ANGULAR DISTRIBUTIONS IN THE PROCESS OF PAIR PRODUCTION BY CHARGED PARTICLES

V. D. MIKHAĬLOV

Moscow Engineering Physics Institute

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The angular distributions of the secondary particles from production of electron-positron pairs by a fast charged particle in the field of a nucleus are calculated. Distributions are given both for the particles of the pair and for the original particle after the collision.

1. INTRODUCTION

THE production of electron-positron pairs in collisions of fast particles was studied theoretically long ago (first by Bhabha[1,2] before the development of the Feynman technique). In Bhabha's papers the incident particle was regarded as a classical charge, and naturally his result for the total cross section should be correct for cases in which the pair that is produced receives a relatively small energy. It was found subsequently from experiments with cosmic rays[3] that the cross section calculated by Bhabha did not give agreement with experiment when the incident particle was a high-energy electron.

Recently the cross section for pair production by a high-energy charged particle has been calcuated again, with the Feynman technique, by Japanese theorists (Murota et al.[3]), and also by Ternovskiĭ.[4] It turns out that all authors get the same result for the cross section in the region where the energy of the pair $\omega = (\epsilon_+ + \epsilon_-) \ll E/\mu$ (the pair is less "relativistic" than the parent particle); here μ is the mass of the incident particle and E is its energy. For $\omega \gg E/\mu$, Murota et al. get a result different from Bhabha's, and Ternovskiĭ obtained still higher accuracy. In none of these papers, however, was any special effort made to get the angular distribution of the secondary particles (this affects the nature of the calculations). Using the results of the papers cited[1-4], we have obtained the angular distributions of the secondary particles from the process of pair production by a fast charged particle (of mass $\mu \ll E$).

In the relativistic region ($E \gg \mu$, $\epsilon_\pm \gg 1$) the differential cross section for this process, in lowest order of perturbation theory, can be written as follows:

$$d\sigma = \frac{E}{|p|}\Sigma\,|K|^2\,\delta(E - E' - \varepsilon_+ - \varepsilon_-)\frac{dp_+dp_-dp'}{\varepsilon_+\varepsilon_-(2\pi)^9};$$

$$K \equiv -Ze^4\frac{16\pi^2 i}{q^2k^2}\bar{u}(p')\gamma_\mu u(p)$$

$$\times\,\bar{u}(p_-)\left[\gamma_\mu\frac{t(\gamma, p_- - k) - 1}{D_-}(e\gamma)\right.$$

$$\left.+\,(e\gamma)\,\frac{t(\gamma, k - p_+) - 1}{D_+}\gamma_\mu\right]u(-p_+),$$

where p is the initial and p′ the final momentum of the incident particle, and

$$q = p - p' - p_+ - p_-,\quad k = p - p',\quad D_\pm \doteq (k - p_\pm)^2 + 1.$$

(A system of units is used in which $\hbar = c = m_e = 1$.) Here only two Feynman diagrams have been taken into account (Fig. 1). For the justification for this, see, e.g.,[3].

It is necessary next to carry out averaging and summing over polarizations of the particles. This is done in the standard way, and therefore we shall not deal with it.

Since, as can be seen from the expression for the cross section, in the collision of relativistic particles only small angles are important, we shall set $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{1}{2}\theta^2$. It will be more convenient for us to study the angular distribution in the form of the distribution of transverse components of the momenta of the particles after the collision. If the momentum of a particle after the collision is p′, then because of the assumed small-

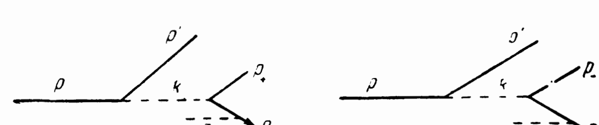

FIG. 1

ness of the angles the transverse component is $p_\perp \approx p'\theta$, $p_\perp = (p'_x, p'_y, 0)$. For small energy transfers $p_\perp \approx (p_x, p_y, 0)$, where p is the momentum of the primary particle before the collision. We also introduce analogous notations for the electron-positron pair.

