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EFFECT OF THE MEDIUM ON THE PROPERTIES OF $K^0\overline{K}^0$ MESON PAIRS

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The effect of coherent interaction of K^0 and $\overline{K}^0$ mesons with the medium on the properties of $K^0\overline{K}^0$ pairs is considered. A set of basis functions for such pairs in the medium is found, and some specific features of interference in $K^0\overline{K}^0$, $K_1^0K_2^0$, $K_1^0(K_2^0)$, and $K^0(\overline{K}^0)$ systems are noted. The accuracy of the usual approximation [1-3] in which K^0 mesons are regarded as free is estimated.

1. The main properties of $K^0\overline{K}^0$ meson pairs were considered in detail in earlier work [1,3], and the specific features of the Pais-Piccioni process for such pairs were indicated. It must be noted that all the deductions of these papers pertain, strictly speaking, to free $K^0\overline{K}^0$ meson pairs which do not interact with matter. For free neutral K mesons, the basic quasistationary basis states are K_1 and K_2. CP-parity conservation allows a $K^0\overline{K}^0$ pair with even orbital momentum to decay only into K_1K_1 or K_2K_2, while a $K^0\overline{K}^0$ pair with odd orbital angular momentum can decay only into K_1K_2. In the case of free neutral K mesons, the transitions $K_1 \to K_2$ and $K_2 \to K_1$ are impossible.

Under real conditions, the production and decay of $K^0\overline{K}^0$ pairs takes place in some specific medium. If the K^0 and $\overline{K}^0$ mesons interact with the atoms of the medium in different fashions, regeneration of the type $K_1 \to K_2$ becomes possible besides the $K^0 \to \overline{K}^0$ transitions [4,5], i.e., CP parity is not conserved.

The quasistationary basis states for the neutral K meson in a medium are no longer K_1 and K_2, but their linear combinations, which we denote by L_1 and L_2. To determine the time dependence of the wave function of the $K^0\overline{K}^0$ pair in the medium, it is necessary to expand the wave function of the pair at the instant of regeneration in terms of products of the type

$$L_1(p)L_1(q),\quad L_2(p)L_2(q),\quad L_1(p)L_2(q),\quad L_2(q)L_1(p)$$

and take into account the exponential time development of the states L_1 and L_2 (p and q—momenta of the first and second K meson, respectively). On the other hand, the detectors at our disposal can record a neutral K meson only in the states K^0, $\overline{K}^0$ or K_1, K_2. This gives rise to "beats" of the Pais-Piccioni type.

Unlike in the case of a free particle, the Pais-Piccioni process for a $K^0\overline{K}^0$ pair produced in a dense medium has a much more complicated character. The purpose of the present paper is to investigate the influence of the medium on interference phenomena in $K^0\overline{K}^0$ systems and to estimate the accuracy of the usual approximation within the framework of which the K^0 mesons are regarded as free.

2. Explicit expressions for L_1 and L_2 were actually derived by Good [4-5]. They can be written in the form

$$L_1(t) = L_1(0)e^{-\chi_1 t},\qquad L_2(t) = L_2(0)e^{-\chi_2 t};$$

$$L_1(0) = K_1 + RK_2 = 2^{-1/2}((1 - iR)K^0 + (1 + iR)\overline{K}^0),$$
$$L_2(0) = RK_1 + K_2 \equiv i2^{-1/2}((1 + iR)K^0 - (1 - iR)\overline{K}^0). \tag{1}$$

In the general case the formulas for R, χ_1, and χ_2 are rather complicated. However, they simplify appreciably in the approximation $R \ll 1$, or

$$(4\pi N\hbar\tau_1 / m)N|f_{12}| \ll 1. \tag{2}$$

Here and below N—number of atoms per cm^3 of the medium, m—mass of K^0 meson, τ_1 (τ_2)—lifetime of the $K_1^0(K_2^0)$ meson, $f_{12} = (A(K^0) - A(\overline{K}^0))/2$—regeneration amplitude, equal to half the difference of the K^0 and $\overline{K}^0$ meson zero-angle scattering amplitudes. Inequality (2) is actually satisfied for all real media. With this,

