EMISSION OF ELECTROMAGNETIC WAVES FROM A PLASMA LAYER AT TWICE THE PLASMA FREQUENCY

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We consider the emission of electromagnetic waves from a plasma layer at twice the plasma frequency. A general expression is obtained for the flux density of electromagnetic radiation. It is shown that a thin plasma layer (thickness small compared with the wavelength) primarily radiates waves with p-polarization; the energy radiated per unit time and unit volume from a thin layer (as defined above) is found to be greater than from a thick layer.

AAMODT and Drummond[1] have shown that the nonlinear interaction of plasma waves in an infinite plasma leads to the excitation of electromagnetic waves with frequency equal to twice the plasma frequency: $\omega = 2\omega_0$.

A plasma can be regarded as infinite if the wavelength of the electromagnetic radiation, which is of order c/ω_0, is small compared with the characteristic plasma dimension a: $a \gg c/\omega_0$. One expects that the finite dimensions of the plasma will have an important effect on the electromagnetic radiation when $a \lesssim c/\omega_0$ (this is the case in many experiments, for example [2]). In the present work we investigate electromagnetic radiation at twice the plasma frequency from a plane plasma layer of thickness 2a.

1. It is assumed that the ions are of infinite mass and uniformly distributed over the layer. Neglecting the thermal motion of the electrons 1) we can write the equation of motion for the electron component of the plasma in the form

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{e}{m}\mathbf{E} - \frac{e}{mc}[\mathbf{vH}],$$

$$\frac{\partial n}{\partial t} + \operatorname{div} n\mathbf{v} = 0, \qquad (1)^*$$

where n and v are respectively the electron density and velocity while the remaining notation is conventional. The electric and magnetic fields are determined from Maxwell's equations:

$$\operatorname{div}\mathbf{H} = 0, \qquad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t},$$

$$\operatorname{div}\mathbf{E} = 4\pi e(n_0 - n),$$

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} - \frac{4\pi}{c}en\mathbf{v}, \qquad (2)^*$$

where n_0 is the ion density in the layer.

We assume that $\mathbf{H} = 0$, $\mathbf{E} = 0$, $\mathbf{v} = 0$, $n = n_0$ in the unperturbed state. From (1) and (2) we obtain an equation describing the electric field perturbation **E**:

$$\Delta\mathbf{E} - \operatorname{grad}\operatorname{div}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi e^2 n_0}{c^2}\mathbf{E}$$
$$= \frac{4\pi e n_0}{c^2}(\mathbf{v}\nabla)\mathbf{v} + \frac{4\pi e^2 n_0}{mc^3}[\mathbf{vH}] + \frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{v}\operatorname{div}\mathbf{E}) \qquad (3)\dagger$$

($n_0 = 0$ outside the plasma).

We are interested in the excitation of electromagnetic waves due to the nonlinear interactions of the plasma waves. Hence the perturbations in **E** and **v** caused by the plasma waves are substituted in the nonlinear terms on the right side of (3) (the magnetic field associated with the plasma wave is zero).

We now write (3) in the form

$$\Delta\mathbf{E} - \operatorname{grad}\operatorname{div}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi e^2 n_0}{mc^2}\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial \mathbf{j}_{\text{ext}}}{\partial t},$$

where $(4\pi/c^2)(\partial \mathbf{j}_{\text{ext}}/\partial t)$ denotes the right side of (3) and we have substituted the perturbations in **E** and **v** characteristic of the plasma waves. Thus the problem is reduced to that of determining the electromagnetic field produced by the specified externally produced current $\mathbf{j}_{\text{ext}}$. We note that $\mathbf{j}_{\text{ext}}$ is nonvanishing only within the plasma layer.

The x axis of the coordinate system is chosen to be perpendicular to the layer and the origin is located midway between the planes defining the layer edges. We now introduce the electrostatic

1) This approach is valid if the wavelength of the perturbations is appreciably greater than the Debye length.

*[vH] = v × H.

*rot = curl.

