ON STABILIZATION OF FLUTE INSTABILITY OF A PLASMA BY AN INHOMOGENEOUS ELECTRIC FIELD

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Stabilization of flute instability by an inhomogeneous electric field is investigated in a two-fluid hydrodynamics model of a plasma. Cases of plane and cylindrical geometry of an inhomogeneous plasma are considered. Dispersion relations are derived in the geometric optics approximation for the flute-instability spectra and local criteria are established for stabilization of the flute instabilities. It is demonstrated that effective stabilization of plasma surface instabilities is possible if the dimension of the electric field inhomogeneity is smaller than the characteristic dimension of the plasma inhomogeneity (i.e. the transverse dimension of the plasma), and if the field itself is stronger than or comparable with the thermal field. Stabilization is independent of the sign of the electric field and is due to distortion of the flute shape within a time shorter than the development time of the flute instability.

1. INTRODUCTION

ONE of the most dangerous forms of instability, which determines the lifetime of a plasma in an installation with positive curvature of the force lines of the retaining magnetic field ("probkotron"—pyrotrons), is flute instability [1,2]. In view of its long wavelength (the wavelength of the flute perturbations exceeds the Larmor radius of the ions), flute instability leads to decay of the plasma as a whole within a time comparable with the instability development time. Flute instability of a plasma in a pyrotron is very difficult to stabilize. Mechanisms for the stabilization of flute instability considered by various authors [3-8] are either little effective under real conditions, or cannot be used from the energy point of view (they lead to strong cooling of the plasma in the installation).

We call attention in this paper to a simple and, in our opinion, very effective mechanism for stabilizing flute instability of a plasma by an inhomogeneous electric field. Such a field, as a rule, always exists in a plasma. It can be brought about, in particular, by a decrease in the particle density towards the boundary of the plasma and by the different values of the Larmor radii of the electrons and ions. The electric field is usually concentrated at the surface of the plasma, within a region of the order of or somewhat larger than the Larmor radius of the ions. The inhomogeneous electric field can also be produced in the plasma artificially. The point is that when a plasma is in a strong magnetic field, the equalization of the charge density transversely to the magnetic field is very slow (it is determined by the diffusion spreading time) and consequently the state of a plasma with an arbitrarily inhomogeneous electric field and non-zero space charge can be regarded as quasi-stationary.

The physical picture of the stabilizing action exerted by the inhomogeneous electric field on the flute instability of a plasma is the following. Owing to the different azimuthal drift of the electrons and the ions in the curvilinear magnetic field, the charges separate in the density fluctuations (flutes) produced in the plasma, and this leads to the appearance of an azimuthal polarization field and a radial drift of the plasma. When an inhomogeneous electric field is present in the plasma, the resultant flute will rotate at a speed that varies along the radius. The latter leads to a distortion of the shape of the flute and by the same token to a decrease in the azimuthal component of the polarization electric field in the flute. At sufficiently large inhomogeneity of the radial electric field, the azimuthal component of the polarization field can even reverse sign, thus leading to stabilization of the flute instability. This will occur, obviously, under conditions when the time of distortion of the flute shape is shorter than the time of development of the flute instability, i.e.,

$$\left(\frac{\partial v_E}{\partial r}\right)^2 \sim \frac{v_E^2}{L_E^2} > \frac{g^*}{L}. \qquad (1)$$

Here v_E—velocity of the inhomogeneous electric drift (rotation) of a plasma with inhomogeneity dimension $\sim L_E$; g^*—effective gravitational field, which takes into account both the curvature of the magnetic-field force lines and the centrifugal force acting on the rotating plasma; finally, L—characteristic dimension of the inhomogeneity, on the order of the transverse plasma dimension.

