ANOMALOUS SKIN EFFECT IN FERROMAGNETIC METALS IN A STRONG MAGNETIC FIELD [1)]

A. Ya. BLANK, M. I. KAGANOV, and YU LU

Institute of Radiophysics and Electronics, Academy of Sciences, Ukrainian S.S.R.; Physicotechnical Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor June 1, 1964

J. Exptl. Theoret. Phys. (U.S.S.R.) **47**, 2168-2177 (December, 1964)

The high-frequency properties of ferromagnetic metals in a strong magnetic field ($R \ll l$, R is the Larmor orbit radius and l is the mean free path) are investigated in the anomalous skin effect region ($kR \gg 1$, $k = 2\pi/\lambda$, λ is the electromagnetic wavelength). It is shown that weakly damped coupled spin and electromagnetic waves may exist in the metal near the ferromagnetic resonance and antiresonance frequencies and the cyclotron resonance frequency. It is found that the ferromagnetic resonance frequency shifts and the shape of the resonance curve changes if spatial dispersion of the magnetic permeability is taken into account. The possibility of the cyclotron frequency or of any of its harmonics being equal to the ferroresonance or antiresonance frequencies is considered. Equality of the frequencies should significantly change the spectrum of the weakly damped waves. The existence of weakly damped waves causes the impedance to have a peculiar frequency dependence, which has been calculated for a stationary magnetic field parallel to the surface of the metal.

1. INTRODUCTION

As is well known, the high-frequency properties of ferrodielectrics are not very sensitive to exchange effects (see, for example, [1]). In metals, if the effective permeability is large and the anisotropy energy is small, the "exchange" term in the Landau-Lifshitz equation can become comparable with or even larger than the "Zeeman" term, owing to the strong inhomogeneity of the magnetic field in the skin layer. The exchange interaction leads to a shift of the resonant frequency and to additional broadening of the resonant line. These effects were observed by Rado and Weertman in permalloy with vanishingly small anisotropy[2,3]. Ferromagnetic resonance under such conditions was called by them spin-wave resonance. The study of spin-wave resonance is of interest in connection with the possibility of determining the exchange constant α from experimental data. We note that the exchange effects are significant also for thin films, in which resonance with standing spin waves was observed[4,5].

The macroscopic theory of spin-wave resonance in a magnetic field parallel to the surface of the metal was developed by Ament and Rado[6] under the assumption that the skin effect has a normal character. A study[7] of spin-wave resonance under normal skin effect and general boundary conditions has made it possible to obtain agreement with experiment[3]. Spin-wave resonance at low temperatures, when the skin effect becomes essentially anomalous, was investigated by V. Gurevich[8] in magnetic fields perpendicular and parallel to the surface of the metal.

No account was taken in the above-mentioned investigations of the influence of the magnetic field on the electric conductivity; this neglect is justified if the radius of the electron orbit R is large compared with the effective mean free path l ($R \gg l$). However, this condition may not be satisfied at helium temperatures.

In this paper we investigate the high-frequency properties of a ferromagnetic metal with anomalous skin effect in a strong magnetic field, when the opposite inequality holds true

$$R \ll l. \tag{1}$$

Closely linked with the evaluation of the influence of the magnetic field on the electric conductivity of the metal is the possible propagation of weakly damped waves. As shown by many authors[9-11], if condition (1) is fulfilled weakly damped waves (helical and magnetohydrodynamical) can propa-

[1)]Some of the results of this were obtained by one of the authors (Yu Lu) during his stay at the Khar'kov University (1961).

gate in the metal and make it transparent in the corresponding frequency region, as well as cause other resonance effects. Weakly damped electromagnetic waves in a nonferromagnetic metal were investigated in the region of the anomalous skin effect by Kaner and Skobov[12].

In ferromagnetic conductors, the weakly damped electromagnetic waves are strongly linked with oscillations of the magnetic moment. The propagation of such coupled waves was considered by Stern and Callen[13] and by one of the authors[14] for the normal skin effect, when the role of the exchange is insignificant.