†$\Delta\mathbf{E} = \nabla^2\mathbf{E}$.

potential associated with the plasma wave x and expand it in a Fourier series in the x-coordinate and a Fourier integral over the y and z-coordinates:

$$\varphi = (2\pi)^{-1}\int d\mathbf{k}e^{i\mathbf{kr}}(\varphi_{\mathbf{k}}(x)\,e^{-i\omega_0 t}+\varphi_{-\mathbf{k}}(x)\,e^{i\omega_0 t}),$$

$$\varphi_{\mathbf{k}}(x) = \sum_{p=0}^{\infty}\left(\varphi_{\mathbf{k}p}^{(a)}\sin\frac{p\pi}{a}x+\varphi_{\mathbf{k}p}^{(s)}\cos\frac{(p+1/2)\pi}{a}x\right). \quad (4)$$

Here, k is a two-dimensional vector in the plane perpendicular to the x axis

The function

$$\mathbf{E}_{\mathbf{k}}(x) = (2\pi)^{-1}\int e^{-i\mathbf{kr}}\mathbf{E}d\mathbf{r}$$

satisfies the equation

$$\left(i\mathbf{k}+\xi\frac{\partial}{\partial x}\right)\left[i(\mathbf{kE_k})+\frac{\partial E_{\mathbf{k}x}}{\partial x}\right]+\left(k^2-\frac{\partial^2}{\partial x^2}\right)\mathbf{E_k}$$
$$+\frac{\omega_0^2\theta(x)}{c^2}\mathbf{E_k}+\frac{1}{c^2}\frac{\partial^2\mathbf{E_k}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial\mathbf{j}_{\text{ст}\,\mathbf{k}}}{\partial t},$$
$$\theta(x) = \begin{cases}1, & |x|<a,\\ 0, & |x|>a,\end{cases} \quad (5)$$

where ξ is a unit vector in the x-direction. In what follows we shall drop the subscript "ext" on the externally produced current.

The Fourier form of the external current j_k is related to φ_k as follows:

$$\mathbf{j_k} = \mathbf{j_k}^{(+)}e^{-2i\omega_0 t}+\mathbf{j_k}^{(-)}e^{2i\omega_0 t}, \qquad \mathbf{j_k}^{(-)} = \mathbf{j}_{-\mathbf{k}}^{(+)*},$$

$$\mathbf{j_k}^{(+)} = -i(4\pi)^{-2}\frac{e}{m\omega_0}\int d\mathbf{k}'d\mathbf{k}''\delta(\mathbf{k}'+\mathbf{k}''-\mathbf{k})$$
$$\times\left\{i\left[\mathbf{k}'\varphi_{\mathbf{k}'}\frac{d^2\varphi_{\mathbf{k}''}}{dx^2}+\mathbf{k}''\varphi_{\mathbf{k}'}\frac{d^2\varphi_{\mathbf{k}'}}{dx^2}+\frac{\mathbf{k}}{2}\frac{d\varphi_{\mathbf{k}'}}{dx}\frac{d\varphi_{\mathbf{k}''}}{dx}\right.\right.$$
$$\left.-\left(\frac{\mathbf{k}(\mathbf{k}'\mathbf{k}'')}{2}+\mathbf{k}'k''^2+\mathbf{k}''k'^2\right)\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}''}\right]+\xi\left[\frac{d}{dx}\right.$$
$$\times\left(\frac{3}{2}\frac{d\varphi_{\mathbf{k}'}}{dx}\frac{d\varphi_{\mathbf{k}''}}{dx}-\frac{(\mathbf{k}'\mathbf{k}'')}{2}\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}''}\right)$$
$$\left.\left.-k''^2\varphi_{\mathbf{k}''}\frac{d\varphi_{\mathbf{k}'}}{dx}-k'^2\varphi_{\mathbf{k}'}\frac{d\varphi_{\mathbf{k}''}}{dx}\right]\right\}. \quad (6)$$

In the expression for j_k we have omitted nonoscillating terms since these make no contribution to the radiation.

