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SUPPRESSION OF INELASTIC CHANNELS IN RESONANT NUCLEAR SCATTERING IN CRYSTALS

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The resonance scattering of a particle is treated for the case of a crystal containing nuclei with a lowlying level. It is shown that in certain cases, when the Bragg condition is satisfied, a situation may occur where the amplitudes for the formation of a compound (excited) state in the incident and diffracted waves are equal in magnitude but opposite in sign. Because of the obvious coherence of the two waves, in this case the probability for formation of the compound nucleus is zero, and thus the inelastic channels disappear and the crystal becomes nonabsorbing (partially or completely).

A special analysis is made of the effect of vibration of the nuclei in the lattice. It is found that in the case of narrow lines ($\Gamma \ll \omega_p$, where ω_p is the characteristic frequency of the phonon spectrum), temperature and any other nuclear oscillations have practically no effect. In the case of broad lines, vibration of the nuclei partially destroys the effect, which then becomes temperature dependent. The problem is studied in detail for the case of resonance scattering of γ quanta in the presence of internal conversion. It is assumed that the γ quanta are produced in the source in a decay accompanied by the Mössbauer effect. It is shown that in an M1 transition, for γ quanta with a particular polarization, the effect described occurs for Bragg scattering from any crystal plane. For the other polarization, or for both polarizations in the case of an E2 transition, special conditions must exist in order for the effect to occur.

1. INTRODUCTION

A characteristic feature of resonant nuclear interaction at low energies is the formation of a "compound nucleus" (excited nucleus) with its subsequent decay, which can be realized through different channels.

In general the ratio of the widths Γ_1 and Γ_2 for the elastic and inelastic channels is arbitrary. Suppose that $\Gamma_1/\Gamma_2 \ll 1$. Then it is easily understood, considering the large value of the resonant cross section, that even a very thin layer of material will practically completely absorb the primary beam of particles (with an energy spread within the width of the resonance level), converting them into other particles (e.g., neutrons into γ quanta, γ quanta into conversion electrons).

The question arises whether it is possible to suppress the inelastic channels, and thus change a strongly absorbing medium into one which only scatters. At first glance the question seems absurd, since this would require forbidding the formation of the compound (excited) nucleus. But this is precisely the situation that can be realized if the scattering occurs from a strictly regular arrangement of the system of nuclei.

Under ordinary conditions, the interactions of nuclei in matter with the incident particles occur completely independently, and their collective action manifests itself only in the drop in intensity of the primary beam with thickness. But for scattering by crystals, if the wave length of the particles is comparable with the interatomic spacing, or less, the picture changes fundamentally. Now the ψ function of the particles at each point in the material can be formed as the result of collective action of the whole regular system, and need not coincide with the original plane wave. This permits us to pose the question of finding conditions under which the ψ function at each nucleus would be such that the probability for formation of a compound nucleus would become zero.

In an ideal crystal there is an abrupt readjustment of the ψ function when the Bragg condition is fulfilled, when we know that a state of definite energy consists of a superposition of two plane waves.

It turns out that in various cases one can have the situation where in a sufficiently thick crystal the amplitude for compound nucleus formation has the same value for the initial and diffracted waves, but with opposite signs. Because of the obvious coherence of the two waves, the probability of formation of the compound nucleus then actually drops to zero. In the simplest cases this situation corresponds to a vanishing of the total ψ function at all the nodes of the lattice (in the case of interaction with γ quanta, to the vanishing at the lattice sites of the electric or magnetic field intensity).

Thus, in resonance scattering by an ideal crystal, when the Bragg condition is fulfilled, the inelastic channels may be suppressed (completely or partially), and the material may change from being strongly absorbing to being (completely or partially) transparent.

It is of interest to emphasize that this cannot be achieved in irregular systems, and consequently the resulting yield of the nuclear reaction depends essentially on how the nuclei are arranged in space.

We must mention that the picture given here has a quite close analogy in ordinary atomic scattering of x rays in a crystal. As was first observed experimentally by Borrmann[1] and later explained by Laue,[2] the ordinary absorption of x rays in a crystal because of interaction with the electrons decreases when the Bragg condition is realized—the so-called "anomalous transmission." It is clear that here, in contrast to the nuclear case, we can have only a partial reduction of the absorption, because of the finite dimensions of the atoms.

At first glance it seems that vibration of the atoms in the crystal, which occurs even at T = 0, should always lead to some disturbance of this picture and, as a consequence, to a partial reestablishment of the inelastic channels. However, as the present work shows, this statement is strictly speaking not true. Thus, in the case of narrow resonances ($\Gamma \ll \omega_p$, where ω_p is a characteristic frequency for the phonon spectrum), the effect under very specific conditions turns out to be independent of the vibration of the nuclei and consequently of the temperature (we are referring to the effect when the Bragg condition is satisfied exactly—cf. below).

In this paper the problem under consideration is analyzed in detail for the case of resonance scattering of γ quanta by nuclei having a lowlying isomeric level. It is assumed that the decay of the isomeric level in the crystal gives a finite probability for the Mössbauer effect, so that the source emits γ quanta with a narrow spread in energy corresponding to the width of the level. The ratio of the elastic width Γ_1 to the conversion width Γ_2 is assumed to be arbitrary.

2. DEFINITION OF THE CURRENT

Let us consider a crystal containing nuclei with a lowlying resonance level ω_0. Suppose that there is a flux incident on this crystal of γ quanta having a narrow spread in energy around ω_0 of the order of the level width Γ. To describe the radiation field inside the crystal we use the usual set of Maxwell equations. Then, converting to space and time Fourier components, we have

$$(k^2 - \omega^2/c^2)\mathbf{E}(\mathbf{k}, \omega) - \mathbf{k}(\mathbf{kE}(\mathbf{k}, \omega)) = 4\pi c^{-2}\omega i \mathbf{j}(\mathbf{k}, \omega), \tag{2.1}$$

$$\mathbf{j}(\mathbf{k}, \omega) = \sum_n \mathbf{j}_n(\mathbf{k}, \omega). \tag{2.2}$$

Here $\mathbf{j}_n(\mathbf{k}, \omega)$ is the Fourier component of the current density produced by the n-th resonant nucleus in the crystal. (We neglect the electronic part of the current as well as the current corresponding to the nonresonant nuclei.)