2. THE DISTRIBUTION OF THE PRIMARY PARTICLES

We first investigate the distribution of the transverse components of the momentum of the primary particle (for example, a muon of mass μ) after collision with the external field of a nucleus Z. To do so we integrate the given differential cross section over the angles of the electron and positron. A similar integration is done in[4]. The result is

$$d\sigma = \frac{Z^2e^8}{3(2\pi)^5}\frac{L}{M^4}\frac{dp_+dp_-p_\perp dp_\perp}{\omega^2(p_\perp{}^2E^2+\mu^2\omega^2)^2}$$
$$\times\left\{\frac{\mu^2\omega^2(p_\perp{}^2+\omega^2)+2E^4p_\perp{}^2}{2\omega^2}[p_\perp{}^2+\omega^2+2M^2(p_+{}^2+p_-{}^2)]\right.$$
$$\left.+\frac{4p_+{}^2p_-{}^2(p_\perp{}^2E^2+\mu^2\omega^2)^2}{(p_\perp{}^2+\omega^2)^2}\right\},$$

where

$$M^2 = 1+\frac{p_+p_-}{E^2\omega^2}(E^2p_\perp{}^2+\mu^2\omega^2) \qquad (E\approx p),$$
$$L = \ln\frac{p_+p_-}{M(\omega^2+p_\perp{}^2)^{1/2}}.$$

Let us perform the integration over the electron and positron momenta p_+ and p_-. To do so we deal separately with the regions where the energy transferred to the pair has values $\omega \ll E/\mu$ (region 1) and where $\omega \gg E/\mu$ (region 2). It can be seen at once that in region 1, owing to the factor $(p^2E^2+\mu^2\omega^2)^2$ in the denominator, the important values of $p_\perp$ are those up to $p_\perp \sim 1$. In region 1 we have $\epsilon_\pm\mu/E \ll 1$, and consequently M^2 can be taken in the form

$$M^2 \approx 1 + p_+p_-\omega^{-2}p_\perp{}^2.$$

Since the largest contribution to the integral comes from the region where $\epsilon_+ \sim \epsilon_- \sim \omega/2$, in the factor L/M^4 we neglect the dependence on p_+ or p_-, and set $p_+p_- \approx \frac{1}{4}\omega^2$. Performing the integration over the momentum of one of the particles of the pair, we get

$$d\sigma = \frac{Z^2e^8}{3(2\pi)^5}\frac{L}{M^4}\frac{p_\perp dp_\perp}{(p_\perp{}^2E^2+\mu^2\omega^2)^2}\frac{d\omega}{\omega^2}$$
$$\times\left\{\frac{\mu^2\omega^2(p_\perp{}^2+\omega^2)+2E^4p_\perp{}^2}{2\omega}\left(\frac{7}{3}\omega^2+\frac{1}{5}p_\perp{}^2\omega^2+p_\perp{}^2\right)\right.$$
$$\left.+\frac{(\mu^2\omega^2+p_\perp{}^2E^2)^2}{(p_\perp{}^2+\omega^2)^2}\frac{2}{15}\omega^5\right\}.$$

The probabilities of large momentum transfers are of particular interest. For $p_\perp \gtrsim 1$ we have $p_\perp^2E^2 \gg \mu^2\omega^2$. Therefore, setting $L \approx \ln\omega$ (correct up to a factor in the argument of the logarithm), we find

$$d\sigma = \frac{Z^2e^8}{3(2\pi)^5}\frac{dp_\perp}{p_\perp}\left\{\frac{7}{3}+\frac{1}{3}p_\perp{}^2\right\}\frac{1}{(1+\frac{1}{4}p_\perp{}^2)^2}$$
$$\times\int_1^{E/\mu}\frac{\ln\omega d\omega}{\omega} = \frac{Z^2e^8}{36\pi^5}\frac{7+p_\perp{}^2}{(4+p_\perp{}^2)^2}\ln^2\frac{E}{\mu}\frac{dp_\perp}{p_\perp}.$$

Then for large $p_\perp$

$$d\sigma = \frac{Z^2e^8}{36\pi^5}\ln^2\frac{E}{\mu}\frac{dp_\perp}{p_\perp{}^3}.$$

A simple expression can also be obtained for extremely small transverse momenta $p_\perp < \mu/E$. Here $p_\perp^2E^2 < \mu^2\omega^2$, and on integrating we get

$$d\sigma = \frac{Z^2e^8}{3(2\pi)^5}p_\perp dp_\perp\left(\frac{1}{15}\ln^2\frac{E}{\mu}+\frac{7}{48}p_\perp{}^2\frac{E^4}{\mu^4}\right)$$

(a term $\sim p_\perp^4$ has been dropped), for $p_\perp < \mu/E < 1$.

