THE RELATION BETWEEN POLARIZATION EFFECTS OF PROCESSES INVOLVING A SINGLE ISOBAR

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The relation between polarization effects of the following processes, which proceed via a single isobar, is discussed: 1) $\gamma + N \rightarrow V + N$, 2) $\gamma + N \rightarrow \pi + N$, 3) $\pi + N \rightarrow V + N$, and 4) $\pi + N \rightarrow \pi + N$, where V is a vector meson. Decay of the vector meson into two of three pseudoscalar particles is considered.

INTRODUCTION

As is well known, a connection exists between the amplitudes and cross sections of different processes that proceed via the same Breit-Wigner level[1,2]. There exist, for example, relations between the amplitudes of three processes (AB → AB; AC → AC; AB → AC) of which two are elastic. In this article we discuss relations between the polarization effects of the following processes: 1) photoproduction of a vector meson from a nucleon, 2) photoproduction of a pion from a nucleon, 3) production of a vector meson in πN interactions, and 4) elastic πN interaction.

The particle energy is equal in the center of mass system (c.m.s.) to the mass of the isobar, so that all the processes proceed via the same isobar. We have to consider four processes rather than three, since it is impossible to observe the elastic interaction between a nucleon and a vector meson.

We note that the production of a vector meson can be represented as the decay of an isobar if the vector-meson momentum $k \gtrsim j\mu$ (μ—pion mass). In addition, the width of the meson resonance should be such that during the time of the meson decay the nucleon and meson move apart by a distance at least of the order of $1/\mu$. Isobar decay into a nucleon and a vector meson was not investigated experimentally, but experiments in which different isobars were observed do not exclude the possibility of such a decay, since in some cases[3,4] the investigations concerned the cross section of elastic πp scattering, and in others[5] the total cross section.

Since the vector-meson polarization can be deduced from its decay, we consider in the first section the decay of a meson with specified polarization matrix; in the second section we discuss the relations between the polarization effects of the aforementioned processes.

1. DECAY OF POLARIZED VECTOR MESON

The polarization matrix of bosons was used in the literature in various forms[6]. We shall use the matrix in a different form. The meson is described by a vector V_i satisfying the relation

$$V_i k_i = 0, \tag{1}$$

where k_i—meson momentum. The polarization matrix is of the form

$$\rho_{ij} = \overline{V_i V_j^*} = w_1 A_i^{(1)} A_j^{(1)} + w_2 A_i^{(2)} A_j^{(2)} + w_3 A_i^{(3)} A_j^{(3)} + \frac{1}{2m} \varepsilon_{ijlm} k_l r_m, \tag{2}$$

where m—meson mass. The vectors $A_i^{(\mu)}$ satisfy the condition (1), and also the condition

$$A_i^{(\mu)} A_i^{(\nu)} = \delta_{\mu\nu}. \tag{3}$$

Thus, $A_i^{(2)}$ has only one degree of freedom if $A_i^{(1)}$, and $A_i^{(3)}$ is uniquely defined if $A_i^{(1)}$ and $A_i^{(2)}$ are given.

The quantities w_μ obviously satisfy the relation

$$\sum_\mu w_\mu = 1. \tag{4}$$

If $w_1 > w_2 > w_3$, then $A_i^{(1)}$ yields, in the rest system, the direction of most probable alignment of the meson, and $A_i^{(3)}$ the least probable one. $A_i^{(2)}$ specifies the direction of the most probable alignment in the plane perpendicular to $A_i^{(1)}$, and also the direction of the least probable alignment in a plane perpendicular to $A_i^{(3)}$. The w_μ are the corresponding probabilities. The vector r_i is the average spin. It satisfies condition (1) and does not exceed unity in absolute value.

Let us consider the decay of a vector meson into two pseudoscalar particles. This situation arises in the case of the ρ meson (decay into two pions; see, for example[7,8]), and also in the case of the φ meson (decay into two K mesons[9,10]).

The matrix element has obviously the form

$$S = Dp_iV_i, \tag{5}$$

where D—constant, p_i—momentum of one of the pseudoscalar particles, V_i—vector describing the meson and satisfying (1). Using the polarization matrix (2), we obtain

$$|S|^2 = |D|^2\{w_1(A^{(1)}p)^2 + w_2(A^{(2)}p)^2 + w_3(A^{(3)}p)^2\}. \tag{6}$$

On the other hand, we can raise the following question: what is the probability that a meson having a polarization matrix (2) will have an alignment along the direction of the unit vector B_i? The answer is given by formula (6), in which we replace p_i by B_i (without $|D|^2$). This comparison can be interpreted in the following fashion: when the vector meson decays in the rest system into two pseudoscalar particles, the decay products move apart along the direction of its alignment.