We use the fact that the total momentum of the nucleus in the crystal and the momentum of the γ quanta are negligibly small compared to the momentum of the nucleons in the nucleus. Then we have for the Fourier components of the current density operator for the n-th nucleus,

$$\hat{\mathbf{j}}_n(\mathbf{k}, \omega) = e^{-i\mathbf{kr}_n}\Big[\hat{\mathbf{j}}_1(\mathbf{k}) - \sum_a e^{-i\mathbf{k}\rho_a}\frac{e_a^2}{Mc}\mathbf{A}(\mathbf{r}_n + \rho_a, \omega)\Big], \tag{2.3}$$

$$\hat{\mathbf{j}}_1(\mathbf{k}) = \sum_a e^{-i\mathbf{k}\rho_a}\Big(\frac{e_a}{M}\hat{\mathbf{p}}_a + ic[\mathbf{k}\hat{\mu}_a]\Big). \tag{2.4}^*$$

In (2.3) and (2.4), $\mathbf{r}_n$ denotes the coordinate of the center of gravity of the nucleus, while ρ_a is the relative coordinate of the nucleon in the nucleus, with its corresponding momentum operator $\mathbf{p}_a$. All the other notations are standard.

As our gauge condition we set the scalar potential equal to zero. Then $\mathbf{A}(\mathbf{k}, \omega) = -ic\omega^{-1}\mathbf{E}(\mathbf{k}, \omega)$. Then, within the framework of standard perturbation theory, in the linear approximation in the fields, we have the following expression for the average value of the current density corresponding to an individual nucleus:

$$j_n^i(\mathbf{k}, \omega) = \frac{i}{(2\pi)^3\omega}\int d^3k' E^l(\mathbf{k}', \omega) \times \Big\{\sum_s \frac{\overline{(\hat{j}_1^i(\mathbf{k})e^{-i\mathbf{kr}_n})_{0s}(\hat{j}_1^{l*}(\mathbf{k}')e^{i\mathbf{k}'\mathbf{r}_n})_{s0}}}{\omega - \omega_{s0} + i\Gamma_s/2} + \frac{Ze^2}{M}\delta^{il}\overline{e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}_n}}\Big\}. \tag{2.5}$$

In this expression the index s refers to the intermediate state, while the bar above denotes an av-

*$[\mathbf{k}\mu_a] = \mathbf{k} \times \mu_a$.

erage over the initial state. Γ_S is the width of the resonance level of the nucleus in the crystal, and is in general different from the total width for an isolated nucleus. (In (2.5) and from now on, $\hbar = 1$.)

We note that the second term in (2.5) can be dropped, since it is small compared to the first.

At first we neglect the vibration of the nuclei in the crystal. Then the summation over s in (2.5) reduces to a summation over the components of the multiplet (index ζ), corresponding to the excited nuclear state. The level may be degenerate, or it may be split because of the hyperfine interaction. As for the widths of the individual components, we shall always assume that they are identical.[1)]

We make use of the fact that

$$\sum_n e^{i(\mathbf{k}'-\mathbf{k})\mathbf{r}_n} = \eta \frac{(2\pi)^3}{V_0} \sum_{\mathbf{K}} \delta(\mathbf{k}'-\mathbf{k}-\mathbf{K}) S(\mathbf{K}),$$

$$S(\mathbf{K}) = \sum_j{}' e^{i\mathbf{K}\mathbf{r}_j}. \tag{2.6}$$

Here V_0 is the volume of a unit cell; $\mathbf{K}$ is the reciprocal lattice vector multiplied by 2π; η is the relative concentration of the isotope with the resonant scattering level; $\mathbf{r}_j$ is the relative radius vector of the j-th nucleus in the unit cell. (The summation over j is taken only over the nuclei of the scattering element.) Then, substituting (2.5) in (2.2), we find

$$j^i(\mathbf{k}, \omega) = \sum_{\mathbf{K}} \sigma_\omega^{il}(\mathbf{k}, \mathbf{k}+\mathbf{K}) E^l(\mathbf{k}+\mathbf{K}, \omega),$$

$$\sigma_\omega^{il}(\mathbf{k}, \mathbf{k}+\mathbf{K}) = \eta \frac{i}{\omega V_0} S(\mathbf{K}) \sum_\zeta \frac{\overline{(\hat{j}_1^i(\mathbf{k}))_{0\zeta} (\hat{j}_1^{l*}(\mathbf{k}+\mathbf{K}))_{\zeta 0}}}{\omega - \omega_{\zeta 0} + i\Gamma/2} \tag{2.7}$$

(cf. the analogous relations for the electron current density in a crystal[5]).

We now proceed to determine the average current density, taking account of the vibration of the nuclei in the crystal. In this case the index s in (2.5), in addition to the multiplet sublevel label ζ, also characterizes the set of phonon occupation numbers $\{n^s\}$, where

$$\omega_{s0} = \omega_{\zeta 0} + \sum_\beta \omega_\beta (n_\beta^s - n_\beta^0)$$

(β labels the normal modes).

We introduce explicitly the vector displacement $\mathbf{u}_n$ of the nucleus (relative to its equilibrium position $\mathbf{R}_n$): $\mathbf{r}_n = \mathbf{R}_n + \mathbf{u}_n$. Then expression (2.5) is transformed to the following:

$$j_n^i(\mathbf{k}, \omega) = \frac{i}{(2\pi)^3 \omega} \int d^3k' E^l(\mathbf{k}', \omega) e^{i(\mathbf{k}'-\mathbf{k})\mathbf{R}_n}$$

$$\times \sum_{\zeta\{n^s\}} \frac{\overline{(\hat{j}_1(\mathbf{k}))_{0\zeta} (\hat{j}_1^{l*}(\mathbf{k}'))_{\zeta 0} [(e^{-i\mathbf{k}\mathbf{u}_n})_{\{n_0\}\{n_s\}} (e^{i\mathbf{k}'\mathbf{u}_n})_{\{n_s\}\{n_0\}}]}}{\omega - \omega_{\zeta 0} - \sum_\beta \omega_\beta (n_\beta^s - n_\beta^0) + i\Gamma/2}. \tag{2.8}$$

It is easy to show directly that the product of the matrix elements in the square brackets is independent of the label of the unit cell. For the total current density in the crystal this leads immediately to an expression of the form of (2.7), where for the σ_ω^{il}, after the standard calculation of the matrix elements in the square brackets (cf., for example,[6]) we have

$$\sigma_\omega^{il}(\mathbf{k}, \mathbf{k}_1) = \eta \frac{i}{\omega V_0} \sum_{\zeta j} \overline{(j_1^i(\hat{\mathbf{k}}))_{0\zeta} (j_1^{l*}(\mathbf{k}_1))_{\zeta 0}} e^{i(\mathbf{k}_1-\mathbf{k})\mathbf{R}_j}$$

$$\times \exp\left\{-\frac{1}{2} Z_j(\mathbf{k}) - \frac{1}{2} Z_j(\mathbf{k}_1)\right\} \Big\langle \frac{1}{\omega - \omega_{\zeta 0} + i\Gamma/2}$$

$$+ \frac{1}{1!} \sum_{\beta, n_\beta^s} \frac{(\mathbf{k}\mathbf{u}_{j\beta})_{n_\beta^0 n_\beta^s} (\mathbf{k}_1 \mathbf{u}_{j\beta})_{n_\beta^s n_\beta^0}}{\omega - \omega_{\zeta 0} - \omega_\beta (n_\beta^s - n_\beta^0) + i\Gamma/2} + \ldots \Big\rangle. \tag{2.9}$$