Let us consider the region 2, where $\omega \gg E/\mu$, but still $\omega \ll E$. In this region the diagrams that have been discarded can have an important influence. Still, as is shown in[4], for the muon, as a nuclearly nonactive particle for $\omega \ll E$, the diagrams we are using are nevertheless the decisive ones. In this region, unlike region 1, values of the transverse momenta clear up to $p_\perp \sim \mu$ are important.

In region 2 the quantity M^2 will have a somewhat different form:

$$M^2 = \frac{p_+p_-}{E^2\omega^2}(\mu^2\omega^2+p_\perp{}^2E^2),$$
$$L = \frac{1}{2}\ln\frac{p_+p_-E^2\omega^2}{(\omega^2+p_\perp{}^2)(p_\perp{}^2E^2+\mu^2\omega^2)}$$

The integration over $dp_\perp$ is done in an analogous way. We get

$$d\sigma = \frac{Z^2e^8}{3(2\pi)^5}\frac{LE^4p_\perp dp_\perp\omega^2d\omega}{(\mu^2\omega^2+p_\perp{}^2E^2)^4}\left\{\frac{\mu^2\omega^2(p_\perp{}^2+\omega^2)+2E^4p_\perp{}^2}{2E^2\omega^4}\right.$$
$$\times\left[\frac{16E^2}{\omega}(p_\perp{}^2+\omega^2)+\frac{16}{3}\omega(p_\perp{}^2E^2+\omega^2\mu^2)\right]$$
$$\left.+\frac{4\omega(p_\perp{}^2E^2+\mu^2\omega^2)^2}{(p_\perp{}^2+\omega^2)^2}\right\}.$$

It is easy to do the integration for small momenta $p_\perp < 1$:

$$d\sigma = \frac{Z^2e^8}{6\pi^5}\ln\frac{E}{\mu}p_\perp dp_\perp\left\{\frac{1}{16}+10p_\perp{}^2\right\}.$$

For $\mu < p_\perp < E/\mu$ we have

$$d\sigma = \frac{28}{9}\frac{Z^2e^8}{(2\pi)^5}\ln\frac{E}{2p_\perp\mu}\ln\mu\frac{dp_\perp}{p_\perp^3}.$$

3. THE DISTRIBUTION OF THE PARTICLES OF THE PAIR

In this section we shall obtain the distribution of the particles of the electron-positron pair. For this it is most convenient to use the method by which the calculations were done in Bhabha's papers. In doing so we must remember that this method is known to apply in the region 1 which is of most interest, and in which Bhabha's results agree with those of all the other papers. The cross section is here written in the form

$$d\sigma = \frac{Z^2e^8}{4\pi^4}\int\int\int\int\Big[\int\limits_{-\infty}^{+\infty}\int \frac{S'\,dp_x'\,dp_y'}{\{p_\perp'+p_\perp^++p_\perp^-)^2+\delta^2\}^2\{p_\perp'^2+\varepsilon^2\}^2}\Big]$$

$$\times\, dp_x^+\,dp_y^+\,dp_z^+\,dp_x^-\,dp_y^-\,dp_z^-,$$

where V is the relative velocity of the particles; $p^\mp$, $\epsilon^\mp$ are the momentum and energy of the electron (positron); $p_\perp^\mp$ are the transverse components of the momenta of these particles, and

$$\delta = \frac{1+(p_\perp^-)^2}{2\varepsilon_-}+\frac{1+(p_\perp^+)^2}{2\varepsilon_+},\qquad \varepsilon = \frac{(\varepsilon_++\varepsilon_-)\mu}{E},$$

$$S' = \frac{8}{\varepsilon_+^2D_+^2}[\{1+(p_\perp'+p_\perp^+)^2\}\{1+(p_\perp^+)^2\}]$$

$$-\frac{16}{\varepsilon_+D_+\varepsilon_-D_-}[1+\{(p_\perp'+p_\perp^+,\ p_\perp'+p_\perp^-)$$

$$+(p_\perp^+,\ p_\perp^-)+(p_\perp^--p_\perp^+)^2\}$$

$$-(p_\perp'+p_\perp^+,\ p_\perp'+p_\perp^+)(p_\perp^-,\ p_\perp^+)$$

$$+(p_\perp'+p_\perp^-,\ p_\perp^+)(p_\perp'+p_\perp^+,\ p_\perp^-)$$

$$+(p_\perp'+p_\perp^-,\ p_\perp^-)(p_\perp'+p_\perp^+,\ p_\perp^+)]$$

$$+\frac{8}{\varepsilon_-^2D_-^2}[\{1+(p_\perp'+p_\perp^-)^2\}\{1+(p_\perp^-)^2\}];$$