We now find the function $\mathbf{E}_k(x,t)$ which satisfies (5). Taking the scalar product of (5) with ξ, k/k, and $\mathbf{e} = \xi\times k/k$, we have

$$\frac{1}{c^2}\frac{\partial^2 E_{\mathbf{k}x}}{\partial t^2}+\frac{\omega_0^2\theta(x)}{c^2}E_{\mathbf{k}x}+i\frac{\partial}{\partial x}(\mathbf{kE_k})+k^2E_{\mathbf{k}x}$$
$$= -\frac{4\pi}{c^2}\frac{\partial j_{\mathbf{k}x}}{\partial t}, \quad (7)$$

$$\frac{1}{c^2}\frac{\partial^2(\mathbf{kE_k})}{\partial t^2}+\frac{\omega_0^2\theta(x)}{c^2}(\mathbf{kE_k})+ik^2\frac{\partial E_{\mathbf{k}x}}{\partial x}$$
$$-\frac{\partial^2(\mathbf{kE_k})}{\partial x^2} = -\frac{4\pi}{c^2}\frac{\partial(\mathbf{kj_k})}{\partial t};$$

$$\frac{1}{c^2}\frac{\partial^2(\mathbf{eE_k})}{\partial t^2}+\frac{\omega_0^2\theta(x)}{c^2}(\mathbf{eE_k})+\left(k^2-\frac{\partial^2}{\partial x^2}\right)(\mathbf{eE_k})$$
$$= -\frac{4\pi}{c^2}\frac{\partial(\mathbf{ej_k})}{\partial t}. \quad (8)$$

The equations in (7) can be solved independently of (8). The system in (7) describes the radiation of waves with p-polarization (the electric vector has a component perpendicular to the layer) while (8) describes the radiation of waves with s-polarization (electric vector parallel to the layer).

2. Let us consider the radiation of the p-polarized waves. We seek a solution of (7) in the form

$$\mathbf{E_k} = \mathbf{E_k}^{(+)}e^{-i\omega t}+\mathbf{E_k}^{(-)}e^{i\omega t}, \qquad \omega = 2\omega_0.$$

Then

$$\left(\frac{\omega_0^2}{c^2}\theta(x)-\frac{\omega^2}{c^2}+k^2\right)E_{\mathbf{k}x}^{(+)}+ik\frac{d}{dx}E_{\mathbf{k}}^{(+)} = \frac{4\pi i\omega}{c^2}j_{\mathbf{k}x}^{(+)},$$

$$\left(\frac{\omega_0^2}{c^2}\theta(x)-\frac{\omega^2}{c^2}\right)E_{\mathbf{k}}^{(+)}+ik\frac{dE_{\mathbf{k}x}^{(+)}}{dx}-\frac{d^2E_{\mathbf{k}}^{(+)}}{dx^2} = \frac{4\pi i\omega}{c^2}j_{\mathbf{k}}^{(+)},$$

$$E_{\mathbf{k}}^{(+)}\mathbf{k} = (\mathbf{E_k}^{(+)}\mathbf{k}), \qquad j_{\mathbf{k}}^{(+)}\mathbf{k} = (\mathbf{j_k}^{(+)}\mathbf{k}). \quad (9)$$

The equation for $E_k^{(-)}$ is written in exactly the same way.

At the surfaces x = ±a the functions $E_k^{(+)}$ and $E_k^{(-)}$ satisfy the conditions

$$\{E_{\mathbf{k}}^{(+)}\} = 0, \quad (10)$$

$$\{\varepsilon E_{\mathbf{k}x}^{(+)}\} = 4\pi\sigma_{\mathbf{k}}^{(+)}, \quad (11)$$

where $\sigma_k^{(+)}$ is the density of surface charge while $\epsilon = 1 - \omega_0^2\theta(x)/\omega^2$ is the dielectric constant; (the curly brackets denote a jump in the corresponding quantity). The first boundary condition expresses the continuity of the tangential component of the electric field. The second condition relates the jump in the normal component of the electric induction $D_x = \epsilon E_x$ with the surface density of externally specified charges.[3] It follows from the equation of continuity for the specified currents that the surface charge density can be expressed in terms of the normal component of the density of specified current at the boundary:

$$i\omega\sigma_{\mathbf{k}}^{(+)} = j_{\mathbf{k}x}|_\Gamma,$$

i.e., the boundary condition (11) can be rewritten in the form

$$\{\varepsilon E_{\mathbf{k}x}^{(+)}\} = -\frac{4\pi i}{\omega}j_{\mathbf{k}x}. \quad (12)$$

We now find the solution of (9) subject to the boundary conditions (10) and (12) which satisfies the radiation condition

$$E_{\mathbf{k}}^{(+)},\ E_{\mathbf{k}x}^{(+)} \propto e^{i\varkappa x}, \quad x\to+\infty;$$
$$E_{\mathbf{k}}^{(+)},\ E_{\mathbf{k}x}^{(+)} \propto e^{-i\varkappa x}, \quad x\to-\infty.$$