Here $\langle\ \rangle$ denotes a thermal average over the phonon occupation numbers;

$$\mathbf{u}_{j\beta} = (2M_j \omega_\beta N)^{-1/2} [\mathbf{v}_j(\beta) a_\beta + \mathbf{v}_j^*(\beta) a_\beta^+]$$

is the displacement of the j-th nucleus of the unit cell in the β-th normal mode; $\mathbf{v}_j(\beta)$ is the complex polarization vector;

$$Z_j(\mathbf{k}) = \frac{1}{2M_j} \sum_\beta \frac{|\mathbf{k}\mathbf{v}_j(\beta)|^2}{\omega_\beta} (2\langle n_\beta \rangle + 1). \tag{2.10}$$

In the case of narrow nuclear levels ($\Gamma \ll \omega_p$, ω_p is a characteristic frequency for the phonon spectrum), which is the case of principal interest for scattering of γ quanta, one can actually neglect all the terms in the series (2.9) compared to the first term. The expression then simplifies markedly:

$$\sigma_\omega^{il}(\mathbf{k}, \mathbf{k}_1) = \eta \frac{i}{\omega V_0} \left\{\sum_j e^{i(\mathbf{k}_1-\mathbf{k})\mathbf{R}_j} e^{-1/2[Z_j(\mathbf{k}) + Z_j(\mathbf{k}_1)]}\right\}$$

$$\times \sum_\zeta \overline{(\hat{j}_1^i(\mathbf{k}))_{0\zeta} (\hat{j}_1^{l*}(\mathbf{k}_1))_{\zeta 0}} \Big/ (\omega - \omega_{\zeta 0} + i\Gamma/2). \tag{2.11}$$

In the opposite limiting case, when $\Gamma \gg \omega_p$, one can neglect the phonon energies in the denominators of the terms of the series in (2.8). The series is then summed, and we again get an expression of the form of (2.11), but with the substitution

$$\exp\{-{}^1/_2 Z_j(\mathbf{k}) - {}^1/_2 Z_j(\mathbf{k}_1)\} \to \exp\{-{}^1/_2 Z_j(\mathbf{k}_1 - \mathbf{k})\}. \tag{2.12}$$

If we now substitute (2.7) in (2.1) and take account of (2.9) (and also (2.10)–(2.12)), we arrive at a closed set of equations determining the elec-

1)Regarding the validity of these assumptions cf.[3,4]

tromagnetic field in the crystal. It should be mentioned that averaging the current over the vibrations of the nuclei in the crystal is completely adequate for a problem in which one treats elastic scattering. In fact, the characteristic of elastic scattering is a very long time of interaction and, consequently, a time-averaged picture of the scatterer.

For a lowlying isomeric nuclear level one has in most cases a magnetic dipole (M1) or electric quadrupole (E2) transition. To avoid introducing complications because of generality, we shall limit ourselves to specific calculations of (2.11) for these two cases. We note that the M1 and E2 transitions already give an electrodynamics which includes spatial dispersion.

We limit ourselves to the case where there is no hyperfine splitting (complete degeneracy). Then the denominator in (2.11) has a fixed value, and we need only compute the sum

$$I^{il} = \sum_{\zeta} (\hat{j}_1{}^i(\mathbf{k}))_{0\zeta} (\hat{j}_1{}^{l*}(\mathbf{k}_1))_{\zeta 0}. \tag{2.13}$$

For magnetic dipole and electric quadrupole interactions, the expression (2.13) is a bilinear combination of the vectors $\mathbf{k}$ and $\mathbf{k}_1$ (cf. (2.4)). In the presence of complete degeneracy, symmetry arguments give the following general form for this bilinear combination:

$$I^{il} = b(\mathbf{k}\mathbf{k}_1)\delta^{il} + b_1 k^i k_1{}^l + b_2 k_1{}^i k^l. \tag{2.14}$$

In the case of magnetic dipole interaction ($\hat{\mathbf{j}}_1(\mathbf{k}) = ic\mathbf{k}\times\hat{\boldsymbol{\mu}}$) one gets directly $b_1 = 0$, $b_2 = -b$, and so

$$I^{il} = b[(\mathbf{k}\mathbf{k}_1)\delta^{il} - k_1{}^i k^l]. \tag{2.14'}$$

The constant b can be related to the probability of emission of a γ quantum by the excited nucleus in an M1 transition, and consequently, to the corresponding width Γ_1. One can show that

$$b = \frac{c^5}{4\omega_0{}^3}\frac{2J+1}{2J_0+1}\Gamma_1, \tag{2.15}$$

where J_0 and J are the angular momenta of the normal and excited states, respectively, and ω_0 is the frequency of the excited state.

Similarly, for electric quadrupole interaction ($j_1^i(\mathbf{k}) = -i\omega_0 k^m \hat{Q}^{im}$) we have, in the case of complete degeneracy, $b_1 = -2b$, $b_2 = b$ and consequently,

$$I^{il} = b[(\mathbf{k}\mathbf{k}_1)\delta^{il} + k_1{}^i k^l - {}^2/_3 k^i k_1{}^l], \tag{2.14''}$$

where b has the same value (2.15).

3. SOLUTION OF THE DIFFRACTION PROBLEM

We now turn to the system of equations (2.1), (2.7), and (2.11), and consider the case when the value of $\mathbf{k}$ is such that for one of the diffracted waves the wave vector $\mathbf{k}_1 = \mathbf{k} + \mathbf{K}_1$ has a value close to that obtained from the exact Bragg condition $k_1^2 = k^2$.