The quantities $(\partial v_E/\partial r)^{-1}$ and $(L/g^*)^{1/2}$ characterize, respectively, the time of distortion of the shape of the flute and the time of development of the flute instability in the plasma [1]. In the case of planar geometry of the inhomogeneous plasma we have $g^* = g \sim v_{Ti}^2/R$, where $v_{Ti} = (T_i/M)^{1/2}$ is the thermal velocity of the ions and R is the radius of curvature of the magnetic-field force lines. On the other hand, in the case of cylindrical geometry $g^* = g + v_E^2/r$, and the second term in this expression is the centrifugal acceleration of the particles in the rotating plasma.

The foregoing is only a qualitative description of the stabilizing action of the inhomogeneous electric field on the flute instability of the plasma in the pyrotron. We shall illustrate this effect quantitatively, using a model of two-fluid hydrodynamics of a cold collisionless plasma. Such a model was already used to investigate flute instability [4,6]. It is suitable for the description of plasma oscillations with wavelength larger than the particle Larmor radius. This condition is assumed satisfied. It is also assumed that all the remaining dimensions of the plasma inhomogeneity exceed the ion Larmor radius.

2. STABILIZATION OF FLUTE INSTABILITY IN AN INHOMOGENEOUS PLASMA WITH PLANAR GEOMETRY

In this section we consider the influence of an inhomogeneous electric field on the spectrum of the flute plasma oscillations in the case of planar geometry. The use of planar geometry will allow us to separate in pure form the effect due to the action of the inhomogeneous electric field on plasma flute instability due to the curvature of the magnetic field force lines, excluding by the same token the effects connected with plasma rotation. The role of the latter will be clarified in the next section, where the case of cylindrical geometry of the inhomogeneous plasma is considered.

Following the earlier papers [4], we direct the z axis along the external magnetic field $\mathbf{B}_0$, and the x axis along the inhomogeneous electric field $\mathbf{E}_0(x)$. To take into account the curvature of the force lines of the magnetic field $\mathbf{B}_0$, we introduce a gravitational field g, parallel to the x axis and equal in order of magnitude to $\sim v_{Ti}^2/R$. The equation of two-fluid hydrodynamics, describing the cold plasma, are written in this case in the form (for a single species of particles)

$$\frac{\partial N}{\partial t} + \operatorname{div} N\mathbf{v} = 0,$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = \mathbf{g} + \frac{e}{m}\left(\mathbf{E} + \frac{[\mathbf{vB}]}{c}\right), \qquad (2)^*$$

where N—density of the number of charged particles of one species.

According to these equations, the equilibrium stationary state of the plasma is determined under the foregoing conditions by the drift velocity $\mathbf{v}_0$:

$$\mathbf{v}_0 = \mathbf{v}_E + \mathbf{v}_g = \frac{1}{\Omega B_0}\left[\left(\mathbf{g} + \frac{e}{m}\mathbf{E}_0\right)\mathbf{B}_0\right]$$

$$= \left(0, -\frac{eE_0/m + g}{\Omega}, 0\right), \qquad (3)$$

where Ω—Larmor frequency and v_g and v_E—respectively the velocities of the gravitational and electrical drift of the particles. From Maxwell's equations we obtain here the following conditions for plasma equilibrium:

$$\frac{dE_0}{dx} = 4\pi \sum eN_0, \quad \frac{d}{dx}(B_0^2 - E_0^2) = 8\pi g \sum mN_0. \quad (4)$$

Here N_0—density of the number of particles of one species in the equilibrium state, and the summation extends over all species of charged particles in the plasma. The electron and ion particle-number densities in the plasma are not equal (the plasma is not quasi-neutral): $N_{0W} \neq N_{0i}$.