Here, $\kappa = (\omega^2/c^2 - k^2)^{1/2}$ is the wave vector component perpendicular to the layer outside the plasma. This solution is given by

$$E_k^{(+)} = Ae^{-i\kappa(x+a)} \int_{-a}^{a} dx' \left\{ \left(\frac{\kappa\gamma}{\alpha} - 1\right) e^{-i\alpha(x'-a)} [-\alpha j_k^{(+)} + k j_{kx}^{(+)}] + \left(\frac{\kappa\gamma}{\alpha} + 1\right) e^{i\alpha(x'-a)} [\alpha j_k^{(+)} + k j_k^{(+)}], \quad x < -a,\right.$$

$$E_k^{(+)} = Ae^{i\kappa(x-a)} \int_{-a}^{a} dx \left\{ \left(\frac{\kappa\gamma}{\alpha} - 1\right) e^{i\alpha(x'+a)} [-\alpha j_k^{(+)} - k j_{kx}^{(+)}] + \left(\frac{\kappa\gamma}{\alpha} + 1\right) e^{-i\alpha(x'+a)} [\alpha j_k^{(+)} - k j_{kx}^{(+)}], \quad x > a;\right.$$

$$A = -\frac{4\pi\omega}{\omega^2 - \omega_0^2} \left[\left(\frac{\kappa\gamma}{\alpha} + 1\right)^2 e^{-2i\alpha a} - \left(\frac{\kappa\gamma}{\alpha} - 1\right)^2 e^{2i\alpha a} \right]^{-1},$$

$$\gamma = \frac{\omega^2 - \omega_0^2 - k^2c^2}{\omega^2 - k^2c^2} \frac{\omega^2}{\omega^2 - \omega_0^2}, \qquad \alpha = \left(\kappa^2 - \frac{\omega_0^2}{c^2}\right)^{1/2}.$$

Here

$$E_{kx}^{(+)} = \frac{k}{\kappa} E_k^{(+)}, \quad x < -a; \quad E_{kx}^{(+)} = -\frac{k}{\kappa} E_k^{(+)}, \quad x > a. \tag{13}$$

The quantity α denotes the x-component of the wave vector in the plasma. To define $E_k^{(-)}$ we make use of the relation $E_k^{(-)} = E_{-k}^{(+)*}$. Thus, the problem of determining the electromagnetic field outside the plasma is solved in principle.

In order to conserve space the actual calculations will be carried out only for the case of a thin plasma layer: $a\omega_0/c \ll 1$. Expanding in terms of the parameter $a\omega_0/c$ in (13), to first order we have

$$E_k^{(+)} = \pm \frac{ike \exp(\mp i\kappa(x \pm a))}{4m\omega_0^2} \sum_{p,q} \iint dk' dk'' \delta(k' + k'' - k) \times (-1)^{p+q} \left(\varphi_{k'p}^{(a)} \varphi_{k''q}^{(s)} \frac{p\pi}{a} \left(q + \frac{1}{2}\right) \frac{\pi}{a} + \varphi_{k'p}^{(s)} \varphi_{k''q}^{(a)} \frac{q\pi}{a} \left(p + \frac{1}{2}\right) \frac{\pi}{a} \right).$$

The upper sign in the factor in front of the sum is taken when $x < -a$ and the lower sign when $x > a$.

The time-averaged energy flux of the electromagnetic radiation from the plasma layer in the range $(k, k+dk)$ is given by[4]

$$\bar{S}_k dk = \mathbf{n} \frac{c}{4\pi L^2} (\mathbf{E}_k^{(+)} \mathbf{E}_k^{(+)*} + \mathbf{E}_k^{(-)} \mathbf{E}_k^{(-)*})\, dk = \mathbf{n} \frac{c}{4\pi L^2} \frac{\kappa^2 + k^2}{\kappa^2} (E_k^{(+)} E_k^{(+)*} + E_k^{(-)} E_k^{(-)*})\, dk,$$

where L is the normalized length in the y and z directions and $\mathbf{n}$ is a unit vector in the direction of propagation. Using the expressions obtained above for $E_k^{(+)}$ and $E_k^{(-)}$ we have