Then, as usual, we need only retain two vector equations of the whole set (where the interaction with the individual nucleus is assumed to be small):

$$\begin{aligned}(k^2/\varkappa^2 - 1)E^i(\mathbf{k}) &= g_{00}{}^{il}E^l(\mathbf{k}) + g_{01}{}^{il}E^l(\mathbf{k}_1),\\ (k_1{}^2/\varkappa^2 - 1)E^i(\mathbf{k}_1) &= g_{10}{}^{il}E^l(\mathbf{k}) + g_{11}{}^{il}E^i(\mathbf{k}_1);\end{aligned} \tag{3.1}$$

$$g_{\alpha\beta}{}^{il} = (4\pi i\omega/c^2\varkappa^2)\sigma_\omega{}^{il}(\mathbf{k}_\alpha, \mathbf{k}_\beta) + (k_\alpha{}^i k_\alpha{}^l/\varkappa^2)\delta_{\alpha\beta},$$

$$\varkappa = \omega/c, \quad \mathbf{k}_0 \equiv \mathbf{k}, \quad \alpha, \beta = 0, 1. \tag{3.2}$$

We take the x axis perpendicular to the $(\mathbf{k}, \mathbf{k}_1)$ plane. In accordance with (2.11), (2.13), (2.14'), and (2.14''), we can separate out a system of two scalar equations in (3.1) for the x components of the fields:

$$\begin{aligned}(k^2/\varkappa^2 - 1)E^x(\mathbf{k}) &= g_{00}{}^{xx}E^x(\mathbf{k}) + g_{01}{}^{xx}E^x(\mathbf{k}_1),\\ (k_1{}^2/\varkappa^2 - 1)E^x(\mathbf{k}_1) &= g_{10}{}^{xx}E^x(\mathbf{k}) + g_{11}{}^{xx}E^x(\mathbf{k}_1).\end{aligned} \tag{3.3}$$

In the fixed coordinate system the remaining system of four equations does not simplify. But one can readily convince oneself that the components of both fields which lie in the $(\mathbf{k}, \mathbf{k}_1)$ plane and are perpendicular respectively to $\mathbf{k}$ ($E'(\mathbf{k})$) and $\mathbf{k}_1$ ($E'(\mathbf{k}_1)$), are again related by a system of two equations.

We then get for the magnetic dipole case

$$\begin{aligned}(k^2/\varkappa^2 - 1)E'(\mathbf{k}) &= g_{00}E'(\mathbf{k}) + g_{01}E'(\mathbf{k}_1),\\ (k_1{}^2/\varkappa^2 - 1)E'(\mathbf{k}_1) &= g_{10}E'(\mathbf{k}) + g_{11}E'(\mathbf{k}_1).\end{aligned} \tag{3.4}$$

Taking account of (3.2), (2.11), and (2.14'), we have for narrow nuclear levels ($k \approx k_1 \approx \kappa$),

$$g_{\alpha\beta} = -\frac{4\pi b}{c^2 V_0}\eta\left(\sum_j e^{i(\mathbf{k}_\beta - \mathbf{k}_\alpha)\mathbf{R}_j} e^{-\frac{1}{2}[Z_j(\mathbf{k}_\alpha) + Z_j(\mathbf{k}_\beta)]}\right)\frac{1}{\omega - \omega_0 + i\Gamma/2}. \tag{3.5}$$

Here

$$g_{\alpha\beta}{}^{xx} = g_{\alpha\beta}\cos\varphi_{\alpha\beta}, \quad \varphi_{\alpha\beta} = \widehat{\mathbf{k}_\alpha, \mathbf{k}_\beta}. \tag{3.5'}$$

We note that in general the magnetic field in the crystal is no longer strictly transverse. But one can show, by using the actual form of the tensor $g_{\alpha\beta}^{il}$ from (3.2), that the longitudinal components are proportional to the square of the interaction (g_{00}^2), so they can be neglected within the framework of our treatment. This greatly simplifies the treatment of the quadrupole case, and we again obtain for the transverse components, lying in the $(\mathbf{k}, \mathbf{k}_1)$ plane the system of equations (3.4), but with the substitution

$$g_{\alpha\beta} \to g_{\alpha\beta} \cos 2\varphi_{\alpha\beta}. \tag{3.5''}$$

Thus, in all cases we get the same set of homogeneous equations of the form of (3.4). In particular, we note that such a system is characteristic for the dynamical theory of x rays.[7]

Suppose that a flux of γ rays with wave vector $\boldsymbol{\kappa}$ falls on a crystal which is in the form of a plate. After undergoing a weak refraction at the boundary, the incident flux spreads through the crystal with the wave vector $\mathbf{k}$, where

$$\mathbf{k} = \boldsymbol{\varkappa} + \varkappa\delta\mathbf{n}, \tag{3.6}$$

where $\mathbf{n}$ is along the interior normal to the crystal surface. We introduce the notation

$$k = \varkappa(1+\varepsilon_0), \quad k_1 = \varkappa(1+\varepsilon_1); \qquad |\varepsilon_0|, |\varepsilon_1| \ll 1. \tag{3.7}$$

There is a simple relation between δ and ϵ_0:

$$\delta \cong \varepsilon_0/\gamma_0, \quad \gamma_0 = \cos\Theta_0, \quad \Theta_0 = \widehat{\boldsymbol{\varkappa}, \mathbf{n}}. \tag{3.8}$$

On the other hand, from the relation $\mathbf{k}_1 = \mathbf{k} + \mathbf{K}_1$ we easily find

$$\varepsilon_1 = \alpha/2 + \gamma_1\varepsilon_0/\gamma_0, \quad \alpha = \mathbf{K}_1(\mathbf{K}_1 + 2\boldsymbol{\varkappa})/\varkappa^2,$$

$$\gamma_1 = \cos\Theta_1, \quad \Theta_1 = \widehat{\mathbf{k}_1, \mathbf{n}}. \tag{3.9}$$

We substitute (3.7) and (3.9) in (3.4). The condition for a nontrivial solution of this equation leads to a quadratic equation for ϵ_0, whose roots are

$$\begin{aligned} \varepsilon_0^{(1,2)} &= {}^1\!/_4(g_{00} + \beta g_{11} - \beta\alpha) \pm {}^1\!/_4\{(g_{00} + \beta g_{11} - \beta\alpha)^2 \\ &\quad + 4\beta[g_{00}\alpha - (g_{00}g_{11} - g_{01}g_{10})]\}^{1/2}, \\ \beta &= \gamma_0/\gamma_1. \end{aligned} \tag{3.10}$$

If we denote the wave vectors corresponding to the solutions (3.10) by $\mathbf{k}^{(1,2)}$ and $\mathbf{k}_1^{(1,2)}$, then we have the following expression for the components of the electric field corresponding to polarization s:

$$\begin{aligned} \mathbf{E}_s(\mathbf{r}) &= \mathbf{e}_s[E_s^{(1)}\exp(i\mathbf{k}^{(1)}\mathbf{r}) + E_s^{(2)}\exp(i\mathbf{k}^{(2)}\mathbf{r})] \\ &\quad + \mathbf{e}_{1s}[E_{1s}^{(1)}\exp(i\mathbf{k}_1^{(1)}\mathbf{r}) + E_{1s}^{(2)}\exp(i\mathbf{k}_1^{(2)}\mathbf{r})]. \end{aligned} \tag{3.11}$$

Here (cf. (3.4))

$$E_{1s}^{(1,2)} = \frac{2\varepsilon_0^{(1,2)} - g_{00}}{g_{01}} E_s^{(1,2)}$$

(for $s = 1$, $\mathbf{e}, \mathbf{e}_1 \parallel \mathbf{x}$; for $s = 2$, $\mathbf{e} \perp \mathbf{x} \times \mathbf{k}$, $\mathbf{e}_1 \perp \mathbf{x} \times \mathbf{k}_1$).