As will be shown below, spatial dispersion of the magnetic permeability and of the electric resistivity can give rise to weakly damped coupled spin and electromagnetic waves. The existence of weakly damped waves determines the unique dependence of the surface impedance of ferromagnetic metals on the frequency and on the magnetic field

2. WEAKLY DAMPED WAVES

Let us consider an unbounded ferromagnetic metal, in which the propagation of a plane monochromatic wave of frequency ω obeys the Maxwell equations

$$\mathrm{rot}\,\mathbf{h} = 4\pi\mathbf{j}/c, \qquad \mathrm{rot}\,\mathbf{e} = i\omega\mathbf{b}/c; \qquad (2)^*$$

$$j_i = \sigma_{ik}e_k, \qquad b_i = \mu_{ik}h_k, \qquad (3)$$

where σ_{ik} and μ_{ik} are the electric conductivity and magnetic permeability tensors, respectively.

We consider in what follows the propagation of a wave transverse to a constant field $\mathbf{H}(\mathbf{k}\perp\mathbf{H})$. We direct the x axis along $\mathbf{H}$ and the z axis along $\mathbf{k}$. In this system of coordinates, the magnetic permeability tensor can be written in the form

$$\mu_{ik} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu_1 & i\mu_2 \\ 0 & -i\mu_2 & \mu_1 \end{pmatrix},$$

$$\mu_1 = \frac{\Omega\Omega_1 - (\omega - i\lambda)^2}{\Omega^2 - (\omega - i\lambda)^2}, \qquad \mu_2 = \frac{4\pi\gamma M_0(\omega + i\lambda)}{\Omega^2 - (\omega - i\lambda)^2}. \qquad (4)$$

Here

$$\Omega = \gamma H + \alpha k^2, \ \Omega_1 = \Omega + 4\pi\gamma M_0, \ \mathbf{M}_0$$

M_0—saturation magnetic moment, $\gamma = ge/2mc$ (g—spectroscopic factor), m—mass of free electron, $\alpha = \Theta_C a^2/\hbar$—exchange-interaction constant (Θ_C—Curie temperature, a—lattice constant), λ—Bloch relaxation constant.

*rot = curl.

The electric conductivity tensor in a strong magnetic field, in the case of the anomalous skin effect

$$kR \gg 1 \qquad (5)$$

was calculated by Kaner and Skobov[12]. For an isotropic electron energy spectrum this tensor takes the form

$$\sigma_{xx} = \sigma_{yy} = \frac{3\pi i N e^2}{4m^* v k}\,\mathrm{ctg}\,\frac{\pi(\omega + i\nu)}{\omega_c},$$

$$\sigma_{zz} = \frac{3Ne^2(\nu - i\omega)}{m^* k^2 v^2}\left[1 - \frac{\pi(\omega + i\nu)}{2kv}\,\mathrm{ctg}\,\frac{\pi(\omega + i\nu)}{\omega_c}\right], \qquad (6)^*$$

where N—concentration, v—electron velocity on the Fermi boundary, ν—effective collision frequency, $\omega_C = eB/m^*c$—cyclotron frequency, $B = H + 4\pi M_0$—magnetic induction, and m^*—effective mass of the electron.[2)]

The off-diagonal components of the tensor σ_{ik} are negligibly small in the approximation where $kR \gg 1$[12]. Since the tensor σ_{ik} is diagonal, the dispersion equations for the extraordinary and ordinary waves separate:

$$k^2 = \frac{4\pi i\omega}{c^2}\sigma_{yy}(\omega, k), \qquad (7)$$

$$k^2 = \frac{4\pi i\omega}{c^2}\frac{\det\mu_{ik}}{\mu_{zz}}\sigma_{xx}(\omega, k). \qquad (8)$$

The extraordinary wave (7), in which the magnetic field is parallel to the constant field **H**, does not interact with the oscillations of the magnetic moment. In the ordinary wave (8), the field components e_y and e_z vanish.