We now consider the decay of an ω meson into three pions[11]. The decay is determined by one form factor Φ, and the matrix element is of the form

$$S = \Phi(t_1, t_2)n_iV_i, \tag{7}$$

where $t_1 = p_1p_2$, $t_2 = p_1p_3$, n_i—vector normalized to unity and proportional to $\epsilon_{ijlm}p_{1j}p_{2l}p_{3m}$ (p_1, p_2, p_3—pion momenta). Then

$$|S|^2 = |\Phi(t_1, t_2)|^2\{w_1(A^{(1)}n)^2 + w_2(A^{(2)}n)^2 + w_3(A^{(3)}n)^2\}. \tag{8}$$

Since n_i is in no manner connected with t_1 and t_2, we can again state that the meson decay is uniquely related to its polarization state: in the rest system the normal to the plane in which the pions are scattered is directed along the alignment of the ω meson.

2. CONNECTION BETWEEN THE POLARIZATION EFFECTS OF PROCESSES PROCEEDING VIA A SINGLE ISOBAR

If the different processes proceed via a single isobar with spin j and parity $P = (-1)^{j\pm 1/2}$, then[12] we have the following:

1. The elastic πN interaction is described by a single helicity partial amplitude

$$\frac{1}{\sqrt{2}}\left\{\left\langle\frac{1}{2}\right| \pm \left\langle-\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}\right\rangle \pm \left|-\frac{1}{2}\right\rangle\right\}\frac{1}{\sqrt{2}} = f^{\pm}. \tag{9}$$

2. The photoproduction of the pion from the nucleon is described by two helicity partial amplitudes

$$\frac{1}{\sqrt{2}}\left\{\left\langle\frac{1}{2}\right| \pm \left\langle-\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, -1\right\rangle \mp \left|-\frac{1}{2}, 1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_2{}^{\pm}.$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle\frac{1}{2}\right| \pm \left\langle-\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, -1\right\rangle \mp \left|-\frac{1}{2}, 1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_2{}^{\pm}. \tag{10}$$

3. The photoproduction of the vector meson in πN interactions is described by three amplitudes:

$$\frac{1}{\sqrt{2}}\left\{\left\langle 1, \frac{1}{2}\right| \mp \left\langle -1, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}\right\rangle \pm \left|-\frac{1}{2}\right\rangle\right\}\frac{1}{\sqrt{2}} = \bar{f}_1{}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle -1, \frac{1}{2}\right| \mp \left\langle 1, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}\right\rangle \pm \left|-\frac{1}{2}\right\rangle\right\}\frac{1}{\sqrt{2}} = \bar{f}_2{}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle 0, \frac{1}{2}\right| \mp \left\langle 0, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}\right\rangle \pm \left|-\frac{1}{2}\right\rangle\right\}\frac{1}{\sqrt{2}} = \bar{f}_3{}^{\pm}. \tag{11}$$

Finally, the photoproduction of the vector meson is described by six amplitudes:

$$\frac{1}{\sqrt{2}}\left\{\left\langle 1, \frac{1}{2}\right| \mp \left\langle -1, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, 1\right\rangle \mp \left|-\frac{1}{2}, -1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{11}{}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle -1, \frac{1}{2}\right| \mp \left\langle 1, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, 1\right\rangle \mp \left|-\frac{1}{2}, -1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{21}{}^{\pm}$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle 0, \frac{1}{2}\right| \mp \left\langle 0, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, 1\right\rangle \mp \left|-\frac{1}{2}, -1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{31}{}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle 1, \frac{1}{2}\right| \mp \left\langle -1, -\frac{1}{2}\right|\right\}S(E, j) \times\left\{\left|\frac{1}{2}, -1\right\rangle \mp \left|-\frac{1}{2}, 1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{12}{}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle -1,\ \frac{1}{2}\right| \mp \left\langle 1,\ -\frac{1}{2}\right|\right\} S(E,j)$$

$$\times \left\{\left|\frac{1}{2},\ -1\right\rangle \mp \left|-\frac{1}{2},\ 1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{22}^{\pm},$$

$$\frac{1}{\sqrt{2}}\left\{\left\langle 0,\ \frac{1}{2}\right| \pm \left\langle 0,\ -\frac{1}{2}\right|\right\} S(E,j)$$

$$\times \left\{\left|\frac{1}{2},\ -1\right\rangle \pm \left|-\frac{1}{2},\ 1\right\rangle\right\}\frac{1}{\sqrt{2}} = f_{32\pm}. \qquad (12)$$

Since all these processes proceed via a Breit-Wigner level, the following connection exists between them:

$$f_{mn}^{\pm} = \bar{f}_m^{\pm} f_n^{\pm} / f^{\pm}, \qquad (13)$$

where m runs through the values 1 to 3, and n through the values 1 and 2.