For definiteness let us consider the case of transmission, when $\mathbf{k}_1$ and $\mathbf{k}$ form an acute angle with $\mathbf{n}$, and impose the obvious boundary conditions (where we neglect the small difference between $\mathbf{E}$ and $\mathbf{D}$)

$$E_{1s}^{(1)} + E_{1s}^{(2)} = 0, \qquad E_s^{(1)} + E_s^{(2)} = \mathscr{E}_{0s}.$$

After some simple calculations we get the final expression for the electric field in the crystal (cf. (3.6)):

$$\begin{aligned} E_s(\mathbf{r}) = \mathscr{E}_{0s}e^{i\varkappa\mathbf{r}} \frac{1}{2(\varepsilon_{0s}^{(2)} - \varepsilon_{0s}^{(1)})} \Bigg\{ \mathbf{e}_s \Bigg[&(2\varepsilon_{0s}^{(2)} - g_{00}) \exp\left(\frac{i\varkappa\varepsilon_{0s}^{(1)}}{\gamma_0}\mathbf{nr}\right) \\ &- (2\varepsilon_{0s}^{(1)} - g_{00}) \exp\left(\frac{i\varkappa\varepsilon_{0s}^{(2)}}{\gamma_0}\mathbf{nr}\right)\Bigg] \\ &- \mathbf{e}_{1s}e^{i\mathbf{K}_1\mathbf{r}} g_{10}^s \beta \left[\exp\left(\frac{i\varkappa\varepsilon_{0s}^{(1)}}{\gamma_0}\mathbf{nr}\right) - \exp\left(\frac{i\varkappa\varepsilon_{0s}^{(2)}}{\gamma_0}\mathbf{nr}\right)\right]\Bigg\}. \end{aligned} \tag{3.12}$$

We have introduced the index s on $g_{\alpha\beta}$ and ϵ_0 to emphasize that these quantities have different values for the two polarizations. We note that in accordance with (3.5), (3.5′), and (3.5″), the quantities g_{00} and g_{11} have the same value for both polarizations.

All the physical results are actually contained in this expression. Despite its apparent complexity, the analysis of specific cases presents no difficulty.

We consider the case where the deviation from the Bragg condition is large, i.e., $\alpha \gg |g_{\alpha\beta}|$. We then have for the roots of (3.10)

$$\varepsilon_{0s}^{(1)} \approx {}^1\!/_2 g_{00}, \qquad \varepsilon_{0s}^{(2)} \approx -{}^1\!/_2\beta\alpha. \tag{3.13}$$

It is easy to see that with such a value for the roots the amplitude of the reflected wave is $\sim g_{10}^{s}/\alpha$ and its effect is negligibly small. From the form of (3.13) it immediately follows that the amplitude of the incident wave in (3.12), corresponding to the second root, is small. As a result (3.12) is transformed into the simple expression

$$\mathbf{E}_s(\mathbf{r}) = \mathbf{e}_s\mathscr{E}_{0s}\exp\left(i\varkappa\mathbf{r} + i\frac{\varkappa g_{00}}{2\gamma_0}\mathbf{nr}\right). \tag{3.14}$$

The damping of this wave is obviously determined by the imaginary part of g_{00}.

For the narrow lines we are considering, making use of (3.5), (3.5′), (3.5″) and (2.15), we find

$$\varkappa \operatorname{Im} g_{00} = \frac{\eta}{V_0}\sigma_t(\omega) \sum_j \exp(-Z_j(\mathbf{k})), \tag{3.15}$$

where σ_t is the total resonance cross section for the nucleus in the crystal:

$$\sigma_t = \frac{\pi c^2}{2\omega_0^2}\frac{2J+1}{2J_0+1}\frac{\Gamma\Gamma_1}{(\omega-\omega_0)^2 + (\Gamma/2)^2}. \tag{3.16}$$

But the product

$$\sigma_t \sum_j e^{-Z_j(\mathbf{k})} = \sigma_t$$

determines the total resonance cross section per unit cell when we take account of nuclear vibrations. From (3.14) and (3.15) it follows that the intensity will be damped according to the law

$$I \sim \exp\left(-\frac{\eta}{V_0}\bar{\sigma}_t\frac{\mathbf{nr}}{\gamma_0}\right), \qquad (3.17)$$

in complete agreement with the usual picture.

In the case of broad lines we must use (3.5) for $g_{\alpha\beta}$, but with the substitution (2.12). The damping of the intensity is then again described by (3.17) if we make the substitution $\bar{\sigma}_t \rightarrow \nu\sigma_t$ (where ν is the number of atoms per unit cell of the element containing the resonant isotope). It is interesting that, in contrast to the preceding case, the damping is in general independent of temperature and is determined entirely by the nuclear cross section σ_t. We note that this result is in agreement with the optical theorem, since the forward scattering amplitude $f(0, \omega)$, because of the potential nature of the scattering, is determined just by σ_t (since the large value of Γ has precisely the effect of making the scattering have potential character so far as phonon excitations are concerned). Conversely one can show that in the case of a narrow resonance, $f(0, \omega)$ depends on the temperature and is proportional to $\bar{\sigma}_t$.

Now we consider the case when the deviation from the Bragg condition is small, i.e., $\alpha < |g_{00}|$, $|g_{11}|$, and the crystal is sufficiently thin so that $|\kappa\epsilon_0^{(1,2)}l/\gamma_0| \ll 1$ (where l is the thickness of the crystal plate). Then the exponents in parentheses in (3.12) can be expanded in series and we get for the electric field of the diffracted wave,

$$\mathbf{E}_{1s} = \mathbf{e}_{1s}\mathscr{E}_{0s}\frac{i\varkappa l}{2\gamma_1}g_{10}{}^s\exp\{i(\varkappa + \mathbf{K}_1)\mathbf{r}\},$$

or, converting to intensities,

$$I_{1s} = I_{0s}|g_{10}{}^s|^2(\varkappa l)^2/4\gamma_1^2. \qquad (3.18)$$

For simplicity, we consider a monatomic crystal.