$$\bar{S}_k = \overline{S_k^{(+)}} + \overline{S_k^{(-)}},$$

$$\overline{S_k^{(+)}} = \mathbf{n} \frac{c}{4\pi L^2} \frac{k^2 e^2}{16\pi^2 m^2 \omega_0^4} \sum_{p_1 q_1 p_2 q_2} \int dk_1' dk_1'' dk_2' dk_2'' \times \delta(k_1' + k_1'' - k)\, \delta(k_2' + k_2'' - k)\, (-1)^{p_1+q_1+p_2+q_2}$$
$$\times \left(\varphi_{k_1'p_1}^{(a)} \varphi_{k_1''q_1}^{(s)} \frac{p_1\pi}{a} \left(q_1 + \frac{1}{2}\right) \frac{\pi}{a} + \varphi_{k_1'p_1}^{(s)} \varphi_{k_1''q_1}^{(a)} \frac{q_1\pi}{a} \times \left(p_1 + \frac{1}{2}\right) \frac{\pi}{a} \right) \left(\varphi_{k_2'p_2}^{(a)} \varphi_{k_2''q_2}^{(s)} \frac{p_2\pi}{a} \left(q_2 + \frac{1}{2}\right) \frac{\pi}{a} + \varphi_{k_2'p_2}^{(s)} \varphi_{k_2''q_2}^{(a)} \frac{q_2\pi}{a} \left(p_2 + \frac{1}{2}\right) \frac{\pi}{a} \right). \tag{14}$$

We now average (14) over the phases of the perturbations of the potential φ_{kp} using the relation

$$\langle \varphi_{k_1'p_1}^{(s)} \varphi_{k_2'p_2}^{(s)*} \varphi_{k_1''q_1}^{(a)} \varphi_{k_2''q_2}^{(a)*} \rangle = |\varphi_{k_1'p_1}^{(s)}|^2 |\varphi_{k_1''q_1}^{(a)}|^2 \quad \delta_{k_1'k_2'} \delta_{p_1p_2} \delta_{k_1''k_2''} \delta_{q_1q_2}.$$

Assuming that $\delta_{k_1'k_2'} = (2\pi/L)^{-2} \delta(k_1' - k_2')$ we have

$$\bar{S}_{kp} = \mathbf{n} \frac{k^2}{\kappa^2} \sum_{p,q} \int dk' dk'' \delta(k' + k'' - k) R(k', k'', p, q) \times (N_{k'p}^{(a)} N_{k''q}^{(s)} + N_{k'p}^{(s)} N_{k''q}^{(a)});$$

$$R(k', k'', p, q) = 16\pi \frac{e^2\hbar^2}{m^2 c} \times \frac{(p\pi/a)^2 (q + 1/2)^2 \pi^2/a^2}{[k'^2 + (p\pi/a)^2][k''^2 + (q + 1/2)^2 \pi^2/a^2]},$$

$$N_{kp}^{(a)} = \frac{[k^2 + (p\pi/a)^2] |\varphi_{kp}^{(a)}|^2}{4\pi L^2 \hbar\omega_0},$$

$$N_{kp}^{(s)} = \frac{[k^2 + (p + 1/2)^2 \pi^2/a^2] |\varphi_{kp}^{(s)}|^2}{4\pi L^2 \hbar\omega_0}. \tag{15}$$

The quantity N_{kp} denotes the number of plasma waves characterized by k, p per unit volume. The index s (or a) refers to waves in which the perturbation and potential is an even (odd) function of x.

Proceeding in the same way as above we can obtain an expression for the energy flux density associated with the s-polarized waves. An estimate of the intensity of the s-wave indicates that $S_{ks} \sim S_{kp}(\alpha\omega_0/c)^2 \ll S_{kp}$. Consequently, the predominant radiation from a thin plasma layer is p-waves.

Now let us consider a typical experimental situation. The characteristic wave vector of the plasma waves k_0 is appreciably greater than ω_0/c. In this case we can write $\mathbf{k}'' = -\mathbf{k}' + \mathbf{k} \approx -\mathbf{k}'$ in (15) and the expression for S_{kp} is simplified appreciably:

$$S_{kp} = \mathbf{n} \frac{k^2}{\kappa^2} \sum_{p,q} \int dk' R(k', k', p, q) (N_{k'p}^{(a)} N_{k'q}^{(s)} + N_{k'p}^{(s)} N_{k'q}^{(a)}). \tag{16}$$

We note that the dependence on wave vector is now contained only in the factor in front of the integral.