Relation (13) enables us to predict all the polarization effects of the process (12), if complete information is obtained concerning the amplitudes of processes (9)—(11). On the other hand, if measurement of the part of the polarization effects of the processes (9)—(11) yields incomplete information concerning their amplitudes, it is still possible to predict the corresponding polarization effects of process (12). For example, if all the measurements are made in processes (9)—(11) with unpolarized nucleons, then these measurements still enable us to predict the polarization effects of process (12) when the nucleons are not polarized. We present the simplest results, when the photon and the nucleons are not polarized:

$$d\sigma_{\pi N}^{\mathrm{el}} = M^{-2}|f|^2\left(j+\frac{1}{2}\right)$$

$$\times \left(P_{j-1/2}(z)P'_{j+1/2}(z) - P'_{j-1/2}(z)P_{j+1/2}(z)\right)d\Omega, \qquad (14)$$

$$d\sigma_{\gamma N\to\pi N} = \frac{p}{2kM^2}\{P'^2_{j-1/2}\left(|f_1|^2 + J^2|f_2|^2\right) + P'^2_{j+1/2}$$

$$\times \left(|f_1|^2 + J^{-2}|f_2|^2\right) - 2zP'_{j-1/2}P'_{j+1/2}\left(|f_1|^2 + |f_2|^2\right)\}\, d\Omega, \qquad (15)$$

$$d\sigma^{\parallel}_{\pi N\to VN} = \frac{K}{M^2 p}|\bar{f}_3|^2\left(j+\frac{1}{2}\right)$$

$$\times \left(P_{j-1/2}P'_{j+1/2} - P'_{j-1/2}P_{j+1/2}\right)d\Omega\,\frac{do}{4\pi/3}, \qquad (16)$$

$$d\sigma^{\perp}_{\pi N\to VN} = \frac{K}{2pM^2}(M_1 + M_2)\,d\Omega\,\frac{do}{4\pi/3}, \qquad (16')$$

$$d\sigma^{n}_{\pi N\to VN} = \frac{K}{2pM^2}(M_1 - M_2)\,d\Omega\,\frac{do}{4\pi/3}, \qquad (16'')$$

$$d\sigma^{\parallel}_{\gamma N\to VN} = \frac{K}{2kM^2}\{P'^2_{j-1/2}\left(|f_{31}|^2 + J^2|f_{32}|^2\right)$$

$$+ P'^2_{j+1/2}\left(|f_{31}|^2 + J^{-2}|f_{32}|^2\right) - 2zP'_{j-1/2}P'_{j+1/2}$$

$$\times \left(|f_{31}|^2 + |f_{32}|^2\right)\}\, d\Omega\,\frac{do}{4\pi/3}, \qquad (17)$$

$$d\sigma^{\perp}_{\gamma N\to VN} = \frac{K}{4kM^2}(N_1 + N_2)\,d\Omega\,\frac{do}{4\pi/3}, \qquad (17')$$

$$d\sigma^{n}_{\gamma N\to VN} = \frac{K}{4kM^2}(N_1 - N_2)\,d\Omega\,\frac{do}{4\pi/3}. \qquad (17'')$$

$$J = \left[\left(j+\frac{3}{2}\right)\Big/\left(j-\frac{1}{2}\right)\right]^{1/2}, \qquad (18)$$

$$M_2 = 2\,\mathrm{Re}\,\bar{f}_1\bar{f}_2^*\left(J^{-1}P_{j-1/2}P'_{j+1/2} - JP'_{j-1/2}P_{j+1/2}\right)$$

$$\times\left(j+\frac{1}{2}\right), \qquad (19)$$

$$M_1 = P'^2_{j-1/2}\left(|\bar{f}_1|^2 + J^2|\bar{f}_2|^2\right) + P'^2_{j+1/2}\left(|\bar{f}_1|^2 + J^{-2}|\bar{f}_2|^2\right)$$

$$- 2zP'_{j-1/2}P'_{j+1/2}\left(|\bar{f}_1|^2 + |\bar{f}_2|^2\right), \qquad (19')$$

$$N_1 = P'^2_{j-1/2}\left[|f_{11}|^2 + |f_{22}|^2 + J^2\left(|f_{21}|^2 + |f_{12}|^2\right)\right]$$

$$+ P'^2_{j+1/2}\left[|f_{11}|^2 + |f_{22}|^2 + J^{-2}\left(|f_{21}|^2 + |f_{12}|^2\right)\right]$$