From (3.5) it follows that in the case of narrow resonance lines, the intensity of scattered γ quanta is determined by the product of the probabilities for the Mössbauer effect in absorption (wave vector $\mathbf{k}$) and emission (wave vector $\mathbf{k}_1$), which agrees with the known result (cf, for example,[18]).

In the case of broad lines, using (3.5) and making the replacement (2.12), we find

$$I_{1s} \sim \left|\sum_j \exp\{i\mathbf{K}_1\mathbf{R}_j - {}^1/_2 Z_j(\mathbf{K}_1)\}\right|^2,$$

which is in complete agreement with the results corresponding to potential scattering (cf., for example,[6]).

We note that $I_{1s} \sim \eta^2 l^2/V_0$. Such a dependence on the number of particles is directly related to the coherent character of the scattering.

4. SUPPRESSION OF INELASTIC CHANNELS. ANALYSIS OF RESULTS

We now consider the situation which arises when the parameter

$$\Delta^s = g_{00}g_{11} - g_{01}{}^s g_{10}{}^s \qquad (4.1)$$

is small. If the deviation from the Bragg condition is also small ($\alpha \ll |g_{00}|$, $|g_{11}|$), the first term in the square root in (3.10) will be large compared to the second, and we have approximately

$$\varepsilon_{0s}{}^{(1)} = -\frac{\beta}{2}\frac{g_{00}\alpha - \Delta^s}{g_{00} + \beta g_{11}} - \frac{\beta^3}{2}\frac{g_{00}g_{11}\alpha^2}{(g_{00}+\beta g_{11})^3} + \frac{\beta^2}{2}\frac{[(\Delta^s)^2 - (g_{00}-\beta g_{11})\alpha\Delta^s]}{(g_{00}+\beta g_{11})^3}$$

$$\varepsilon_{0s}{}^{(2)} = \frac{1}{2}\left[g_{00} + \beta g_{11} - \beta\alpha + \beta\frac{g_{00}\alpha - \Delta^s}{g_{00}+\beta g_{11}}\right] \qquad (4.2)$$

From (4.2) it follows immediately that two of the four waves in (3.12), corresponding to the wave vectors $\mathbf{k}^{(2)}$ and $\mathbf{k}_1^{(2)}$, are damped with practically the same decrement as the intensity in (3.17). Thus the total intensity in the crystal, at a depth sufficient for complete damping of these two waves, is described by the expression

$$\mathbf{E}_s(\mathbf{r}) = \mathscr{E}_{0s}\exp\left(i\varkappa\mathbf{r} + i\frac{\varkappa\varepsilon_{0s}^{(1)}}{\gamma_0}\mathbf{nr}\right) \times \frac{\beta}{2(\varepsilon_{0s}^{(2)} - \varepsilon_{0s}^{(1)})}\left\{\mathbf{e}_s\left(g_{11} - \frac{\beta g_{11}\alpha + \Delta^s}{g_{00}+\beta g_{11}}\right) - \mathbf{e}_{1s}g_{10}^s e^{i\mathbf{K}_1\mathbf{r}}\right\}. \qquad (4.3)$$

Let $\Delta^S = 0$ and $\alpha = 0$. Then $\epsilon_{0S}^{(1)} = 0$ and the field amplitude (4.3) is not damped at all. Thus under these conditions the part of the intensity associated with the field (4.3) propagates through the crystal without absorption, while the inelastic channel corresponding to conversion appears completely suppressed.

The question arises whether one can have the situation where the quantity Δ^S given by (4.1) goes to zero. Consider magnetic dipole interaction. Using (3.5) we easily establish that for the field components for which the polarization vectors lie in the $(\mathbf{k}, \mathbf{k}_1)$ plane ($s = 2$), the parameter Δ^S in (4.1) is rigorously zero if the unit cell contains only one atom of the element of interest. Thus, for magnetic dipole interaction a part of the intensity really travels through the crystal without absorption.

Suppose the crystal has cubic symmetry with one atom per unit cell. Then $Z(\mathbf{k})$ is independent of direction relative to the crystal axes and $Z(\mathbf{k}) = Z(\mathbf{k}_1)$. Then for magnetic dipole interaction and $s = 2$,

$$g_{10} = g_{01} = g_{00} = g_{11}. \tag{4.4}$$

If we use (4.3), it is not difficult to see that when the Bragg condition is strictly satisfied ($\alpha = 0$), the expression is curly brackets is equal simply to

$$g_{00}(\mathbf{e} - \mathbf{e}_1 e^{i\mathbf{K}_1\mathbf{r}}).$$

If we write the amplitude of the magnetic field, we get, instead of this expression,

$$g_{00}([\mathbf{ek}] - [\mathbf{e}_1\mathbf{k}_1]e^{i\mathbf{K}_1\mathbf{r}}). \tag{4.5}$$

It is easy to see that (4.5) is strictly zero at the lattice sites. Consequently, for this polarization the nodes of the magnetic field coincide with the nodes of the crystal lattice. This is why the magnetic dipole interaction does not result in the formation of an excited nucleus, thus suppressing the inelastic conversion channel. It is interesting that this situation is realized for Bragg reflection from any crystal plane.

If the crystal is not cubic then we know that the probability for the Mössbauer effect is anisotropic.[9] Then, even with one atom per unit cell the relation (4.4) is spoiled, although for $s = 2$ we again have $\Delta^s = 0$. Because (4.4) is violated, when $\alpha = 0$ the nodes of the lattice will no longer be nodes of the magnetic field, as is clearly seen from (4.3). There will still be an effect of suppression of the inelastic channels. The physical nature of this result is interesting.

Consider the amplitude A for formation of the excited state of the nucleus. In a crystal this amplitude can be written as a product of the corresponding amplitude for a rigidly bound nucleus, A_n, and the anisotropic amplitude $A(\mathbf{k})$, which describes a transition diagonal in the phonons when the individual nucleus in the lattice receives a momentum $\mathbf{k}$. In the case of magnetic interaction the quantity A_n is proportional to the magnetic field intensity at the nucleus.

