form

$$\Phi_k = \tfrac{1}{2}\eta\dot{\mathbf{M}}^2. \qquad (7)$$

As a simple example we shall find the magnitude of Φ_H. The equatorial moment of inertia may in principle be written, for a homogeneously magnetized body of arbitrary shape, in the form

$$J_H = \int r^2 dm = \int r^2 \frac{H^2}{8\pi c^2}\, dv, \qquad (8)$$

where r is the distance of the volume element dv from the instantaneous axis of rotation and the integration is taken over all space. Thus, for a homogeneously magnetized sphere of radius r_0 with magnetic moment $\mathbf{M}$, (8) gives us

$$J_H = \frac{4}{5}\frac{M^2}{c^2 r_0}, \qquad (9)$$

and therefoie

$$\Phi_H = \frac{2}{5}\frac{M^2\sin^2\vartheta}{c^2 r_0}\omega^2. \qquad (10)$$

*[MH] = M × H

The inertia field is defined by the relation $H_{in} dM = -d\Phi_k$ or

$$\mathbf{H}_{in}\dot{\mathbf{M}} = -\frac{d\Phi_k}{dt} = -\eta\dot{\mathbf{M}}\ddot{\mathbf{M}}. \quad (11)$$

(We consider only the case in which η does not depend on the direction of M.) Therefore we must choose the inertia field to be parallel to $\ddot{\mathbf{M}}$, so that we get

$$\mathbf{H}_{in} = -\eta\ddot{\mathbf{M}}. \quad (12)$$

Thus, the equation of motion has the form

$$\dot{\mathbf{M}} = -\gamma[\mathbf{M}, \mathbf{H}_0 - \alpha\dot{\mathbf{M}} + \beta\ddot{\mathbf{M}} - \eta\ddot{\mathbf{M}}]. \quad (13)$$

Dividing (13) by the volume of the body in question, we get

$$\dot{\mathbf{M}}_0 = -\gamma[\mathbf{M}_0, \mathbf{H}_0 - \alpha V\dot{\mathbf{M}}_0 + \beta V\ddot{\mathbf{M}} - \eta V\ddot{\mathbf{M}}]. \quad (14)$$

It can be seen from the structure of (13) that it may refer equally well to a volume element of the body or to the body as a whole, if the magnitude of the magnetic moment of the element in question, or of the whole body, is conserved.

The conditions for the magnetization to remain homogeneous were considered in a previous paper by the author.[3] First of all, of course, it is necessary that H_0 be sufficiently homogeneous over the volume considered. The friction and inertia fields are, generally speaking, inhomogeneous over the volume of the body, but provided the inhomogeneity is weak the exchange interaction acts to prevent the homogeneity of magnetization being destroyed, so that in practice the effect of the friction and inertia fields is averaged over the whole volume. In this case the effective $\mathbf{H}_f$ and $\mathbf{H}_{in}$ may be defined from (4) and (11), with P and Φ_k being taken as the integrals of the power dissipation and kinetic energy.

The coefficients α, β, and η are in general functions of the size and shape of the body, the electric, elastic and other properties, and may also depend on the rate of charge of M and the initial conditions. The calculation of each of these coefficients, and an estimate of whether it need be taken into account, can be carried out individually in any given case.

We may compare (13) with equations obtained previously for the motion of M. The equations of Landau and Lifshitz[1], Gilbert[4], Bloch[5] and others do not contain inertial terms and therefore may be used only to describe uniform precession (resonance phenomena); they cannot describe transient processes.

As can be seen from (13), the term $\alpha\dot{\mathbf{M}}$ corresponds to the form of the dissipative term given by Gilbert[4]. The dissipative term in the equations of Landau and Lifshitz, which does not take the form of the dissipation into account, leads to various unphysical consequences. For instance, for the motion of M in a thin film (with $H_e \ll 4\pi M_0$, $\alpha\gamma M > 1$) the Landau-Lifshitz equation predicts an increase of the velocity of rotation of M with increasing friction, although in the case considered (motion of M in a plane) the friction can only retard the reversal process.

Dissipative and inertial terms of the form $(\dot{\mathbf{M}} \times \ddot{\mathbf{M}})$ and $(\mathbf{M} \times \ddot{\mathbf{M}})$ were obtained by Ginzburg[6] by taking into account the action of the field produced by the moving magnetic moment on the moment itself.

We shall give a few formulae derived from (13) which are useful in applications. First, the resonance frequency ω for precession of M in a sphere is given by:

$$\omega[1 + (p + r\omega^2)^2] = \gamma H_e \text{ (no inertia)},$$
$$\omega[1 + s\omega\cos\vartheta] = \gamma H_e \text{ (no friction)}, \quad (15)$$

where

$$p = \alpha\gamma M_0 v, \quad r = \beta\gamma M_0 v, \quad s = \eta\gamma M_0 v,$$

and ϑ is the precession angle.

For the case of a sudden reversal of magnetization in a thin plate or film, we get the following expression for the value of the azimuthal angle φ of the vector M (here φ is measured with respect to the direction of "easy" magnetization, which is taken to lie in the plane of the film, while the impulsive field H_e is oriented in the direction antiparallel to this):

$$-r\dddot{\varphi} + s\ddot{\varphi} + r\dot{\varphi}^3 + p\dot{\varphi} = \gamma(H_e\sin\varphi - H_a\cos\varphi\sin\varphi + H_\perp\cos\varphi), \quad (16)$$

where $s = \eta\gamma M_0 v$, $H_\perp$ is the field perpendicular to H_e, which is necessary to conserve the "rigidity" of M[3], and $H_a \cos\varphi$ is the anisotropy field.

An order of magnitude estimate shows that the inertial term becomes important for fast reversal processes.

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