$$R(p) = \frac{4\pi N\hbar\tau_1}{m}f_{12}(p)\frac{1 - 2i\delta}{1 + 4\delta^2},$$

$$\chi_1(p) = \frac{im_{K_1}}{\gamma(p)} + \frac{1}{2\gamma(p)}\frac{1}{\tau_1} + \frac{1}{4}(\sigma_1(p) + \sigma_2(p))v(p),$$

$$\chi_2(p) = \frac{im_{K_2}}{\gamma(p)} + \frac{1}{2\gamma(p)}\frac{1}{\tau_2} + \frac{1}{4}(\sigma_1(p) + \sigma_2(p))v(p). \tag{3}$$

Here σ_1 and σ_2—total scattering cross sections of the K^0 and $\overline{K}^0$ mesons, respectively, $v = pc/(p^2 + m^2c^2)^{1/2}$—velocity of particle with momentum p, $\gamma(p) = (1 - v^2(p)/c)^{-1/2}$, and $\delta = (m_{K_1} - m_{K_2})c^2\tau_1$.

It is easy to see that the wave functions corresponding at the initial instant of time to the states K_1 and K_2 are expressed in terms of L_1 and L_2 in the following manner:

$$K_1 \to \frac{1}{1-R^2} L_1(0)e^{-\chi_1 t} - \frac{R}{1-R^2} L_2(0)e^{-\chi_2 t},$$

$$K_2 \to -\frac{R}{1-R^2} L_1(0)e^{-\chi_1 t} + \frac{1}{1-R^2} L_2(0)e^{-\chi_2 t}. \quad (4)$$

Assume now that the $K^0\overline{K}^0$ pair is produced with even orbital angular momentum. Then the initial wave function has the form

$$\psi_c = 2^{-1/2}\{K_1(p)K_1(q) + K_2(p)K_2(q)\}. \quad (5)$$

Taking the time dependence into account, we get

$$\begin{aligned}\psi_c = {} & 2^{-1/2}(1 - R^2(p))^{-1}\{(1 + R(p)R(q)) \\ & \times [L_1^{(p)}(0)L_1^{(q)}(0)\exp(-\chi_1(p)T - \chi_1(q)\Theta) \\ & + L_2^{(p)}(0)L_2^{(q)}(0)\exp(-\chi_2(p)T - \chi_2(q)\Theta)] \\ & - (R(p) + R(q))[L_1^{(p)}(0)L_2^{(q)}(0) \\ & \times \exp(-\chi_1(p)T - \chi_2(q)\Theta) \\ & + L_2^{(p)}(0)L_2^{(q)}(0)\exp(-\chi_2(p)T - \chi_2(q)\Theta)]\}. \quad (6)\end{aligned}$$

Here T—time interval elapsed from the instant of production of the pair to the registration of the K meson with momentum p, and Θ—time interval elapsed from the instant of pair production to the registration of the K meson with momentum q. The proper times corresponding to p and q will be denoted, as in [2,3], by τ and θ:

$$\tau = T/\gamma(p), \quad \theta = \Theta/\gamma(q).$$

If the $K^0\overline{K}^0$ pair is produced with odd orbital angular momentum, for example as a result of φ-meson decay, the initial function is written in antisymmetrical form

$$\psi_a = (K_1(p)K_2(q) - K_1(q)K_2(p))/\sqrt{2}. \quad (7)$$

Account of the time dependence leads to the expression

$$\begin{aligned}\psi_a = {} & 2^{-1/2}(1 - R^2(p))^{-1}(1 - R^2(q))^{-1}\{(1 + R(p)R(q)) \\ & \times [L_1^{(p)}(0)L_2^{(q)}(0)\exp(-\chi_1(p)T - \chi_2(q)\Theta) \\ & - L_1^{(q)}(0)L_2^{(p)}(0)\exp(-\chi_2(p)T - \chi_1(q)\Theta)] \\ & + (R(p) - R(q))[L_1^{(p)}(0)L_1^{(q)}(0) \\ & \times \exp(-\chi_1(p)T - \chi_1(q)\Theta) \\ & - L_2^{(p)}(0)L_2^{(q)}(0)\exp(-\chi_2(p)T - \chi_2(q)\Theta)]\}. \quad (8)\end{aligned}$$

With the aid of (1) we can express ψ_a and ψ_s in terms of different combinations of the paired products of the K_1, K_2, K^0, and $\overline{K}^0$ wave functions. The squares of the moduli of the coefficients for each possible combination yield the probability of observation of the $K^0\overline{K}^0$ pair in the corresponding state at the proper times τ and θ.