Integrating the component normal to the layer $S_{\mathbf{k}p}$ with respect to $\mathbf{k}$ between the limits $k = 0$ and $k = 2\omega_0/c$ we obtain the total energy flux radiated from the layer:

$$\bar{S} = \frac{32\pi}{3}\left(\frac{\omega_0}{c}\right)^2 \sum_{p,q} \int d\mathbf{k}' R(\mathbf{k}', \mathbf{k}', p, q)\,(N^{(a)}_{\mathbf{k}'p} N^{(s)}_{\mathbf{k}'q} + N^{(s)}_{\mathbf{k}'p} N^{(a)}_{\mathbf{k}'q}). \quad (17)$$

We now estimate the flux $\bar{S}$ when the plasma-wave energy is distributed uniformly over the spectrum from $k = 0$, $p = 0$ to $k = k_0$, $p = p_0$ assuming that the relation $N^{(a)}_{\mathbf{k}p} = N^{(s)}_{\mathbf{k}p} = N_{\mathbf{k}p}$ is satisfied. In this case the occupation number $N_{\mathbf{k}p}$ is determined from the relation

$$\hbar\omega_0 \sum_{p=0}^{p_0} \int_0^{k_0} d\mathbf{k}\,(N^{(a)}_{\mathbf{k}p} + N^{(s)}_{\mathbf{k}p}) = 2\pi k_0^2 p_0 \hbar\omega_0 N_{\mathbf{k}p} = W,$$

where W is the energy density of the plasma waves. Thus

$$N_{\mathbf{k}p} = W / 2\pi k_0^2 p_0 \hbar\omega_0.$$

When $p_0 \gg 1$ the summation with respect to p and q in (17) can be replaced by an integration. A simple calculation yields

$$\bar{S} = \frac{128}{3} c \left(\frac{\omega_0}{k_0 c}\right)^2 W \frac{W}{m n_0 c^2} I(k_0, p_0),$$

$$I(k_0, p_0) = \int_0^1 \left(1 - \frac{a k_0}{p_0 \pi}\eta \tan^{-1}\frac{p_0\pi}{a k_0 \eta}\right)^2 \eta\, d\eta. \quad (18)$$

If $p_0\pi/a \gtrsim k_0$, then $I \approx 1/2$.

3. Let us now compare (18) with the results of Aamodt and Drummond[1] who assume that the plasma is infinite. These authors found the energy radiated per unit volume of plasma per unit time:

$$\varepsilon \sim 30\omega_0 \left(\frac{\omega_0}{c k_0}\right)^3 W \frac{W}{m n_0 c^2}. \quad (19)$$

When $p_0\pi/a \approx k_0$ the form of the spectral distribution we have chosen corresponds to that used by Aamodt and Drummond. In this case an estimate of ϵ from (18) yields

$$\varepsilon = \frac{S}{2a} = \frac{32}{3}\omega_0 \frac{c}{a\omega_0}\left(\frac{\omega_0}{c k_0}\right)^2 W \frac{W}{m n_0 c^2}. \quad (20)$$

Comparing (19) and (20) we note that a unit volume of a thick plasma layer radiates appreciably less energy per unit time than a unit volume of a thin layer.

It is of interest to examine the results that have been obtained. It follows from (4), (6), and (13) that the amplitude of the electromagnetic wave is proportional to a sum of integrals of the form

$$\int_{-a}^{a} (B e^{i\alpha x} + C e^{-i\alpha x}) \exp\left[i\frac{\pi}{a}\left(p + q + \frac{1}{2}\right)x\right] dx,$$

where B and C are constants. In the case of a thick layer, where $a\omega_0/c \gg 1$, we can allow the limits of integration to go to infinity as an approximation. In this case the integral is nonvanishing only if $\pi(p+q+1/2)/a = \pm a$. In the opposite limiting case, where $a\omega_0/c \ll 1$, p and q are not subject to these strict requirements; the integral is nonvanishing over a wide range of values of p and q and a larger number of plasma waves can contribute to the excitation of an electromagnetic wave with a given wave vector.

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