Using (4) and (6), and neglecting dissipation, we represent (8) in the form

$$k^3 = -\frac{\omega_0^2}{c^2 v}\,\omega\,\frac{\omega_a^2 + \alpha\omega_2 k^2 - \omega^2}{\omega_r^2 + \alpha\omega_1 k^2 - \omega^2}\,\pi\,\mathrm{ctg}\,\pi\frac{\omega}{\omega_c}, \qquad (9)$$

where $\omega_0^2 = 3\pi Ne^2/m$ is of the order of the square of the plasma frequency, and

$$\omega_a = \gamma B, \quad \omega_r = \gamma(HB)^{1/2}, \quad \omega_1 = \gamma(H + B), \quad \omega_2 = 2\gamma B.$$

Estimates show that the "exchange" components are significant only in the direct vicinity of the resonance ω_r and antiresonance ω_a, and can be neglected in a qualitative investigation of the

*ctg = cot.

2)We have assumed that the magnetic field acting on the conduction electron is $B = H + 4\pi M_0$. This, of course, is valid, since, first, the anomalous Hall effect is very small at low temperatures [16] and, second, the investigations of the de Haas–van Alphen effect in ferromagnetic metals confirm this point of view [17].

spectrum (9). The spectrum depends here essentially on the relation between the quantities ω_r, ω_a, and ω_c. The spectral relationship for several cases in which the frequencies ω_r, ω_a, and ω_c are quantities of the same order is shown schematically in Fig. 1. As is clear from the figure, weakly damped waves can exist near the ferromagnetic resonance frequencies ω_r, antiresonance frequencies ω_a, and also near the cyclotron frequency ω_c. The latter case was considered by Kaner and Skobov[12] for ordinary metal.

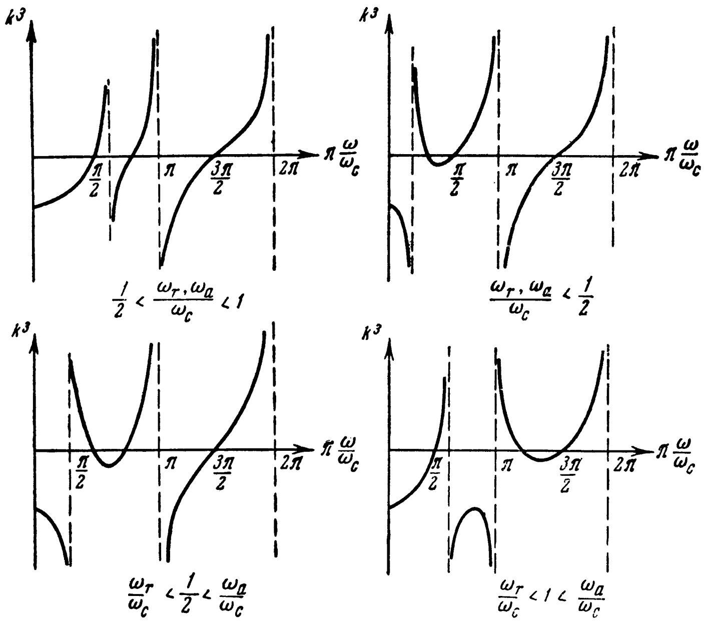

FIG. 1

Investigating the waves near the characteristic frequencies ω_r and ω_a, we assume, only for the purpose of simplifying the notation, that the latter are small compared with the cyclotron frequency.