We now consider experiments in which K^0_{e3} or $K^0_{\mu 3}$ decays are recorded; this corresponds to a classification of the pair in accordance with the states K^0 and $\overline{K}^0$. Expressions for

$$w(K^0(p)K^0(q)\tau\theta), \quad w(K^0(p)\overline{K}^0(q)\tau\theta),$$
$$w(\overline{K}^0(p)K^0(q)\tau\theta), \quad w(\overline{K}^0(p)\overline{K}^0(q)\tau\theta)$$

(the notation is the same as in [2,3]) are quite cumbersome, and will not be presented here. Each such expression can be represented in the form $w = w_0 + w_{int}$, where the w_0 terms pertain to $K^0\overline{K}^0$ pairs moving in vacuum, while the w_{int} terms take into account the coherent interaction of the K^0 and $\overline{K}^0$ mesons with the medium. The w_{int} terms depend on the signs of both the real and imaginary parts of the regeneration amplitude f_{12}, and their ratio to the w_0 terms is of the order of $|R|$ [see formulas (2) and (3)]. Owing to the presence of the w_{int} terms, the Pais-Piccioni terms for the indicated experiments in a medium exhibit some interesting peculiarities.

A. Regardless of whether the $K^0\overline{K}^0$ pair has initially a definite orbital angular momentum or whether it is a superposition of states with even and odd orbital momenta, "beats" are produced, depending on the sign of the difference of the masses of the K^0_1 and K^0_2 mesons. "Beats" of this type do not occur in vacuum for states with definite orbital angular momentum [2,3].

B. In experiments in which the state of the second meson is not identified, the "beats" do not vanish. Then the expressions

$$w(K^0(p)K^0(q)\tau\theta) + w(K^0(p)\overline{K}^0(q)\tau\theta),$$
$$w(\overline{K}^0(p)\overline{K}^0(q)\tau\theta) + w(\overline{K}^0(p)K^0(q)\tau\theta)$$

contain terms proportional to $\cos(\Delta m\tau)$, $\sin(\Delta m\tau)$, $\cos(\Delta m\theta)$, $\sin(\Delta m\theta)$, $\sin[\Delta m(\tau - \theta)]$, and $\cos[\Delta m(\tau - \theta)]$. Their order of magnitude is $|R|$, and as $|R| \to 0$ the beats drop out. For pairs produced with a definite orbital angular momentum in the medium, we have

$$w(K^0(p)K^0(q)\tau\theta) \neq w(\overline{K}^0(p)\overline{K}^0(q)\tau\theta),$$
$$w(K^0(p)\overline{K}^0(q)\tau\theta) \neq w(\overline{K}^0(p)K^0(q)\tau\theta), \quad (9)$$

whereas in vacuum the probabilities are equal under the same conditions. The difference between $w(K^0(p)K^0(q)\tau\theta)$ and $w(\overline{K}^0(p)K^0(q)\tau\theta)$, as shown by calculations, include terms that are linear in R (having the order of magnitude $|R|$).

Thus, with respect to the experiments considered here, a $K^0\overline{K}^0$ pair with definite orbital angular momentum has in a medium properties that are possessed in vacuum only by superpositions of states with even and odd orbital angular momenta.