In accordance with (4.3), when $\alpha = \Delta = 0$ the amplitude for formation of the excited state in the incident wave will be proportional to

$$A \sim [\mathbf{ek}]g_{11}e^{-Z(\mathbf{k})/2}, \tag{4.6$'$}$$

and in the diffracted wave,

$$A \sim [\mathbf{e}_1\mathbf{k}]g_{10}e^{-Z(\mathbf{k}_1)/2}e^{i\mathbf{K}_1\mathbf{r}}. \tag{4.6$''$}$$

But according to (3.5)

$$g_{11}e^{-Z(\mathbf{k})/2} = g_{10}e^{-Z(\mathbf{k}_1)/2},$$

and at the lattice sites the amplitudes (4.6′) and (4.6″) are equal in magnitude but opposite in sign.

Thus we get an interesting picture. In a noncubic monatomic crystal the incident and diffracted waves have different field amplitudes. But the values of these amplitudes are such that they determine equal values of the amplitude for formation of the excited state of the nucleus in both waves. The coherence of the two waves and the presence of opposite signs for the amplitudes (4.6′) and (4.6″) causes the vanishing of the probability for formation of the excited nucleus.

For the electric field components corresponding to polarization perpendicular to the $(\mathbf{k}, \mathbf{k}_1)$ plane ($s = 1$), the parameter Δ^s does not vanish strictly. But for both the M1 and E2 cases, conditions can occur where Δ^s is small. In fact we can conclude from (3.5′) and (4.1) that the necessary condition for this is that $|\cos\varphi_{01}|$ be close to unity. When the Bragg condition is fulfilled

$$\cos\varphi_{01} = 1 - K_1^2/2k^2,$$

and consequently, in order to observe the anomalous transmission of radiation with the polarization $s = 1$, the condition $K_1^2/2k^2 \ll 1$ must be satisfied. At first glance this condition seems to be easily realizable. But there may be purely experimental difficulties associated with the fact that with increasing k the requirements on collimation of the incident beam become stricter (cf. below).

For the polarization $s = 2$, in the case of electric quadrupole interaction we must use (3.5″) for the coefficients $g_{\alpha\beta}$. Now in order for Δ^s to be small, it is necessary that $|\cos 2\varphi_{01}| \approx 1$. In principle, this condition is more easily achieved than the previous case (when $K_1^2/2k^2 \ll 1$ the condition for Δ^s to be small is immediately satisfied for both components).

Thus, for electric quadrupole interaction and for one of the polarizations in the case of magnetic dipole interaction, although the parameter Δ^s does not become exactly zero, one can in principle actually have conditions where the inelastic channels are suppressed to a considerable extent. But unlike the case of M1 with $s = 2$, the realization of these conditions requires a definite choice of crystal planes for reflection, and in general also a specific choice of the crystal.

We note that the results given above are strictly valid for any crystal (any symmetry and any number of atoms per unit cell), so long as the unit cell contains only one atom of the element with the resonant isotope. If this is not the case it may happen that $\Delta^s \neq 0$ even for M1 and $s = 2$, and a separate analysis is required.

We now turn to relation (4.3) and write the expression for the flux of energy transmitted through a plate of thickness l (α and Δ small):

$$\mathbf{I}_s \approx I_{0s} \exp\left(-\frac{2\varkappa l}{\gamma_0} \operatorname{Im} \varepsilon_{0s}^{(1)}\right)$$

$$\times \left[\left|\frac{\beta g_{11}}{g_{00}+\beta g_{11}}\right|^2 \frac{\mathbf{k}}{k} + \left|\frac{\beta g_{10}{}^s}{g_{00}+\beta g_{11}}\right|^2 \frac{\mathbf{k}_1}{k_1}\right]. \tag{4.7}$$

In a monatomic cubic crystal the expression in square brackets simplifies and becomes equal to (cf. (3.5))

$$\left(\frac{\beta}{1+\beta}\right)^2 \left[\frac{\mathbf{k}}{k} + \frac{\mathbf{k}_1}{k_1}\left(\frac{g_{10}{}^s}{g_{00}}\right)^2\right]. \tag{4.7'}$$

From (4.7) we see that the damping of the intensity is determined entirely by $\operatorname{Im} \epsilon_{0S}^{(1)}$. If $\Delta^S = 0$, then in accordance with (4.2), (3.5), and (2.15) ($\operatorname{Im}[g_{00}/(g_{00} + \beta g_{11})] = 0$) we have

$$\frac{2\varkappa l}{\gamma_0} \operatorname{Im} \varepsilon_{0s}^{(1)} = -\alpha^2 \frac{\beta^3 \varkappa l}{\gamma_0} \operatorname{Im} \frac{g_{00} g_{11}}{(g_{00}+\beta g_{11})^3} \equiv -\frac{\alpha^2}{\alpha_0^2}, \tag{4.8}$$

where

$$\alpha_0^2 = \frac{2\pi\eta}{(V_0\varkappa^3)(\varkappa l)} \frac{2J+1}{2J_0+1} \frac{\Gamma_1}{\Gamma} \frac{\gamma_0}{\beta^3} \exp(-2Z(\mathbf{k}) + Z(\mathbf{k}_1))$$

$$\times \{1 + \beta \exp(Z(\mathbf{k}) - Z(\mathbf{k}_1))\}^3. \tag{4.9}$$

From the experimental point of view the quantity α_0 is obviously of decisive importance. In fact in the case of an ideal crystal it determines the condition of collimation of the incident beam, while for slight deviations from ideality it fixes the range of such deviations.

It is very interesting, and somewhat unexpected, that α_0 does not depend on ω. True, this is valid only under the assumptions made above, in particular when $\alpha \ll |g_{00}|, |g_{11}|$, but this only limits the range of possible deviations of ω from ω_0, and does not affect the essential statement. If we use a more exact value of the root $\epsilon_0^{(1)}$, the dependence on ω is very weak for small α, but increases with increasing α. Then an asymmetry in the frequency dependence of the absorption also appears.

Now let us consider the case where $\Delta^S \neq 0$. From (4.2) we see that $\operatorname{Im} \epsilon_0^{(1)}$ contains a term linear in Δ^S, while we should add to the expression (4.8) in the exponent in (4.7) the term

$$\frac{\varkappa l \beta}{\gamma_0} \operatorname{Im} \frac{\Delta^s}{g_{00}+\beta g_{11}} \tag{4.10}$$

(we note that $\operatorname{Im}[(g_{00} - \beta g_{11})\Delta^S/(g_{00} + \beta g_{11})^3] = 0$). In a monatomic cubic crystal, for example for s = 1, this expression is equal to ($\varphi_{01} \ll 1$)

$$\frac{\eta l}{V_0 \gamma_0} \sigma\, e^{-Z(k)} \frac{\beta \varphi_{01}{}^2}{1+\beta} \tag{4.10'}$$

where we have used the notation of (3.16).