Near the frequency ω_r, the wave vector in formula (9) becomes infinite, corresponding to spin-wave resonance. This gives rise to excitation of spin waves with a known dispersion law[1]

$$\omega = \omega_r + \frac{\alpha}{2}\frac{\omega_1}{\omega_r}k^2. \tag{10}$$

When the "detuning" of the resonance is large compared with the exchange term:

$$|\omega_r - \omega| \gg \alpha\omega_2 k^2 / 2\omega_r,$$

the spectrum of the weakly damped wave takes the form (assuming that $\omega \ll \omega_c$)

$$\omega = \omega_r + \frac{1}{2}\frac{\omega_0^2}{c^2R}\frac{\omega_a^2 - \omega_r^2}{\omega_r k^3}. \tag{11}$$

The condition for its existence is

$$k^5 \ll \frac{\omega_0^2}{c^2R}\frac{\omega_a^2 - \omega_r^2}{\alpha\omega_2}.$$

Expressing k as a function of ω by means of (11), we can easily see that the inequality (5) is automatically satisfied in this case. As can be seen from (11), this wave has anomalous dispersion. Its phase velocity is determined by the expression

$$v_{\text{ph}} = \omega_r\left(\frac{Rc^2}{\omega_0^2}\right)^{1/3}\left(\frac{\omega^2 - \omega_r^2}{\omega_a^2 - \omega_r^2}\right)^{1/3}, \tag{12}$$

and the group velocity is

$$v_{\text{g}} = -\frac{3}{2}\frac{\omega_0^2}{c^2R}\frac{\omega_a^2 - \omega_r^2}{\omega_r^5}v_{\text{ph}}^4. \tag{13}$$

Near the antiresonance frequency at $|\omega - \omega_a| \gg \alpha k^2$, there exists a wave with a dispersion law

$$\omega = \omega_a - \frac{(\omega_a^2 - \omega_r^2)}{2\omega_a}\frac{Rc^2}{\omega_0^2}k^3. \tag{14}$$

Inasmuch as the inequality $kR \gg 1$ is violated at the antiresonance point itself, the condition of nearness to this point is

$$\omega_a - \omega \gg 2\pi\gamma M_0(\delta_0 / R)^2, \tag{15}$$

where $\delta_0 = c/\omega_0$—depth of penetration of the field in the metal.

The condition for the existence of the wave (14), which consists in neglecting the exchange effect near the frequency ω_a, leads to the following inequality, which is obviously satisfied,

$$k \gg \frac{\alpha}{2\pi\gamma M_0}\frac{\omega_0^2}{Rc^2}. \tag{16}$$

This wave also has anomalous dispersion. Its phase and group velocities are

$$v_{\text{ph}} = \omega_a\left(\frac{\omega_a^2 - \omega_r^2}{\omega_a^2 - \omega^2}\right)^{1/3}\left(\frac{c_2R}{\omega_0^2}\right)^{1/3}, \tag{17}$$

$$v_{\text{g}} = -\frac{3}{2}(\omega_a^2 - \omega_r^2)\frac{Rc^2}{\omega_0^2}\frac{\omega_a}{v_{\text{ph}}^2}. \tag{18}$$

Finally, near cyclotron frequency, when $\nu \ll n\omega_c - \omega \ll \omega_c$, the expression for the spectrum of the wave takes, in analogy with that given in [12], the form

$$k^3 = \frac{\omega_0^2}{c^2R}\left(1 - \frac{\omega}{n\omega_c}\right)^{-1}\frac{\omega_a^2 - (n\omega_c)^2}{\omega_r^2 - (n\omega_c)^2}. \tag{19}$$

In connection with formula (19), we note the possibility that the cyclotron frequency, or an exact multiple of this frequency, may coincide with the ferromagnetic resonance frequency at certain values of the field H_{cr}, defined by the relation

$$H_{cr} = \frac{4\pi M_0}{(gm^*/2m)^2 n^{-2} - 1}. \quad (20)$$

As can be seen from (20), for the frequencies to coincide it is necessary to satisfy the condition

$$2m \lesssim gm^*. \quad (21)$$

In this case the spectrum of the weakly damped wave is written in the form

$$\omega = \omega_r - \left(\frac{\omega_0^2}{c^2 R} \frac{\omega_a^2 - \omega_r^2}{k^3} \right)^{1/2}. \quad (22)$$