4. We now proceed to experiments corresponding to the classification of the pair with respect to the states K_1 and K_2. We have already noted that K_1K_2 regeneration is impossible in vacuum. Therefore the expressions for

$$w(K_1(p)K_1(q)\tau\theta), \qquad w(K_2(p)K_2(q)\tau\theta),$$
$$w(K_1(p)K_2(q)\tau\theta), \qquad w(K_2(p)K_1(q)\tau\theta)$$

do not contain any "beats" at all, if we disregard the interaction with the medium. When account is taken of the influence of the medium, several specific phenomena can occur:

a) "Beats" are produced, which either depend or do not depend on the sign of the difference between the K_1^0 and K_2^0 masses, for all four experiments in which the states K_1 or K_2 are fixed.

b) These "beats" do not contain terms linear in R, and are of the order of $|R|^2$.

c) If the $K^0\overline{K}^0$ has an even orbital angular momentum at the instant of production, then

$$w(K_1(p)K_2(q)\tau\theta) \sim |R|^2, \tag{10}$$

The corrections to the probabilities $w(K_1(p)K_1(q)\tau\theta)$ and $w(K_2(p)K_2(q)\tau\theta)$ have in this case also the order of magnitude $|R|^2$.

d) If the $K^0\overline{K}^0$ pair is produced with odd orbital angular momentum, then

$$w(K_1(p)K_1(q)\tau\theta) \sim |R|^2,$$
$$w(K_2(p)K_2(q)\tau\theta) \sim |R|^2, \tag{11}$$

and the corrections to the probability $w(K_1(p)K_2(q)\tau\theta)$ are of the order of $|R|^2$.

5. Let us analyze now the role of the medium for experiments that correspond to classification of the states of the pair in terms of $K_1(K_2)$ for momentum p, and $K^0(\overline{K}^0)$ for momentum q. In this case "beats" do not arise at all for a $K^0\overline{K}^0$ pair in vacuum with a definite orbital angular momentum. They occur only for a mixture of even and odd orbital momenta, and it is important that the variables τ and θ separate, as is well known, in the formulas for the probabilities

$$w(K_1(p)K_1(q)\tau\theta), \qquad w(K_1(p)K^0(q)\tau\theta).$$

In the presence of a medium, the "beats" occur also for states with definite initial orbital angular momentum. The variables τ and θ then no longer separate. For the symmetrical wave function (even orbital angular momenta), neglecting the terms quadratic in R, we obtain

$$w\begin{pmatrix} K_1(p)K^0(q)\tau\theta \\ K_1(p)\overline{K}^0(q)\tau\theta \end{pmatrix} = \frac{1}{4}\exp\Big[-\frac{1}{2}N(\sigma_1(p)+\sigma_2(p))x_1 - \frac{1}{2}N(\sigma_1(q)+\sigma_2(q))x_2\Big]\Big\{\exp\Big(-\frac{\tau+\theta}{\tau_1}\Big)\Big[1+\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)} \times(\operatorname{Im}f_{12}^{(q)}+2\delta\operatorname{Re}f_{12}^{(q)})\Big]\pm A\mp B\Big\}, \tag{12}$$

where

$$A=\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)}\exp\Big(-\frac{\tau+\theta}{2}\Big(\frac{1}{\tau_1}+\frac{1}{\tau_2}\Big)\Big) \times(\operatorname{Im}f_{12}^{(p)}-2\delta\operatorname{Re}f_{12}^{(p)})\cos[\Delta m(\tau-\theta)] + (\operatorname{Re}f_{12}^{(p)}-2\delta\operatorname{Im}f_{12}^{(p)})\sin[\Delta m(\tau-\theta)]], \tag{13}$$

$$B=\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)}\exp\Big(-\frac{\tau}{\tau_1}-\frac{\theta}{2}\Big(\frac{1}{\tau_1}+\frac{1}{\tau_2}\Big)\Big) \times[(\operatorname{Im}(f_{12}^{(p)}+f_{12}^{(q)})-2\delta\operatorname{Re}(f_{12}^{(p)}+f_{12}^{(q)}))\cos(\Delta m\theta) + (\operatorname{Re}(f_{12}^{(p)}+f_{12}^{(q)})-2\delta\operatorname{Im}(f_{12}^{(q)}+f_{12}^{(p)}))\sin(\Delta m\theta)]. \tag{14}$$