$$D_+ = (p_\perp'+p_\perp^+)^2$$

$$+\Big(\frac{\varepsilon_++\varepsilon_-}{V}-p_z^+-p^-\Big)\Big(\frac{\varepsilon_++\varepsilon_-}{V}-p_z^++p^-\Big),$$

$$D_- = (p_\perp'+p_\perp^-)^2$$

$$+\Big(\frac{\varepsilon_++\varepsilon_-}{V}-p_z^--p^+\Big)\Big(\frac{\varepsilon_++\varepsilon_-}{V}-p_z^-+p^+\Big).$$

We depart from the procedure in[1] and introduce new variables:

$$\zeta_\perp = p_\perp'+p_\perp^++p_\perp^-,\quad p_\perp' = p_\perp',\quad p_\perp^- = p_\perp^-.$$

It is easy to see that the most important contribution to the integral over $\zeta_\perp$ is from the region near the origin, where $0 < \zeta_\perp < 1$. Therefore we expand the integrand in a series in $\zeta_\perp^2$, and drop the higher-order terms (for more details see[1]). The same applies to the integral over $p_\perp'$.

The integration over the angular parts of the variables $\zeta_\perp$ and $p_\perp'$ can be done simply. We get as the result

$$d\sigma = \frac{2Z^2e^8}{\pi^2}\Big[\int\limits_0^1\frac{\zeta_\perp^2}{(\zeta_\perp^2+\delta^2)^2}d\zeta_\perp^2\int\limits_0^1\frac{p_\perp'^2\,dp_\perp'^2}{(p_\perp'^2+\varepsilon^2)^2}$$

$$\times\int\int\limits_{-\infty}^{\infty}\Big\{\varepsilon_+^2+\varepsilon_-^2+\frac{4\varepsilon_+\varepsilon_-p_\perp^{-2}}{[1+(p_\perp^-)^2]^2}\Big\}$$

$$\times\frac{dp_x^-\,dp_y^-}{\omega^4[1+(p_\perp^-)^2]^2}\Big]\,d\varepsilon_+\,d\varepsilon_-.$$

Here $p_\perp$ is the transverse component of the momentum of the electron.

After integrating over the variables indicated we find

$$d\sigma = \frac{Z^2e^8}{4\pi^2}\ln\delta^2\ln\varepsilon^2\,dp_x\,dp_y\int\limits_1^\infty d\varepsilon_+\Big\{(\varepsilon_+^2+\varepsilon_-^2)+\frac{4\varepsilon_+\varepsilon_-p_\perp^2}{(1+p_\perp^2)^2}\Big\}$$

$$\times\frac{d\varepsilon_-}{\omega^4(1+p_\perp^2)^2}$$

We then carry out the integration over ϵ_+, setting $\epsilon_+ \approx \epsilon_- \approx {}^1\!/_2\omega$ in the argument of the logarithm, as we did before. We get

$$d\sigma = \frac{16Z^2e^8pd_\perp p_\perp}{\pi^2(1+p_\perp^2)^2}\int\frac{d\omega}{\omega^4}\ln\omega\ln\frac{E}{\mu\omega}$$

$$\times\Big\{\frac{2}{3}\omega^3+\frac{2p_\perp^2}{3(1+p_\perp^2)^2}\omega^3\Big\}.$$

Assuming $\ln(E/\mu) \gg 1$, we get the final result:

$$d\sigma = \frac{16}{9\pi}Z^2e^8\frac{p_\perp dp_\perp}{(1+p_\perp^2)^2}\ln^3\frac{E}{\mu}\Big[1+\frac{p_\perp^2}{(1+p_\perp^2)^2}\Big].$$

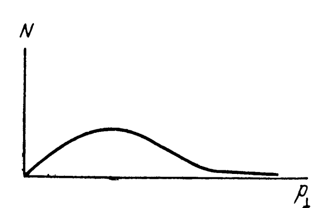

FIG. 2

Accordingly, in the distribution of the transverse components of the electron (positron) momentum the important values of $p_\perp$ are those running up to $p_\perp \sim 1$ (or in ordinary units $p_\perp \sim mc$, where m is the mass of the electron).

The shape of the resulting distribution is shown graphically in Fig. 2.

In conclusion I express my gratitude to I. L. Rozental', and also to G. T. Zatsepin and the members of the seminar he conducts for a discussion.

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89