Comparing with (3.17) we see that for $\alpha = 0$ the logarithmic decrement is $\beta\varphi_{01}^2/(1 + \beta)$ times smaller than for the usual case far from Bragg reflection.

5. THE TEMPERATURE EFFECT

The inclusion of nuclear vibration in the crystal led in the previous sections to the result that the basic parameters $g_{\alpha\beta}$ of the dynamical problem became dependent on the phonon spectrum and the temperature. This dependence was introduced in explicit form through the quantities Z (2.10).

At first glance the nuclear vibrations should spoil the effect of suppression of inelastic channels, at least in the ratio $\overline{u^2}/a^2$ (where $\overline{u^2}$ is the mean square displacement and a is the interatomic spacing). Actually, in the first place the nuclei are no longer strictly at the lattice sites, and secondly one can now have emission or absorption of phonons with simultaneous emission of γ quanta from the nuclei. But, paradoxical as it may seem, in the case of narrow resonance levels such a disturbance of the effect does not occur.

As for the first argument, it is immediately refuted if we remember that elastic scattering corresponds to long interaction times. As a consequence the γ quanta see a time-averaged and, consequently, strictly periodic picture. The second point is more complicated. We first consider the case of narrow resonances when $\Gamma \ll \omega_p$. It is physically obvious that the absorption of γ quanta by such nuclei cannot be accompanied by emission or absorption of phonons. The latter can occur only through decay of the excited state. But under the conditions where the formation of the excited nucleus in the crystal is forbidden, it is obvious that at the same time the emission or absorption of phonons is also forbidden. Thus, in the case of narrow lines the condition $\Delta^S = 0$ (M1, s = 2) is satisfied at any temperature, and consequently, at any temperature with $\alpha = 0$ one should observe the effect of suppression of inelastic channels. Here the dependence of g_{00} and g_{11} on temperature is related to the fact that in this case the total cross section for absorption of a γ quantum by a nucleus in the crystal is temperature dependent: it is determined by the product of σ_t from (3.16) and the probability for the Mössbauer effect f (in the monatomic case, for example, $f = e^{-Z(\mathbf{k})}$).

Thus in the case of narrow resonances, the crystal behaves like a "self-tuning system."

In the case of broad resonances ($\Gamma \gg \omega_p$), the picture changes drastically. The total absorption cross section, and with it g_{00} and g_{11}, do not depend on the temperature and the phonon spectrum. On the other hand, the coefficients g_{01} and g_{10} depend on the Debye-Waller factor (cf. (3.5)) with the change (2.12); in a monatomic crystal, for example, $g_{01} = g_{10} \sim \exp\{-Z(\mathbf{K}_1)/2\}$ and, consequently, depend essentially on the temperature. Because of this, $\Delta^S \neq 0$ even for the case of M1 and s = 2, and there is no complete effect of suppression of inelastic channels. The effect is partially spoiled even at T = 0 because of the zero point vibrations, and this obviously becomes worse with increasing temperature. In a monatomic crystal

$$\Delta = g_{00}^2(1 - e^{-Z(\mathbf{K}_1)}) \approx g_{00}^2 Z(\mathbf{K}_1).$$

The earlier assumption that Δ^S is small corresponds to the condition $Z \ll 1$. On the other hand,

$$Z(\mathbf{K}_1) \sim \overline{u^2}a^{-2}(K_1 a)^2,$$

and the spoiling of the effect is less the heavier the lattice atoms, the higher the temperature and the smaller the minimum reciprocal lattice vector.

6. CONCLUDING REMARKS

The results obtained in the preceding sections show that when the Bragg condition is satisfied we may get a partial or complete suppression of the inelastic channels in nuclear resonance scattering. Although the analysis was carried out for resonant interaction of γ quanta with nuclei, the qualitative results are very general in character, and are applicable to nuclear scattering of other particles.

The possibility of observing the effect experimentally depends primarily on having monocrystals (or single crystal layers) which are highly ideal. If we can get a sufficiently strong source, the choice of the Mössbauer isotope is less critical. Nuclei whose isomeric state decays via an M1 transition have an obvious advantage. The effect can then be observed in reflection from any set of crystal planes (cf. Secs. 3 and 4).

The conditions on collimation and on permissible deviations from ideality also depend to a considerable degree on the quantity α_0 (4.9). Analysis of this expression permits several remarks that are important for the experimental realization of the effect:

a) $\alpha_0 \sim 1/\kappa^2$, so that it is advisable to choose a Mössbauer nucleus with its excited level as low as possible. We note that if we take account of the relation between α and the angle Θ between the vectors $\boldsymbol{\kappa}$ and $\mathbf{K}$ (cf. (3.9)), we see that the true dependence of the angle of collimation on κ is weaker, and actually goes like $1/\kappa$.

b) $\alpha_0 \sim \sqrt{\Gamma_1/\Gamma}$, so in order to have a reasonable limitation on collimation the conversion coefficient should not be too large.

c) The dependence of α_0 on η makes it advisable to use a sample with a high concentration of the corresponding stable isotope. Then if one has a choice one should give preference to crystals with the maximum possible atomic density in the element of interest.

d) From the point of view of collimation the crystal thickness should be as small as possible. The need to include electronic scattering in turn imposes quite strict limitations on the value of l. On the other hand, to observe the effect the crystal thickness must be taken sufficiently large so that when the Bragg condition is violated the primary beam is practically completely absorbed. The appropriate condition is obtained from the requirement that the exponent in (3.17) be large compared to unity. It is easy to see that attempts to reduce l again lead to the requirement of high concentration of the corresponding isotope, low energy of the γ quanta and a sizable value for Γ_1/Γ. The analysis shows that one can easily find a range of thicknesses satisfying both the restrictions on upper and lower limits.

e) In the case of narrow lines, the phonon spectrum and the temperature affect only the conditions of collimation. But these restrictions are not very strong, and actually reduce to the requirement that the Mössbauer effect be sizable. Let us estimate α_0 for the example of Fe^{57} with $E_\gamma = 14.4$ keV ($J_0 = 1/2$, $J = 3/2$, $\Gamma_1/\Gamma \sim 0.1$). We set $\gamma_0 \sim \gamma_1 \sim 1$, $V_0 = 10^{-27}$ cm^3, $Z(\mathbf{k}) = 0$, and take $l = 1\mu$ (at this thickness and far from the Bragg condition, all the radiation is completely absorbed resonantly, while the interaction with the electrons still does not contribute strongly). Under these conditions, $\alpha_0 \sim 1'$, i.e., it has a comparatively large value. Thus the restrictions on the collimation are entirely reasonable.

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