$$- 2zP'_{j-1/2}P'_{j+1/2}\left(|f_{11}|^2 + |f_{22}|^2 + |f_{12}|^2 + |f_{21}|^2\right)$$

$$- |f_{22}|^2\frac{4(1-z^2)}{(j-1/2)(j+3/2)}\left[\frac{j-3/2}{j-1/2}P'_{j-1/2}P''_{j+1/2} + \frac{j+5/2}{j+3/2}\right.$$

$$\left.\times P'_{j+1/2}P''_{j-1/2} - \frac{4z}{(j-1/2)(j+3/2)}P''_{j+1/2}P''_{j-1/2}\right], \qquad (20)$$

$$N_2 = 2\mathrm{Re}\left(f_{11}f_{21}^* + f_{12}f_{22}^*\right)\left(j+\frac{1}{2}\right)$$

$$\times\left(J^{-1}P_{j-1/2}P'_{j+1/2} - JP'_{j-1/2}P_{j+1/2}\right) - 4\,\mathrm{Re}\,f_{12}f_{22}^*$$

$$\times\frac{1-z^2}{(j-1/2)(j+3/2)}\left(JP'_{j-1/2}P''_{j+1/2} + J^{-1}P'_{j+1/2}P''_{j-1/2}\right). \qquad (20')$$

The following notation is used: M—mass of isobar, z = cos ϑ, where ϑ—angle through which the nucleon is scattered in the c.m.s.; K, k, p—absolute values of the momenta of the vector meson, photon, and pion in the c.m.s.; $d\sigma^{\parallel}$—cross section when the vector meson is aligned along its own momentum; $d\sigma^{\perp}$—cross section when the meson is aligned in the plane of the reaction perpendicular to its momentum; $d\sigma^{n}$—cross section when the meson is aligned along the normal to the reaction plane; dΩ—c.m.s. solid-angle element in which the scattered nucleon is situated; do—solid-angle element in which the vector describing the aligned state of the vector meson is situated. As follows from the discussion in Sec. 1, do is an element of the solid angle containing either the meson-decay products (in the case of decay into two particles) or the normal to the plane of scattering of the decay product (in the case of decay to three particles).

Let us write out relations that follow from (13):

$$|f_{mn}|^2 = |\bar{f}_m|^2|f_n|^2|f|^{-2}, \quad f_{mn}f_{m'n}^* = \bar{f}_m\bar{f}_{m'}^*|f_n|^2|f|^{-2}. \tag{21}$$

We note also that formulas (14)—(20) are the same for both parties of the isobar, and in the present case the Minami uncertainty[7] is not lifted.

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Supplement (20 August 1964). In elastic πN interaction, the parity of the isobar is manifest only in the case when the target is polarized and the recoil-nucleon polarization is measured[7]. On the other hand, in the case of the process $\pi + N \rightarrow V + N$, the parity comes into play if the target is polarized, and one measures, for example, the cross section for the production of aligned vector mesons (and the alignment is fixed from the meson decay). An analogous situation takes place also in the processes $\gamma + N \rightarrow \pi + N$ and $\gamma + N \rightarrow V + N$. If the target is polarized then, for example,

$$d\sigma^n_{\pi N\rightarrow VN} = \frac{K}{8pM^2}[M_1 - M_2 \pm (\xi_1 \mathbf{n})2\sin\vartheta P'_{j+1/2}P'_{j-1/2}$$
$$\times (J - J^{-1})\,\mathrm{Im}\,\bar{f}_1^{\pm}\bar{f}_2^{\pm *}]\,d\Omega\,\frac{do}{4\pi/3}$$

where ξ_1—target polarization vector, $\mathbf{n}$—the vector $\mathbf{p}_2 \times \mathbf{p}$ normalized to unity, perpendicular to the plane of the reaction, $\mathbf{p}_2$—momentum of the recoil nucleon, and $\mathbf{p}$—momentum of the incident pion. The sign of $\mathrm{Im}\bar{f}_1^{\pm}\bar{f}_2^{\pm}$ can be determined by measuring the cross section for the production of a vector meson aligned along an axis rotated clockwise about p through $\pi/4$ relative to $\mathbf{n}$:

$$d\sigma^{\pi/4}_{\pi N\rightarrow VN} = \frac{K}{8pM^2}\Big[M_1 + \frac{\xi_1\mathbf{p}}{p}(2j+1)$$
$$\times (J^{-1}P'_{j+1/2}P_{j+1/2} - JP'_{j-1/2}P_{j-1/2})$$
$$\times\ \mathrm{Im}\,\bar{f}_1^{\pm}\bar{f}_2^{\pm *}\Big]\,d\Omega\,\frac{do}{4\pi/3}.$$

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