3. REFLECTION OF ELECTROMAGNETIC WAVE FROM A HALF-SPACE

Let us consider a semi-infinite metal ($z > 0$). Assume again that the x axis is directed along the constant magnetic field **H** and the wave propagates perpendicular to the surface of the metal. The initial system of equations consists of the Maxwell equations:

$$\frac{4\pi}{c} j_x = -\frac{dh_y}{dz}, \qquad \frac{i\omega}{c} b_y = \frac{de_x}{dz}, \quad (23)$$

where $b_y = h_y + 4\pi m_y$, and m_y must be determined from the linearized equation of motion

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma \left[\mathbf{m} + \alpha \frac{\partial^2 \mathbf{m}}{\partial z^2}, \mathbf{H} \right] + \gamma [\mathbf{M}_0, \mathbf{h}] - \lambda \mathbf{m}, \quad (24)^*$$

to which we must add the boundary conditions. The current density j_x is determined from the conduction-electron distribution function, which is a solution of the kinetic equation with corresponding boundary conditions.

However, the calculation of the impedance can be greatly simplified by using for the connection between the current and the electric field a relation that holds true in unbounded space:

$$j_x(k) = \sigma(\omega, k) e_x(k), \quad (25)$$

where $\mathbf{j}(k)$ and $\mathbf{e}(k)$—Fourier components of the current and field, respectively and $\sigma(\omega, k) \equiv \sigma_{xx}(\omega, k)$ is given by (6). Such a substitution, as shown by Azbel' and Kaner[15], leads to the appearance in the impedance of an inessential real factor of the order of unity. The insensitivity to the boundary conditions is manifest, in particular, in the fact that the impedance in the case of anomalous skin effect (for $H = 0$) differs from the impedance in the case of specular and diffuse reflection of electrons from the boundary of the metal only by a factor $8/9$ (see [18,19]).

To calculate the impedance with the aid of (23)—(25) it is necessary to continue the electric and magnetic fields into the region $z < 0$. This can be done in two ways: either by assuming that $e_x(z)$ is an even function and $a_y(z)$ is odd, or vice versa. If we neglect the spatial dispersion of the magnetic susceptibility, then the expressions obtained in this case for the impedance, as expected, differ by an inessential factor.[3)]

For the general boundary condition for the magnetic moment, neither an even nor an odd continuation of the magnetic field allows direct use of the expression for the magnetic susceptibility of an unbounded metal. However, if the alternating part of the magnetic moment vanishes on the boundary, then, an odd continuation of the magnetic field easily yields from (24) the second relation of (3). If $\partial \mathbf{m}/\partial z$ vanishes on the boundary, this relation is obtained with an even continuation of the field h_y.

In the former case the electric field $e_x(z)$ is continued in even fashion. We denote the impedance in this case by Z_p. From (23), (25), and (3) we readily obtain

$$Z_p = \frac{2}{i\pi} \int_0^\infty \frac{\mu(\omega, x)\, dx}{x^2 - \varepsilon(\omega, x)\mu(\omega, x)} \quad (\mathbf{m} = 0 \text{ for } z = 0). \quad (26)$$

Here

$$\varepsilon(\omega, x) = 4\pi i \omega^{-1} \sigma(\omega, x)$$

is the dielectric constant of the metal and $x = ck/\omega$ the refractive index of the wave with wave vector **k**.

In the second case ($Z = Z_n$) we have

$$Z_n^{-1} = \frac{2}{i\pi} \int_0^\infty \frac{\varepsilon(\omega, x)\, dx}{x^2 - \varepsilon(\omega, x)\mu(\omega, x)} \quad \left(\frac{\partial \mathbf{m}}{\partial z} = 0 \text{ for } z = 0 \right). \quad (27)$$

Formulas (26) and (27) have a very broad range of applicability. They can be used, naturally, to calculate the surface impedance of a ferromagnetic dielectric without spatial dispersion of the dielectric constant $[\epsilon(\omega, x) \equiv \epsilon(\omega)]$. When account is taken of the spatial dispersion of the dielectric constant, it should be noted that (26) is valid if $\partial P/\partial z = 0$ on the boundary of the sample, and (27) is valid if $\mathbf{P} = 0$ when $z = 0$ ($\mathbf{P}$—dielectric polarization vector). If the spatial dispersion is not significant $[\epsilon(\omega, x) \equiv \epsilon(\omega), \mu(\omega, x) \equiv \mu(\omega)]$, when we see readily that

$$Z_p = Z_n = (\mu/\varepsilon)^{1/2}. \quad (28)$$

---

*$[\mathbf{M}_0, \mathbf{h}] = \mathbf{M}_0 \times \mathbf{h}$.