To investigate the stability of such an equilibrium plasma state, let us consider small deviations from equilibrium, and let us confine ourselves only to potential perturbations. Using the hydrodynamic equations (2) and Maxwell's equations for the description of plasma potential oscillations, we obtain

$$\Delta\Phi + \sum \frac{1}{\omega'}\left\{\frac{\Omega' k_y \omega_L^2\, \partial\Phi/\partial x - \omega' k_y^2 \omega_L^2 \Phi}{\Omega\Omega' - \omega'^2}\right.$$

$$\left. + \frac{\partial}{\partial x}\frac{\omega' \omega_L^2\, \partial\Phi/\partial x - \Omega\omega_L^2 k_y \Phi}{\Omega\Omega' - \omega'^2} + \frac{\omega_L^2 k_z^2 \Phi}{\omega'}\right\} = 0;$$

$$\omega' = \omega - k_y v_0, \quad \Omega' = \Omega + \partial v_0/\partial x \approx \Omega + \partial v_E/\partial x. \qquad (5)$$

*$[\mathbf{vB}] = \mathbf{v} \times \mathbf{B}$.

Here Φ—potential of the perturbation field, ω—oscillation frequency, $\omega_L = (4\pi e^2 N_0/m)^{1/2}$—Langmuir frequency of particles of one species, and k_y and k_z—respectively the components of the wave vector along the y and z axes.

To determine the eigenvalue spectrum of Eq. (5), we use the geometrical-optics approximation [9] and confine ourselves to low-frequency flute oscillations, for which $k_z = 0$ and $\Omega_i\Omega_i' \gg \omega'^2$. Equation (5) then assumes the form

$$\left(1+\frac{\omega_{Li}^2}{\Omega_i\Omega_i'}\right)\left(k_y^2-\frac{\partial^2}{\partial x^2}\right)\Phi+\sum\frac{k_y\Phi}{\omega'}\frac{\partial}{\partial x}\frac{\omega_L^2}{\Omega'}=0. \quad (6)$$

We note that in writing out this equation we have neglected the inhomogeneity of the particle gravitational drift velocity. This is valid under the condition [4] that $v_A \gg v_{gi}$, where v_A—Alfven velocity.

In the zeroth approximation of geometrical optics we obtain from (6) the following dispersion relation, which determines the spectrum of the low-frequency potential plasma oscillations

$$\int dx\left\{-1-\frac{k_y^{-1}}{1+\omega_{Li}^2/\Omega_i\Omega_i'}\sum\frac{1}{\omega'}\frac{\partial}{\partial x}\frac{\omega_L^2}{\Omega'}\right\}^{1/2}=\frac{\pi n}{|k_y|}, \quad (7)$$

where n is an integer much larger than unity. The integration in (5) extends over the transparency region of the plasma, in which the integrand is positive. In the absence of the inhomogeneous electric field, $\partial E_0/\partial x = 0$, Eq. (7) goes over into the dispersion equation for flute oscillations of a plasma in a gravitational field [4].

The presence of an inhomogeneous field E_0, as can be readily seen from this equation, leads to stabilization of the unstable flute oscillations of a plasma, and the necessary local stabilization condition takes the form

$$\left[\frac{\partial}{\partial x}\left\{\frac{\omega_{Li}^2\,\partial v_E/\partial x}{\Omega_i\Omega_i'}-\frac{c}{B_0}\frac{\partial E_0}{\partial x}\right\}\right]^2$$
$$>4\frac{k_\perp^2 g}{\Omega_i}\left(1+\frac{\omega_{Li}^2}{\Omega_i\Omega_i'}\right)\frac{\partial}{\partial x}\frac{\omega_{Li}^2}{\Omega_i'};$$
$$k_\perp^2=k_y^2+k_x^2\sim k_y^2+\pi^2n^2/L^2. \quad (8)$$

In writing out this inequality, neglecting the gravitational drift of the electrons, we made use of (4) and took account of the fact that $\omega - k_y v_E \gg k_y v_{gi}$ for flute instabilities. It is essential to note that the obtained stabilization condition (8) depends neither on the sign of the inhomogeneous electric field itself (i.e., the sign of the plasma space charge) nor on the sign of its gradient.