Analogously, for the antisymmetrical wave function (odd orbital angular momentum), the calculations yield

$$w\begin{pmatrix} K_1(p)K^0(q)\tau\theta \\ K_1(p)\overline{K}^0(q)\tau\theta \end{pmatrix} = \frac{1}{4}\exp\Big[-\frac{1}{2}N(\sigma_1(p)+\sigma_2(p))x_1 - \frac{1}{2}N(\sigma_1(q)+\sigma_2(q))x_2\Big]\Big\{\Big[1\mp\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)}(\operatorname{Im}f_{12}^{(q)} - 2\delta\operatorname{Re}f_{12}^{(q)})\Big]\exp\Big(-\frac{\tau}{\tau_1}-\frac{\theta}{\tau_2}\Big)\pm C\pm D\Big\}, \tag{15}$$

where

$$C=\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)}\exp\Big(-\frac{\tau+\theta}{2}\Big(\frac{1}{\tau_1}+\frac{1}{\tau_2}\Big)\Big) \times[(\operatorname{Im}f_{12}^{(p)}-2\delta\operatorname{Re}f_{12}^{(p)})\cos[\Delta m(\tau-\theta)] + \sin[\Delta m(\tau-\theta)](\operatorname{Re}f_{12}^{(p)}-2\delta\operatorname{Im}f_{12}^{(p)})], \tag{16}$$

$$D=\frac{8\pi N\tau_1\hbar}{m(1+4\delta^2)}\exp\Big(-\frac{\tau}{\tau_1}+\frac{\theta}{2}\Big(\frac{1}{\tau_1}+\frac{1}{\tau_2}\Big)\Big) \times[(\operatorname{Im}(f_{12}^{(p)}-f_{12}^{(q)})-2\delta\operatorname{Re}(f_{12}^{(p)}-f_{12}^{(q)}))\cos(\Delta m\theta) - (\operatorname{Re}(f_{12}^{(p)}-f_{12}^{(q)})-2\delta\operatorname{Im}(f_{12}^{(p)}-f_{12}^{(q)})\sin(\Delta m\theta)]. \tag{17}$$

Obviously, if the total momentum of the pair is equal to zero we get $D = 0$.

The expressions for

$$w\begin{pmatrix} K_2(p)K^0(q)\tau\theta \\ K_2(p)\bar{K}^0(q)\tau\theta \end{pmatrix}$$

can be readily obtained from (12)—(17) by respectively replacing in all formulas the double signs ± and ∓ by ∓ and ±, replacing τ and θ in the exponentials by θ and τ, replacing Δm by $-\Delta m$ in the trigonometric functions of (13) and (16), and replacing Δm by $-\Delta m$ and θ by τ in the trigonometric functions of (14) and (17).

It is easy to see that the sum

$$w(K_1(p)K^0(q)\tau\theta) + w(K_1(p)\bar{K}^0(q)\tau\theta)$$

does not contain any "beats" that are linear in R, in full agreement with Sec. 4 of this paper (the expression $w(K^0K^0\tau\theta) + w(K^0\bar{K}^0\tau\theta)$ does contain such "beats," see Sec. 3).

Thus, for many experiments with $K^0\bar{K}^0$ pairs in a medium (Secs. 3 and 5), the previously known formulas [2,3] are valid only accurate to terms of order $|R|$, and these terms produce "beats," which depend on the sign of the mass difference of K_1 and K_2, even for states with definite orbital angular momenta. For other experiments, in which the states K_1 and K_2 are fixed, the corrections connected with the account of the medium are of the order of magnitude $|R|^2$ (Sec. 4). The parameter $|R|$ can be represented in the form

$$|R| = 9 \cdot 10^{10} |f_{12}| \rho / A, \qquad (18)$$

where ρ—density of the substance, A—its atomic weight (the quantity δ in (3) was set equal to 1.5). We see therefore that inasmuch as $|f_{12}|$ 10^{-12} cm and $\rho/A \sim 10^{-1}$ for the majority of dense media [6], we have $|R| \sim 10^{-2}$. The net total of the correction terms that are linear in $|R|$ (Secs. 3 and 5) can amount to 10% of the fundamental terms pertaining to the free pair, and generally speaking cannot be neglected.

As regards $|R|^2$, this quantity does not exceed 10^{-4}—10^{-3}. Consequently, accurate to several tenths of 1%, we can assume that when the orbital angular momentum of the pair is even, the pair cannot decay into K_1 and K_2, and for odd orbital angular momentum decay to K_1K_1 and K_2K_2 is impossible.

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