3)An even continuation of the field e corresponds to the very special boundary condition $\partial\psi/\partial z = 0$ when $z = 0$, while the odd corresponds to $\psi = 0$ when $z = 0$ (for the definition of the function ψ see [15]).

Under the assumption used here, namely that the skin effect has an ultra-anomalous character, the dielectric constant is determined, as already mentioned, by the specific electric conductivity [see (6)]. The impedance (26), (27) can then be calculated for several limiting cases, to which we now proceed.

Away from the cyclotron frequency, (we assume for simplicity that $\nu \ll \omega_c$), we represent $\epsilon(\omega, x)$ in the form

$$\varepsilon(\omega, x) = -\frac{\omega_0^2 c}{\omega^2 R}\frac{(\omega^2+\nu^2)^{-1/2}}{x}e^{-i\varphi},$$

$$\operatorname{tg}\varphi = \frac{\nu}{\omega}, \quad 0<\varphi<\frac{\pi}{2}. \qquad (29)^*$$

Neglecting the exchange effect, the magnetic permeability has, accurate to terms of order λ^2, the form

$$\mu = \frac{(\omega_a^2-\omega^2)(\omega_r^2-\omega^2)+2i\lambda\omega(\omega_a^2-\omega_r^2)}{(\omega_r^2-\omega^2)^2+4\lambda^2\omega^2}. \qquad (30)$$

Inasmuch as $\operatorname{Im}\mu > 0$ ($\lambda > 0$, $\omega_a > \omega$), expression (30) can be written in the form

$$\mu = |\mu| e^{i\vartheta}, \quad \operatorname{tg}\vartheta = \frac{2\lambda\omega(\omega_a^2-\omega_r^2)}{(\omega_a^2-\omega^2)(\omega_r^2-\omega^2)} \quad (0<\vartheta<\pi). \qquad (31)$$

With the aid of (29) and (30), using (26), we obtain, far from cyclotron frequency,

$$Z = \frac{4}{3\sqrt{3}}\frac{\omega}{c}|\mu|^{2/3}\left(\frac{vc^2(\omega^2+\nu^2)^{1/2}}{\omega_0^2\omega_c\omega}\right)^{1/3} \times \exp\left\{i\left(\frac{2}{3}\vartheta+\frac{\varphi}{3}-\frac{\pi}{2}\right)\right\}. \qquad (32)$$

Neglecting dissipation the impedance (32) becomes imaginary which corresponds to total reflection of the wave. Near the characteristic frequencies ω_r and ω_a in the region of the existence of weakly damped waves (11) and (14) the impedance becomes complex [as is readily seen from (31), in the region of weakly damped waves $\vartheta = \pi$ and φ can be set equal to zero]. At the same time the phase in (32) becomes equal to $\pi/6$.

The frequency dependence of the real (R_p) and imaginary (X_p) parts of the impedance (32), without account of dissipation, is shown schematically in Fig. 2. An account of dissipation changes the resonance curve in the usual fashion. The spatial dispersion of the magnetic susceptibility, the role of which becomes appreciable near the frequency ω_r, does not make any contribution to the line width, as shown by calculations, but does cause a shift in the resonance frequency. The shifted resonance frequency is