Assuming for estimating purposes that the dimension of the electric field inhomogeneity is $\sim L_E$, and that the dimension of the plasma inhomogeneity is L, we obtain from (8), in the case of a relatively dense plasma in which $c^2 \gg v_A^2$, the following estimate for the stabilization condition:

$$\frac{v_E^2}{L_E^2}>4\frac{k_\perp^2L_E^2}{(1+L_E/L)^2}\frac{g}{L}\gtrsim4\frac{g}{L}\sim4\frac{v_{Ti}^2}{RL}. \quad (9)$$

This inequality coincides with the condition given above for the stabilization of flute instability (1), and corresponds to the distortion of the shape of the flute in the inhomogeneous electric field within a time shorter than the time of development of the instability. From the condition (9) it follows that the smaller L_E, the more effective the stabilization of the flute instability (in this case L_E should remain longer than the wavelength of the flute oscillations $\lambda \sim 1/k_\perp$). If we recognize that within the framework of the model in question L_E is larger than the Larmor radius of the ions, we note that stabilization of the flute instability by an inhomogeneous electric field is actually possible under conditions when this field is larger than or comparable with the thermal field, i.e., $L_E eE_0/T_i \gtrsim 1$.

3. STABILIZATION OF FLUTE INSTABILITY IN AN INHOMOGENEOUS PLASMA WITH CYLINDRICAL GEOMETRY

The case of planar geometry of inhomogeneous plasma considered above has the major shortcoming that it disregards completely the effects connected with the rotation of a plasma in a radial electric field. To take these effects into account, let us consider cylindrical geometry of an inhomogeneous plasma, placed in a longitudinal magnetic field $\mathbf{B}_0$ (directed along the z axis) and a radial electric field $\mathbf{E}_0(r)$. The gravitational field $g \sim v_{Ti}^2/R$, which takes into account the curvature of the force lines of the magnetic field $\mathbf{B}_0$, is also assumed directed along the radius of the plasma cylinder. Under these conditions, the equilibrium state of the plasma [the stationary solution of the system (2)] is determined by the azimuthal drift $v_0 = (0, v_{\varphi 0}, 0)$, which satisfies the following relation (for one species of particles)

$$g+eE_0/m+\Omega v_{\varphi0}+v_{\varphi0}^2/r=0. \quad (10)$$

The last term in the left side of this relation is the centrifugal acceleration of the particles in the rotating plasma.

Assuming the inequality [1)] satisfied

[1)]It follows from this inequality that all the results obtained below are valid away from the plasma cylinder axis, in the direct vicinity of the plasma surface.

$$v_{\varphi 0} / r\Omega \ll 1, \tag{11}$$

we obtain from (10), accurate to terms of first-order in the small parameter (11),

$$v_{\varphi 0} \approx v_{g^*} + v_E = -\frac{g^*}{\Omega} - \frac{eE_0}{m\Omega}, \quad g^* = g + \frac{1}{r} v_E^2. \tag{12}$$

This expression for the particle drift velocity takes into account both the action of the gravitational field g, due to the curvature of the force lines of the retaining magnetic field B_0, and the action of the centrifugal force, which arises when the plasma rotates in a radial electric field. The plasma-equilibrium conditions, obtained from Maxwell's equations are then written in the form

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_0) = 4\pi \sum eN_0,$$

$$\frac{\partial B_0^2}{\partial r} - \frac{1}{r^2}\frac{\partial}{\partial r}(rE_0)^2 = 8\pi g^* \sum mN_0. \tag{13}$$

For small potential oscillations about the equilibrium state of the plasma we obtain from the hydrodynamic equations (2) and Maxwell's equations the following equation:

$$\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\right)\Delta\Phi - \sum \frac{\Phi}{\omega - lv_{\varphi 0}/r} \frac{l}{r}\frac{\partial}{\partial r} \frac{\omega_L^2}{\Omega + \partial v_E/\partial r + v_E/r} = 0, \tag{14}$$

which describes the flute oscillations ($k_z = 0$, $\omega \ll \Omega_i$) of a cylindrical inhomogeneous plasma placed in a radial electric field, in the geometrical-optics approximation. Here $l = 0, 1, 2, \ldots$ — azimuthal wave number.