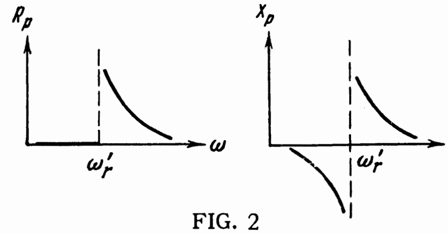

FIG. 2

$$\omega_r' = \omega_r + \frac{5}{4}\left[\frac{8}{27}\frac{\omega_0^4}{c^4R^2}\frac{(4\pi\gamma M_0)^2\omega_a^2}{\omega_r^5}\alpha^3\omega_1^3\right]^{1/5}. \qquad (33)$$

In the vicinity of the shifted resonance frequency ω_r', allowance for the spatial dispersion leads to an appreciable change in the form of the resonance curve. An investigation of the question of the influence of the spatial dispersion of the magnetic susceptibility on the form of the resonance curve will be the subject of a separate communication.

The real part of the impedance differs from zero also near the frequency of cyclotron resonance, owing to the possible propagation of weakly damped waves (19). When $\omega_c - \omega \gg \nu$ we have

$$Z_p = \frac{2}{3\sqrt{3}}\frac{n\omega_c}{c}\left[\frac{(n\omega_c)^2-\omega_a^2}{(n\omega_c)^2-\omega_r^2}\right]^{2/3}\left(\frac{c^2R}{\omega_0^2}\frac{n\omega_c-\omega}{\omega_c}\right)^{1/3} \qquad (34)$$

(in deriving (34) we have assumed for concreteness that $\omega_a, \omega_r < \omega_c$).

Calculation shows that in the cases in question the values of the impedance Z_n differ from Z_p only by a numerical factor, this being an obvious consequence of the insensitivity of the impedance to the boundary conditions for a magnetic moment in the case of the anomalous skin effect.

4. CALCULATION OF THE IMPEDANCE FOR AN ARBITRARY LAW OF CONDUCTION-ELECTRON DISPERSION

At the end of Sec. 2 we pointed to the possible coincidence of the ferromagnetic resonance frequency ω_r and the cyclotron frequency ω_c. To observe this effect when the carrier energy spectrum is isotropic it is necessary to change simultaneously both the frequency of the electromagnetic wave and the magnitude of the applied field **H**. When the energy spectrum is anisotropic, the possibility of realizing this effect is more favorable for then the electron cyclotron frequency, as is well known, depends on the orientation of the constant field relative to the crystallographic axes. At certain directions of the field in a plane parallel to the surface of the metal, the cyclotron

*tg = tan.

frequency may equal the ferromagnetic resonance frequency ω_r. Then, as expected, a sharp change takes place in the shape of the resonance curve.

The electric conductivity tensor of the metal, for $kR \gg 1$ and for arbitrary electron dispersion, was calculated by Kaner and Skobov[12]:

$$\sigma(\omega, k) = \frac{8e^2\omega_c m v_x^2}{h^3|v_y'|k}\int_{-\infty}^{\infty}\frac{dp_x}{\nu - i(\omega - n\omega_c - n\omega_c'' p_x^2/2)}, \quad (35)$$

where all the quantities that depend on p_x are taken at the point $p_x = 0$, $v_y' = \partial v_y/\partial\tau$, τ—dimensionless time of revolution of the electrons on the trajectory, $\omega_c'' = \partial^2\omega_c/\partial p_x^2|_{p_x=0}$. Expression (35) was obtained for an unbounded metal under the assumption that the cyclotron resonance takes place on the central section $p_x = 0$.

Putting $\Delta = (\omega - n\omega_c)/\omega_c$ and $\xi = \nu/\omega_c$, we easily obtain

$$\sigma(\omega, k) = \frac{8\pi e^2 m v_x^2}{h^3|v_y'|}\left(\frac{n\omega_c''}{2\omega_c}\right)^{1/2}\frac{e^{i\psi}}{k|u|}, \quad (36)$$

where the dimensionless parameter u is connected with ξ and Δ in the following fashion:

$$u = \beta + i\xi/2\beta, \qquad \beta = 2^{-1/2}(\Delta + \sqrt{\Delta^2 + \xi^2})^{1/2},$$

and the phase ψ takes the form

$$\operatorname{tg}\psi = \xi/(\Delta + \sqrt{\Delta^2 + \xi^2}), \qquad 0 < \psi < \pi/2. \quad (37)$$

Using (36), (37), and (31) we obtain for the surface impedance (26)

$$Z_p = C(n\omega_c)^{2/3}|\mu|^{2/3}|u|^{1/3}e^{i(\psi + 2\vartheta - \pi)/3},$$
$$C = \frac{1}{3\sqrt{3}}\left[\frac{2h^3|v_y'|}{\pi c e^2 m v_x^2}\left(\frac{1}{2}\frac{n\omega_c''}{\omega_c}\right)^{1/2}\right]^{1/3}. \quad (38)$$

Assume for concreteness that $\Delta > 0$. Setting ν and λ in (35) equal to zero, we write Z_p in the form

$$Z_p = C(n\omega_c)^{2/3}\left|\frac{\omega_a^2 - \omega^2}{\omega_r^2 - \omega^2}\right|^{2/3}\left(\frac{\omega - n\omega_c}{\omega_c}\right)^{1/6}e^{-i\pi/3}. \quad (39)$$

Formula (39) is somewhat arbitrary. The frequency ω in the numerator or in its denominator should be set equal to $n\omega_c$, depending on which of the frequencies, ω_r or ω_a respectively, is equal to the cyclotron frequency. As seen from (39), when $n\omega_c$ coincides with ω_r or ω_a, greatly differing values of the impedance are obtained. Allowance for dissipation leads in the usual fashion to a finite line width.

The authors are grateful to É. A. Kaner for a useful discussion of many problems touched upon in the present article.

[1] Akhiezer, Bar'yakhtar, and Kaganov, UFN **51**, 533 (1960) and **52**, 1 (1960).

[2] G. T. Rado and J. P. Weertman, J. Phys. Chem. Sol. **11**, 315 (1959).

[3] G. T. Rado and J. P. Weertman, Phys. Rev. **94**, 1386 (1954).

[4] C. Kittel, Phys. Rev. **110**, 1295 (1958).

[5] M. H. Seavey and P. E. Tannenwald, Phys. Rev. **97**, 1558 (1958).

[6] W. S. Ament and G. T. Rado, Phys. Rev. **97**, 1558 (1955).

[7] M. I. Kaganov and Yü Lu, Izv. AN SSSR ser. fiz. **25**, 1375 (1961), Columbia Tech. Transl. p. 1388.

[8] V. L. Gurevich, JETP **33**, 1497 (1957), Soviet Phys. JETP **6**, 1155 (1958); ZhTF **28**, 2352 (1958), Soviet Phys. Tech. Phys. **3**, 2159 (1959).

[9] R. G. Chambers and B. K. Jones, Proc. Roy. Soc. **A270**, 417 (1962).

[10] É. A. Kaner and V. G. Skobov, JETP **45**, 610 (1963), Soviet Phys. JETP **18**, 419 (1964).

[11] Bass, Blank, and Kaganov, JETP **45**, 1081 (1963), Soviet Phys. JETP **18**, 747 (1964).

[12] É. A. Kaner and V. G. Skobov, FTT **6**, 1104 (1964), Soviet Phys. Solid State **6**, 851 (1964).

[13] E. A. Stern and E. R. Callen, Phys. Rev. **131**, 512 (1963).

[14] A. Ya. Blank, JETP **47**, 325 (1964), Soviet Phys. JETP **20**, 216 (1965).

[15] M. Ya. Azbel' and É. A. Kaner, JETP **32**, 896 (1957), Soviet Phys. JETP **5**, 730 (1957).

[16] J. Smit, Suppl. Nuovo cimento **6**, 1177 (1957).

[17] J. R. Anderson and A. V. Gold, Phys. Rev. Lett. **10**, 227 (1963).

[18] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. **A195**, 336 (1948).

[19] M. Ya. Azbel' and M. I. Kaganov, DAN SSSR **100**, 437 (1955).

Translated by J. G. Adashko
315