The spectrum of plasma oscillations in the zeroth approximation of geometrical optics is determined by [compare with (7)]

$$\int dr \left\{ -\frac{l^2}{r^2} - \frac{l}{r(1 + \omega_{Li}^2/\Omega_i^2)} \times \sum \frac{1}{\omega - lv_{\varphi 0}/r} \frac{\partial}{\partial r} \frac{\omega_L^2}{\Omega + \partial v_E/\partial r + v_E/r} \right\}^{1/2} = \pi n, \tag{15}$$

where n is an integer much larger than unity. The integration in (15) is over the transparency region of the plasma, located near the surface where the inequality (11) is satisfied.

From the dispersion relation (15) we obtain, under the condition $\omega - lv_E/r \gg lv_{gi}/r$, the following local condition for the stabilization of the flute instability of a cylindrical plasma by an inhomogeneous electric field [compare with condition (8)]:

$$\left[\frac{\partial}{\partial r}\left\{\frac{\omega_{Li}^2}{\Omega_i^2}\left(\frac{\partial v_E}{\partial r} + \frac{v_E}{r}\right) - \frac{c}{B_0 r}\frac{\partial}{\partial r}(rE_0)\right\}\right]^2 > \frac{4k_\perp^2 g^*}{\Omega_i}\left(1 + \frac{\omega_{Li}^2}{\Omega_i^2}\right)\frac{\partial}{\partial r}\frac{\omega_{Li}^2}{\Omega_i};$$

$$k_\perp^2 = \frac{l^2}{r^2} + k_r^2 \sim \frac{l^2}{r^2} + \frac{\pi^2 n^2}{L^2}. \tag{16}$$

In writing out this inequality, as in the planar case, we have neglected the gravitational and centrifugal drift of the electrons and used relation (13). In the case of a relatively dense plasma, in which $c^2 \gg v_A^2$, we obtain from the condition (16) the following estimate for the stabilization of the plasma:

$$\frac{v_E^2}{L_E^2} > 4\frac{k_\perp^2 L_E^2}{(1 + L_E/L)^4}\frac{g^*}{L} \gtrsim 4\frac{g^*}{L} \sim \frac{4}{L}\left(\frac{v_{Ti}^2}{R} + \frac{v_E^2}{L}\right). \tag{17}$$

This inequality, the same as inequality (9), coincides with condition (1) and corresponds to distortion of the shape of the flute in an inhomogeneous electric field within a time shorter than the time of development of the flute instability in a cylindrical plasma placed in a radial electric field. The nature of the flute instability is in this case significantly different from the instability of a planar plasma. The point is that the radial electric field in the cylindrical plasma causes the plasma to rotate, and this in turn can give rise to flute instability of the plasma as a result of the action of the centrifugal force. This circumstance is manifest in the second term of the right side of (17), and causes the effective stabilization of the flute instability by the inhomogeneous electric field in the cylindrical plasma to be possible in the case when the transparency region is located in a narrow layer near the surface of the plasma and $L_E \ll L$. In the general case, however, it is necessary to satisfy the inequality (16).[2)]

In conclusion we note that stabilization of flute instability was observed when a positive potential was applied in the "Ogra" installation [11]. It can

[2)]It must be emphasized that the criteria which we have obtained for the stabilization of flute instability pertain, strictly speaking, to oscillations with sufficiently short wave-lengths, which can be described in the geometrical-optics approximation; they cannot be extended to include long-wave oscillations which are outside the scope of this approximation. We note also that the inhomogeneous electric field and the plasma rotation associated with it exert a stabilizing influence not on all types of plasma instability. Thus, for example, the electric field has no effect whatever on the "universal" instability of an inhomogeneous plasma [9], and the stability of a self-compressing discharge (pinch effect) as shown by Gerjuoy and Rosenbluth [10], becomes even worse as a result of plasma rotation.

also be supposed that the unusually high stability of the plasma in pyrotrons with "hot electrons"[8,12] is due to some degree to the stabilization mechanism indicated above.

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25